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# The Measurability and Vacuum Energy

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**Abstract**

As at the present time the cosmological vacuum energy is the principal candidate for the Dark Energy, its studies are of current importance in theoretical physics. In this paper the expected value of the density of this energy is studied within the **measurability** concept introduced in the earlier works by the author. It is shown that the **measurable** form of General Relativity at low energies $E \ll E_{\ell}$, without additional restrictions, gives the dynamic vacuum energy density, slowly varying in time and very close to this quantity in continuous space-time. The interesting problems associated with the proposed approaches and with the obtained results are formulated.

**Subject Classification:** 04.20.-q,95.36.+x

**Keywords:**gravity,measurability,vacuum energy

## 1 Introduction

In the last 20 years estimation of the expected value of the vacuum energy density $<\rho_{vac}>$ has acquired priority as a problem of fundamental physics due to the fact that the vacuum energy is the principal candidate for the Dark Energy. On the other hand, it is well known that $<\rho_{vac}>\propto \lambda$, where $\lambda$ is the cosmological term in Einstein Equations (for example, review [1]). So, estimation of the expected value of the vacuum energy density $<\rho_{vac}>$ (or the same $\lambda$) becomes an important component in solution of the Dark Energy Problem.

This paper is devoted to derivation of a real estimate of the vacuum energy density $< \rho_{vac} >$ (and hence to a study of the Dark Energy Problem)in terms of the **measurability** concept, put forward in the previous works of the author [2]–[9].
Section 2 briefly presents all the necessary preliminary information. In Section 3 the author considers different approaches to solution of the target problem of this work.
In subsection 3.1 different variants of a dynamical model for $\lambda = \lambda(t)$ are considered. It should be noted that in canonical General Relativity (GR) [10] we have $\lambda = const$ due to the Bianchi identity. In subsection 3.1 it is stated that in this case, in principle, we can take the dynamical model for $\lambda$ but with the additional assumptions. At the same time, these additional assumptions are unnecessary when GR is considered as a low-energy limit of some, more general, theory. Then $\lambda$ naturally becomes the dynamical quantity $\lambda = \lambda(t)$ slowly varying in the present time.
Further (subsection 3.2) it is shown that in the **measurable** form of GR the dynamical cosmological term $\lambda(t)$, slowly varying at low energies $E \ll E_p$, arises naturally to give in the limit $\lambda = const$ in canonical GR [10]. Because of this, the representation of GR in the **measurable** format offers new possibilities for solution of the Dark Energy Problem within the scope of the approaches associated with the vacuum energy.

# 2 Preliminary Information

In this Section we briefly consider some of the results from [2]–[7] which are essential for subsequent studies. Without detriment to further consideration, in the initial definitions we lift some unnecessary restrictions and make important specifications.
Presently, many researchers are of the opinion that at very high energies (Plank's or trans-Planck's) the ultraviolet cutoff exists that is determined by some maximal momentum.
Therefore, it is further assumed that there is a maximal bound for the measurement momenta $p = p_{max}$ represented as follows:

$$p_{max} \doteq p_\ell = \hbar/\ell, \tag{1}$$

where $\ell$ is some small length and $\tau = \ell/c$ is the corresponding time. Let us call $\ell$ the *primary* length and $\tau$ the *primary* time.
Without loss of generality, we can consider $\ell$ and $\tau$ at Plank's level, i.e. $\ell \propto l_p, \tau = \kappa t_p$, where the numerical constant $\kappa$ is on the order of 1. Consequently, we have $E_\ell \propto E_p$ with the corresponding proportionality factor, where $E_\ell \doteq p_\ell c$.

