**Advanced Studies in Theoretical Physics**
**Vol. 15, 2021, no. 4, 201 - 234**
**HIKARI Ltd, www.m-hikari.com**
**https://doi.org/10.12988/astp.2021.91549**

# Property of Tensor Satisfying Binary Law 3

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**Abstract**

I have already reported "Property of Tensor Satisfying Binary Law 2". This article is the article that I revise contents of "Property of Tensor Satisfying Binary Law 2", and increased the report about new characteristics. We may arrive at the deeper understanding in this about Property of Tensor Satisfying Binary Law.

**Keywords:** Tensor, Covariant Derivative

# 1 Introduction

I have already reported "Property of Tensor Satisfying Binary Law 2".[2] This article is the article that I revise contents of "Property of Tensor Satisfying Binary Law 2", and increased the report about new characteristics. I show below the part which I added in this article newly. These are Definision7, Definision12,Definision19,Proposition3,Proposition6. The part that revision was most greatly carried out is Chapter 3.

# 2 Definition

**Definition 1** $\overline{x^\mu}\neq x^\mu, \overline{x^\nu}\neq x^\nu, \overline{x^\mu}=x^\nu, \overline{x^\nu}=x^\mu$ *is established.[1]*
*I named* $\overline{x^\mu}\neq x^\mu, \overline{x^\nu}\neq x^\nu, \overline{x^\mu}=x^\nu, \overline{x^\nu}=x^\mu$ *"Binary Law".[1]*

**Definition 2** *If* $\overline{x^\mu}\neq x^\mu, \overline{x^\nu}\neq x^\nu, \overline{x^\mu}=x^\nu, \overline{x^\nu}=x^\mu$ *is established,* $x_\nu = x^\mu$ *is established.[1]*

**Definition 3** *If* $\overline{x^\mu}\neq x^\mu, \overline{x^\nu}\neq x^\nu, \overline{x^\mu}=x^\nu, \overline{x^\nu}=x^\mu$ *is established,* $x_\mu = x^\nu$ *is established.[1]*

**Definition 4** *If* $\overline{x^\mu}\neq x^\mu, \overline{x^\nu}\neq x^\nu, \overline{x^\mu} = x^\nu, \overline{x^\nu} = x^\mu$ *is established,* $x_\nu = -x_\mu$ *is established.[1]*

**Definition 5** *If* $\overline{x^\mu}\neq x^\mu, \overline{x^\nu}\neq x^\nu, \overline{x^\mu} = x^\nu, \overline{x^\nu} = x^\mu$ *is established,* $x^\nu = -x^\mu$ *is established.[1]*

**Definition 6** *If all coordinate systems* $x^\mu, x^\nu, x^\sigma, x^\lambda, \cdots$ *satisfies* $\overline{x^\mu}\neq x^\mu, \overline{x^\nu}\neq x^\nu, \overline{x^\mu} = x^\nu, \overline{x^\nu} = x^\mu$, *all coordinate systems* $x^\mu, x^\nu, x^\sigma, x^\lambda, \cdots$ *shifts to only two of* $x^\mu, x^\nu$*.[1]*

**Definition 7** *I express* $x^\epsilon$ *when all coordinate systems satisfies Binary Law in* $x^{\nu(\epsilon)}$*.*

**Definition 8** $g^\mu_\mu = 1$,$g^\mu_\nu = 0 : (\mu\neq\nu)$ *is establishment.[3]*

**Definition 9** $\frac{\partial^3 x^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} = M$ *is established for* $\frac{\partial^3 x^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}$*.*

**Definition 10** *The first-order covariant derivative of the covariant vector satisfied*
$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_\tau \Gamma^\tau_{\mu\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_\tau \frac{1}{2} g^{\epsilon\tau}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\epsilon}\right)$*.[4]*

**Definition 11** *The first-order covariant derivative of the contravariant vector satisfied*
$x^\mu_{;\nu} = \frac{\partial x^\mu}{\partial x^\nu} + x^\tau \Gamma^\mu_{\tau\nu} = \frac{\partial x^\mu}{\partial x^\nu} + x^\tau \frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)$*.[4]*