**Explanation.** In the theory under study it is not assumed from the start that there exists some minimal length $l_{min}$ and that $\ell$ is such. In fact, the minimal length is defined with the use of Heisenberg's Uncertainty Principle (HUP) $\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar$ or of its generalization to high (Planck) energies – Generalized Uncertainty Principle (GUP) [11]–[19], for example, of the form [11]

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' l_p^2 \frac{\Delta p}{\hbar}, \tag{2}$$

where $\alpha'$ is a constant on the order of 1. Evidently this formula (2) initially leads to the minimal length $\widetilde{\ell}$ on the order of the Planck length $\widetilde{\ell} \doteq 2\sqrt{\alpha'} l_p$. Besides, other forms of GUP [19] also lead to the minimal length. Thus, we should note that in all the works $l_{min}$ is actually (but not explicitly) introduced on the basis of some measuring procedure (different forms of the Generalized Uncertainty Principle (GUP)). In any form GUP in turn is a high-energy generalization of HUP. But in the original proof of HUP a planar geometry of the initial space-time was actively used [20]. Extension of this principle to other pairs of conjugate variables is also valid only for quantum mechanics in the planar geometry space [21]. As HUP is a local principle, at low energies in the curved space-time, by virtue of Einstein's Equivalence Principle, we can consider that in a fairly small neighborhood of any point the geometry is planar an hence HUP is valid too. But all the results obtained point to the fact that $l_{min}$ should be at a level of $l_p$,i.e. $l_{min} \propto l_p$, or even should be smaller. However, as showed in the papers..., at the Planck scales Einstein's Equivalence Principle (EEP) is obviously inapplicable, and there is no way to use the measuring procedure ignoring the space geometry at these scales. Meantime, none of the GUP forms [19] makes an effort to include it and is hardly completely correct. Moreover, there are some serious arguments against GUP as demonstrated in Section IX of the review paper[19]. The foregoing considerations support argumentation against the introduction of $l_{min}$ from the start.

Because of this, in the present work the validity of this principle is not implied from the start too. GUP is given merely as an example. As $p_{max}$ (1) is taken at Planck's level, it is clear that HUP is inapplicable. Taking this into consideration, the existence of a certain minimal length $\widetilde{\ell}$ is not mandatory. So, we start from the *primary* length $\ell$ and the *primary* time $\tau$. The whole formalism, developed in [2]–[7] on condition that $\ell$ is the minimal length, is valid for the case when $\ell$ is the *primary* length but now we can lift the formal requirement for involvement of $l_{min}$ in the theory from the start.

There is one more barrier for the use of $l_{min}$ in the theory as indicated in [18] and other works (for example, [19]). In the above-mentioned papers, it has been noted that there is a nonzero minimal uncertainty in position, i.e. $l_{min}$ implies that there is no physical state which is a position eigenstate since an

eigenstate would, of course, have zero uncertainty in position. So, in this case in a quantum theory we have the momentum representation rather than the position representation, and the quantum theory becomes very depleted.
The question arises whether the introduction of $p_{max}$ is naturally associated with the involvement of a minimal length. But this is the case only when at the energies $E_{max}$ corresponding to $p_{max}$ we have the substantiated measuring procedure. Unfortunately, this is not the case.
Note that in the canonical QFT in continuous space-time (i.e. without $l_{min}$)[22] –[24] measurements of the contributions in the loop amplitudes involve the standard cut-off procedure for some large (maximal) momentum $p_{cut} \doteq p_{max}$. Then it is demonstrated that the theory at low energies $p \ll p_{cut}$ is in fact independent of the selection of $p_{cut} \doteq p_{max}$. Of course, the theory still remains to be continuous [22] –[24]. In this case we make another step forward, relating the corresponding length $\ell = \hbar/p_{max}$ to $p_{max}$ and constructing on its basis a low-energy theory very close to the initial continuous theory. Now we have the naturally derived parameter $\ell$ for the construction of a high-energy deformation of this theory at the energies $E \approx E_{max}$ within the scope of determining the physical theory deformation [**?**].So, we start from the *primary* length $\ell$ and the *primary* time $\tau$. The whole formalism, developed in [2]–[7] on condition that $\ell$ is the minimal length, is valid for the case when $\ell$ is the *primary* length but now we can lift the formal requirement for involvement of $l_{min}$ in the theory from the start.
Based on $p_{max}$ and $\ell$ in [8], [9], the notions of **primary measurability** and **generalized measurability** have been introduced and studied. Then, in terms of them, the author has constructed in the general form the quantum theory and gravity which at low energies $E \ll E_p$ are close to the corresponding canonical theories in continuous space-time.
In what follows we mainly make references to [7]–[9]. In particular, the basic definitions **Primary Measurability**,**Generalized Measurability**,**Primarily Measurable Quantities**(**PMQ**),**Primarily Measurable Momenta**(**PMM**), **Generalized Measurable Quantities**(**GMQ**) and the like are given in Section II of [7].
The canonical quantum field theory (QFT) [22]– [24] is a local theory *at all energies scales* considered in continuous space-time with a plane geometry, i.e with the Minkowskian metric $\eta_{\mu\nu}(\overline{x})$. But, as it has been already noted in this work and indicated by the author in [8],[9], the space-time metric can not be Minkowskian *at all energies scales* due to the great metric quantum fluctuations arising at the Planck scale $E \approx E_p$ and due to replacement of the known space-time geometry by space-time (quantum) foam [25]–[28].
The Planck scales $E \approx E_p$ present a natural bound for the applicability of EEP. However, it has been noted in [8], [9] that this bound in the general case is not the upper bound. In fact, this bound should satisfy the condition

$E \ll E_p$. As this takes place, the energy scales $E \ll E_p$ are understood as the energies in the interval $0 < E \leq 10^{-2}E_p$.
It is clear that we can suggest relativistic invariance of the theory only within the above-mentioned energy scales.

# 3 Vacuum Energy and Measurability

## 3.1 Dynamical Cosmological Term

The dark matter problem [29], along with the dark energy problem [30], is presently the basic problem in modern fundamental physics, astrophysics, and cosmology. Whereas in the first case real hypotheses have been accepted already [29], the dark energy still remains enigmatic [31]–[34]. But it is the opinion of most researchers that dark energy represents the energy of the cosmic vacuum, its density being associated with the cosmological term $\lambda$ in Einstein's equation [1]. In this respect an important reservation must be made–the point is that most common is the term cosmological constant. Actually, due to the Bianchi identities [10]

$$\nabla_\mu G^\mu{}_\nu = 0, \tag{3}$$

where $G^\mu{}_\nu$ – Einstein equations

$$G^\mu{}_\nu \equiv R^\mu{}_\nu - \frac{1}{2}\,\delta^\mu{}_\nu R = \frac{8\pi G}{c^4}\left[T^\mu{}_\nu + \frac{c^4\lambda}{8\pi G}\,\delta^\mu{}_\nu\right], \tag{4}$$

the energy-momentum tensor $T^\mu{}_\nu$ (energy-momentum density tensor) remains covariantly valid

$$\nabla_\mu T^\mu{}_\nu = 0. \tag{5}$$

From whence it directly follows that the cosmological term $\lambda$ is a constant.
But, as has been rightly noted in several publications (e.g., [35]–[37]), conservation laws (5) are only regulating the energy-momentum exchange between the field sources and gravitational field and are liable to be violated if an independent energy source is existent in the Universe. Such a source may be associated with a time-varying cosmological term. So, in this case it is reasonable to consider $\lambda = \lambda(t)$.
Then the Bianchi identity (3) is replaced by the "generalized Bianchi identity" [36]

$$\nabla_\mu \left[T^\mu{}_\nu + \lambda^\mu{}_\nu\right] = 0, \tag{6}$$

where $\lambda^{\mu}{}_{\nu} = \varepsilon_{\lambda}\delta^{\mu}{}_{\nu}$ is some energy-momentum tensor (referred to as the dark energy-momentum tensor [36]) related to the cosmological term, where

$$\varepsilon_{\lambda} = \frac{c^4\lambda}{8\pi G} \tag{7}$$

is the corresponding energy density.
We are not interested in a diversity of the cosmological models involving the dynamic cosmological term $\lambda = \lambda(t)$ and different relationships between $\lambda$ and $t$. Rather detailed references to such models have been made in the thesis work [38]. Of interest is the problem: how can we obtain the observable value of $\lambda = \lambda_{observe}$ knowing that $\lambda$ is dependent on $t$.
Note that the vacuum energy density $< \rho_{vac} >$ in [1] was evaluated with the use of a simple theoretical quantum field model. According to the modern estimate, a value of $< \rho_{vac} >$ derived directly from QFT (in the absence of supersymmetry) is higher than the experimental value by a factor of $10^{122}$. Specifically, this estimate was derived by simple procedures in [39]:

$$< \rho_{vac} >= 2\int_0^{p_{max}} dp\, \frac{4\pi p^2}{(2\pi\hbar)^3}\, \frac{\hbar\omega}{2}, \tag{8}$$

where $p_{max} \sim P_{pl}$ is the momentum cut-off at Planck scales, as both in [39] and in [1] it is assumed that General Relativity is valid right up to the Planck scales. Proceeding from $p = (\hbar\omega/c)$, we can obtain [39]

$$< \rho_{vac} >= \frac{\hbar\Omega^4}{8\pi^2 c^3}. \tag{9}$$

And then $\Omega$ is given by $\Omega_p$ , where

$$\hbar\Omega_p = E_p = \left(\frac{\hbar c^5}{G}\right)^{1/2}. \tag{10}$$

Then $< \rho_{vac} >$ is higher than the observable value $< \rho_{vac,observe} >$ approximately by a factor of $10^{122}$ [39].
Yet, taking the cosmological term as a dynamic quantity, we can write (8) as

$$< \rho_{vac}(t) >= 2\int_0^{p(t)} dp\, \frac{4\pi p^2}{(2\pi\hbar)^3}\, \frac{\hbar\omega}{2}, \tag{11}$$

that in the very early Universe, at the time close to the Planck's time $t_p$, turns to (8) as follows:

$$< \rho_{vac}(t_p) >= 2\int_0^{P_{pl}} dp\, \frac{4\pi p^2}{(2\pi\hbar)^3}\, \frac{\hbar\omega}{2}. \tag{12}$$

However, it is known that the experimental value $< \rho_{vac,exp} >$ is sooner determined by low momenta and energies (corresponding to large scales) and also by the infrared limit that is given by a radius of the transparent part of the Universe $R_{Univ} \approx 10^{28} cm$. (Here $< \rho_{vac,exp} > \propto \lambda_{observe}$). For such momenta by the uncertainty principle (8)we have

$$< \rho_{vac}(t_{Univ}) >= 2 \int_0^{p_{min}} dp \, \frac{4\pi p^2}{(2\pi\hbar)^3} \, \frac{\hbar\omega}{2}, \tag{13}$$

where $t_{Univ}$–Universe life-time, $p_{min}$ –"minimal" momentum.
As $R_{Univ}/l_p = P_{pl}/p_{min} = 10^{61}$, then by analogy (14)and in the infrared limit we have

$$< \rho_{vac}(t_{Univ}) >= \frac{\hbar\Omega^4_{min}}{8\pi^2 c^3}, \tag{14}$$

where

$$\hbar\Omega_{min} = E_{min}. \tag{15}$$

But in this case $\rho_{vac}(t_{Univ})$ is lower than $\rho_{vac}$ calculated from(8) by a factor of $10^{244}$ rather than by a factor of $10^{122}$.
This result is easily obtained using the Uncertainty Principle for the pair of conjugate variables $(\lambda, V)$,[40]–[43]:

$$\Delta V \approx \frac{\hbar}{\Delta\lambda}, \tag{16}$$

where $\lambda \doteq \lambda(t)$ is the dynamical cosmological term, $V$ is the space-time volume. And $V$ results from the Einstein-Hilbert action $S_{EH}$ [41]:

$$\lambda \int d^4x \sqrt{-g} = \lambda V, \tag{17}$$

where (17) is the term in $S_{EH}$.
In [44],[45] it is shown that, if GUP is valid, then the Uncertainty Principle (16) may be generalized at Planck scales up to the Generalized Uncertainty Principle

$$\Delta V \approx \frac{\hbar}{\Delta\lambda} + \alpha' t_p^2 \overline{V}^2 \frac{\Delta\lambda}{\hbar}, \tag{18}$$

where $\overline{V}$ - spatial part $V$ that, as is assumed, may be extracted explicitly.
(18) is of interest from the viewpoint of two limits [44],[45]:
1)IR - limit: $t \to \infty$
2)UV - limit: $t \to t_{min}$.
In the case of IR-limit we have large volumes $\overline{V}$ and $V$ at low $\Delta\lambda$. Because of this, the main contribution in the right-hand side of (18) is made by the first term, as great $\overline{V}$ in the second term is damped by small $t_p$ and $\Delta\lambda$. Thus, we can derive

$$\lim_{t\to\infty} \Delta V \approx \frac{\hbar}{\Delta\lambda} \tag{19}$$

in accordance with (16), and $\lambda$ is a dynamic value fluctuating around zero.
And for case 2) $\Delta\lambda$ becomes significant

$$\lim_{t\rightarrow t_{min}}\overline{V}=\overline{V}_{min}\sim\overline{V}_p=l_p^3;\lim_{t\rightarrow t_{min}}V=V_{min}\sim V_p=l_p^3t_p. \tag{20}$$

As a result, we have

$$\lim_{t\rightarrow t_{min}}\Delta V=\frac{\hbar}{\Delta\lambda}+\alpha_\lambda V_p^2\frac{\Delta\lambda}{\hbar}, \tag{21}$$

where the parameter $\alpha_\lambda$ absorbs all the above-mentioned proportionality coefficients.
For (21) $\Delta\lambda\sim\lambda_p\equiv\hbar/V_p=E_p/\overline{V}_p$.
It is easily seen that in this case $\lambda\sim m_p^4$, in agreement with the value obtained using a standard (i.e. without super-symmetry and the like) quantum field theory (formulae (8),(12)). Despite the fact that $\lambda$ at Planck's scales (referred to as $\lambda_{UV}$) (21) is also a dynamic quantity, it is not directly related to well-known $\lambda$ (16),(19) (called $\lambda_{IR}$) because the latter, as opposed to the first one, is derived from Einstein's equations

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\lambda g_{\mu\nu}=8\pi GT_{\mu\nu}, \tag{22}$$

were $c=1$.
However, Einstein's equations (22) are not valid at the Planck scales and hence $\lambda_{UV}$ may be considered as some high-energy generalization of the conventional cosmological constant, leading to $\lambda_{IR}$ in the low-energy limit.
As $V\sim tl^3\sim l^4$, where $l$ - characteristic linear dimension $V$, it is directly inferred that in the infrared region $\lambda_{IR}$ calculated for $l=R_{Univ}$ is lower than $\lambda_{UV}$ in $\sim R_{Univ}^4/l_p^4=10^{244}$ – in a perfect agreement with the result obtained by the direct calculations at the beginning of this Section.
In other words, a quantum field theory with UV-cutoff in the assumption that $\lambda$ is a dynamic quantity gives no correct values for $\lambda_{observ}$ as well.
In this way, similar to formula in [46], we obtain

$$\lambda_{VE}=\sqrt{\lambda_{UV}\lambda_{IR}}, \tag{23}$$

where $\lambda_{VE}=\lambda_{observe}$ – cosmological term corresponding to the observable value.
Let us write $<\rho_{vac,observe}>$ as

$$<\rho_{vac,observe}>=2\int_0^{p_{obs}}dp\,\frac{4\pi p^2}{(2\pi\hbar)^3}\,\frac{\hbar\omega}{2}, \tag{24}$$

where $p_{obs}$ is the some momentum value corresponding available $<\rho_{vac,observe}>$.
It should be noted that the Holographic Principle [47] –[49] is helpful during the

solution of the cosmological term problem. Let us begin with the Schwarzschild black holes, whose semiclassical entropy is given by

$$S = \pi R_{Sch}^2/l_p^2 = \pi R_{Sch}^2 m_p^2, \tag{25}$$

where $R_{Sch}$ is the adequate Schwarzschild radius. Then, as it has been pointed out in [50], in case the Fischler - Susskind cosmic holographic conjecture [51] is valid, the entropy of the Universe is limited by its "surface" measured in Planck units [50]:

$$S \leq \frac{A}{4} m_p^2, \tag{26}$$

where the surface area $A = 4\pi R^2$ is defined in terms of the apparent (Hubble) horizon

$$R = \frac{1}{\sqrt{H^2 + k/a^2}} \tag{27}$$

with the curvature $k$ and scale $a$ factors.
Using further the reasoning line of [50] based on the results of the holographic thermodynamics, we can relate the entropy and energy of a holographic system [52]. Similarly, in terms of the $\alpha$ parameter one can easily estimate the upper limit for the energy density of the Universe (denoted here by $\rho_{hol}$):

$$< \rho_{hol} > \leq \frac{3}{8\pi R^2} m_p^2 \tag{28}$$

that is drastically differing from the one obtained with well-known QFT $< \rho_{vac,UV} >$ from formula (12)

$$< \rho_{vac,UV} > \sim m_p^4. \tag{29}$$

Since $m_p \sim 1/l_p$, the right-hand side of (28) is actually nothing else but the energy density in **Holographic Dark Energy Models** [53]–[56].
In fact, the upper limit of the right-hand side of (28) is attainable, as it has been indicated in [50]. The "overestimation" value of $r$ for the energy density $< \rho_{vac,UV} >$, compared to $< \rho_{hol} >$, may be determined as

$$r = \frac{< \rho_{vac,UV} >}{< \rho_{hol} >} = \frac{8\pi}{3} \frac{R_{Univ}^2}{l_p^2} = \frac{8\pi}{3} \frac{S}{S_p}, \tag{30}$$

where $S_p$ is the entropy of the Plank mass and length for the Schwarzschild black hole, $R_{Univ}$, is the radius of the transparent part of the Universe $\approx 10^{28} cm$, $S$ is the entropy of the corresponding Schwarzschild black hole. We can easily calculate (e.g., see [50])

$$r = 5.44 \times 10^{122}. \tag{31}$$

in a good agreement with the astrophysical data.

## 3.2 Gravity in Measurable Picture and Vacuum Energy Density

Let us consider the Vacuum Energy Problem according to the approach with gravity in the **measurable** form [7]. Then, as noted in Section 5 of [7] **measurable** form of GR ($\mathcal{EEM}[\{N\}]$) is the deformation of canonical GR, and the well-known Einstein Equations ($\mathcal{EE}$) (22) in GR arise on passage to the limit

$$\lim_{|\{N\}|\rightarrow\infty}\mathcal{EEM}[\{N\}]=\mathcal{EE}. \tag{32}$$

It is clear that in such consideration the cosmological term $\lambda$ from the start is dependent on the available energy $E$. Besides, this term is dynamic $\lambda=\lambda(t)$ because the energy is always dependent on time to a greater or lesser extent, i.e. $E\doteq E(t)$ at low energies $E\ll E_p$ slowly varying in time $\lambda=\lambda(t)$. Therefore, it can be considered very close to the cosmological constant $\lambda$ in canonical GR. In this case, for sufficiently high $|\{N\}|(|\{N\}|\gg 1)$, $\mathcal{EEM}[\{N\}]$ is very close to $\mathcal{EE}$ and to the Bianchi identities (3), whereas the covariant conservation of the energy-momentum density tensor (formula (5)), to a high accuracy, is the case in $\mathcal{EEM}[\{N\}]$ as well.
Let us revert to Section 3.1. As follows from this Section, when we can derive a correct value for the cosmological term $\lambda$ from QFT within the energy range $E\ll E_\ell$ (or same $E\ll E_p$). But, as noted in [7], in this case the **measurable** form of any theory and, in particular, of both QFT and GR, is close to the canonical form in continuous space-time.
Then, the quantity $<\rho_{vac,observe}>$ from formula (24) is replaced by the quantity

$$<\rho_{vac,observe}>_{meas}\sim\sum_{N=N^*}^{N_*}\frac{4\pi p_N^2 p_{N(N-1)}}{(2\pi)^3}\frac{1}{2}\sqrt{p_N^2+m^2}, \tag{33}$$

were $N^*\gg N_*\gg 1$.
Due to the fact that a lower limit of the integral from (24) equals zero, to obtain an exact **measurable** analog of $<\rho_{vac,observe}>$, in formula (33) it is assumed that $N^*=\infty$. However, in [3], for real physical systems, in quantum consideration we always have $N^*<\infty$ that is associated with selection of some minimal momentum $p_{min}=p_{N^*}\neq 0$. Then in the continuous form $<\rho_{vac,observe}>$ from (24) is replaced by

$$<\rho_{\widetilde{vac,observe}}>\sim\int_{p_{min}}^{p_{obs}}\frac{4\pi k^2 dk}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2}. \tag{34}$$

Obviously, for large but finite $N^*$, the integral in the right-hand side of formula (34) is little different from the corresponding quantity in (24). And in virtue of Section 2 and results of [7], this integral is insignificantly different from the

sum in the right-hand side of formula (33).
As noted in [3], in a **measurable** consideration the wave function and the contributions for all the computational quantities in QFT, from the start, should be subdivided into two main classes:
1) case when the **measurable** form of a theory is very close to its initial (continuous) variant in accordance with the energies $E \ll E_\ell$ ($E \ll E_p$). In this case the **measurable** variant of this theory is considered to be a nearly continuous theory for $|N_{x_\mu}| \gg 1$. And
2) high-energy case when $E \approx E_\ell$ or ($E \approx E_p$). Then in a **measurable** variant the theory becomes actually discrete in accordance with $|N_{x_\mu}| \approx 1$.
In [3] and in [8], [9] it has been substantiated that in the process of calculations within a **measurable** variant of QFT the contributions associated with classes 1) and 2) *cannot* be considered within one and the same sum. Moreover, case 2) we can consider as a high-energy deformation of case 1).
In so doing it is convenient to use the dimensionless quantity $\alpha_l$ as a deformation parameter at the energies $E_l$ determined by the scale of the length $l$ [57],[58]:

$$\alpha_l \doteq \ell^2/l^2. \qquad (35)$$

Clearly, at low energies $E \ll E_\ell$ ($E \ll E_p$) the values of $\alpha_l$ fall within the interval

$$0 < \alpha_l \ll 1. \qquad (36)$$

According to Definition in [9], at low energies $E \ll E_\ell$ (same $E \ll E_p$) all the *observable quantities* are **PMQ** and hence formula (36) may be refined as follows:

$$\alpha_l = \frac{1}{N_l^2} \ll 1, N_l \gg 1, \qquad (37)$$

where we have $l = N_l \ell$ and $N_l$ is an integer number.
It should be noted that the parameter $\alpha_l$ was used in the author's works [59],[60] devoted to the Dark Energy Problem and high-energy (Planck) gravity deformation, though beyond the **measurability** concept.
As demonstrated in [59],[60], in terms of the deformation parameter $\alpha_l$ the basic formulae from the first part of this section look quite natural and simple. Specifically, formula (26) in this case takes the form

$$S \leq \pi \alpha_R^{-1}, \qquad (38)$$

where $\alpha_R = \ell^2/R^2$ for $\ell = l_p$.
In a similar way, for this value of the *primary* length $\ell = l_p$, in terms of $\alpha_l$

we can express some other important quantities used in studies of the Dark Energy Problem. For example, formula (30) is written as

$$r = \frac{< \rho_{vac,UV} >}{< \rho_{hol} >} = \frac{8\pi}{3}\alpha_R^{-1}. \qquad (39)$$

The only difference between definitions of $\alpha_l$ in [59],[60] and in the present work resides in the fact that in the first case $\ell$ in formula (35) is equal to the minimal length $\ell = l_{min}$, whereas in the second case – to the *primary* length $\ell$ that, according to Section 2, is not *necessarily* $l_{min}$. At the same time, this difference is not very important. At low energies $E \ll E_p$ in both cases $\alpha_l$ satisfies the condition of (36). And only at high energies $E \approx E_p$ $(E \approx E_\ell)$ in the first case $\alpha_l$ is at maximum $\alpha_l = 1$, whereas in the second case it is not improbable that $\alpha_l > 1$.
In any consideration,on going from high energies (HE) $E \approx E_\ell$ to low energies (LE) $E \ll E_\ell$, the deformation parameter $\alpha_l$ should be transformed correspondingly:

$$\alpha_l(HE) \stackrel{(E\approx E_\ell)\rightarrow(E\ll E_\ell)}{\Rightarrow} \alpha_l(LE). \qquad (40)$$

Note that in the **measurable** form we exactly know the expression for $\alpha_l$ at low energies $\alpha_l = \alpha_l(LE)$ that is given by formula (37). At high energies in the case of $\alpha_l = \alpha_l(HE)$ we can only assume the general form for $l$ (for example formula (113) in [7]).
Similarly, for the dynamic cosmological term $\lambda = \lambda(t)$ we have

$$\lambda(HE) \stackrel{(E\approx E_\ell)\rightarrow(E\ll E_\ell)}{\Rightarrow} \lambda(LE) \approx \lambda_{observe}, \qquad (41)$$

or same

$$\lambda(\alpha_l) \stackrel{(l\approx \ell)\rightarrow(l\ll \ell)}{\Rightarrow} \lambda(\alpha_l) \approx \lambda_{observe}, \qquad (42)$$

where $\lambda(HE)$ corresponds to the value derived from formula (8).
As, for $l \ll \ell$, it is obvious that $\alpha_l$ from formula (36) is a small parameter, we can perform a series expansion of the quantity $\lambda(\alpha_l)$ in terms of this parameter. In [59] it is shown that, for the general case represented by formulae ((8)–(15)) (disregarding the holographic principle), this series expansion is of the following form:

$$\lambda(\alpha_l) = \alpha_l^2 \lambda(\alpha_\ell) + .... \qquad (43)$$

And in the case when the holographic principle is valid (formulae (26)–(31)), we have

$$\lambda(\alpha_l) = \alpha_l \lambda(\alpha_\ell) + ..., \qquad (44)$$

where dots denote the terms with a higher order of smallness, whereas $\lambda(\alpha_\ell) \doteq \lambda(HE)$ denotes a maximal value of the cosmological term $\lambda$ corresponding to

some initial time $t = t_0$.
It is clear that just formula (44) for $l = R_{Univ}$ provides the correct value of $\lambda$ that is equal to $\lambda_{observ} = \lambda_{DE}$.
Despite the fact that all the results in [59] have been obtained within the well-known QFT in continuous space-time, to a high accuracy, they are valid in the suggested variant of **measurable** (discrete) consideration as all calculations are performed at low energies $E \ll E_p$ when the **measurable** form of the theory is very close to the canonical form in continuous space-time.

# 4 Conclusion

**4.1** The measurable form of GR at low energies $E \ll E_\ell$, without additional restrictions, gives the value for the dynamical vacuum energy density $< \rho_{vac}(t) >$, slowly varying in time and very close to the constant $\lambda = const$ for GR in continuous space-time. Besides, in this case for some models (specifically, those when the Holographic Principle is valid) $< \rho_{vac}(t) >$ takes on a value that is practically coincident with the experimental one.

**4.2** It should be noted that the results obtained for the high-energy **measurable** deformation of GR in Section 7 of [7]), offer considerable possibilities (meanwhile hypothetical) of effective studying of the cosmological term at high energy $E \approx E_\ell$ (same $E \approx E_p$).

**4.3** Let us return to formula (24) or to similar formulae (33),(34). This enables us to formulate the inverse problem: finding of the value for the (energy) momenta $p_{obs}, (E_{obs})$ , for which the left side of (24) contains the modern experimental value of $< \rho_{vac,observe} >$ or same $\lambda_{observe}$.
With the introduction of the quantity

$$R_{obs} \doteq \frac{\hbar}{p_{obs}}, \tag{45}$$

the problem arises: to which extent $R_{obs}$ is related to the Einstein Equivalence Principle boundary applicability [8],[9]?

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