**Definition 12** *The second-order covariant derivative of the covariant vector satisfied*

$$
\begin{aligned}
&x_{\mu;\nu;\sigma} = \frac{\partial x_{\mu;\nu}}{\partial x^\sigma} - x_{\iota;\nu}\Gamma^\iota_{\mu\sigma} - x_{\mu;\iota}\Gamma^\iota_{\nu\sigma}\\
&= \frac{\partial}{\partial x^\sigma}\left(\frac{\partial x_\mu}{\partial x^\nu} - x_\tau \Gamma^\tau_{\mu\nu}\right) - \left(\frac{\partial x_\iota}{\partial x^\nu} - x_\tau\Gamma^\tau_{\iota\nu}\right)\Gamma^\iota_{\mu\sigma} - \left(\frac{\partial x_\mu}{\partial x^\iota} - x_\tau\Gamma^\tau_{\mu\iota}\right)\Gamma^\iota_{\nu\sigma}\\
&= \frac{\partial^2 x_\mu}{\partial x^\nu \partial x^\sigma} - \frac{\partial}{\partial x^\sigma}\left(x_\tau \frac{1}{2} g^{\epsilon\tau}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\epsilon}\right)\right)\\
&-\frac{\partial x_\iota}{\partial x^\nu}\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\mu} - \frac{\partial g_{\mu\sigma}}{\partial x^\epsilon}\right)\\
&+x_\tau \frac{1}{2} g^{\epsilon\tau}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\nu}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\mu} - \frac{\partial g_{\mu\sigma}}{\partial x^\epsilon}\right)\\
&-\frac{\partial x_\mu}{\partial x^\iota}\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)\\
&+x_\tau \frac{1}{2} g^{\epsilon\tau}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^\iota} + \frac{\partial g_{\iota\epsilon}}{\partial x^\mu} - \frac{\partial g_{\mu\iota}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right).[4]
\end{aligned}
$$

**Definition 13** *The second-order covariant derivative of the contravariant vector satisfied*

$$
\begin{aligned}
&x^\mu_{;\nu;\sigma} = \frac{\partial x^\mu_{;\nu}}{\partial x^\sigma} + x^\iota_{;\nu}\Gamma^\mu_{\iota\sigma} - x^\mu_{;\iota}\Gamma^\iota_{\nu\sigma}\\
&= \frac{\partial}{\partial x^\sigma}\left(\frac{\partial x^\mu}{\partial x^\nu} + x^\tau \Gamma^\mu_{\tau\nu}\right) + \left(\frac{\partial x^\iota}{\partial x^\nu} + x^\tau\Gamma^\iota_{\tau\nu}\right)\Gamma^\mu_{\iota\sigma} - \left(\frac{\partial x^\mu}{\partial x^\iota} + x^\tau\Gamma^\mu_{\tau\iota}\right)\Gamma^\iota_{\nu\sigma}\\
&= \frac{\partial^2 x^\mu}{\partial x^\nu \partial x^\sigma} + \frac{\partial}{\partial x^\sigma}\left(x^\tau \frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\right)\\
&+\frac{\partial x^\iota}{\partial x^\nu}\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)
\end{aligned}
$$

$$+x^\tau\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu}+\frac{\partial g_{\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)$$
$$-\frac{\partial x^\mu}{\partial x^\iota}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)$$
$$-x^\tau\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\iota}+\frac{\partial g_{\iota\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\iota}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right).$$ *[4]*

**Definition 14** *The third-order covariant derivative of the contravariant vector satisfied*

$$x^\mu_{;\nu;\sigma;\lambda}=\frac{\partial x^\mu_{;\nu;\sigma}}{\partial x^\lambda}+x^\kappa_{;\nu;\sigma}\Gamma^\mu_{\kappa\lambda}-x^\mu_{;\kappa;\sigma}\Gamma^\kappa_{\nu\lambda}-x^\mu_{;\nu;\kappa}\Gamma^\kappa_{\sigma\lambda}$$
$$=\frac{\partial}{\partial x^\lambda}\left\{\frac{\partial}{\partial x^\sigma}\left(\frac{\partial x^\mu}{\partial x^\nu}+x^\tau\Gamma^\mu_{\tau\nu}\right)+\left(\frac{\partial x^\iota}{\partial x^\nu}+x^\tau\Gamma^\iota_{\tau\nu}\right)\Gamma^\mu_{\iota\sigma}-\left(\frac{\partial x^\mu}{\partial x^\iota}+x^\tau\Gamma^\mu_{\tau\iota}\right)\Gamma^\iota_{\nu\sigma}\right\}$$
$$+\left\{\frac{\partial}{\partial x^\sigma}\left(\frac{\partial x^\kappa}{\partial x^\nu}+x^\tau\Gamma^\kappa_{\tau\nu}\right)+\left(\frac{\partial x^\iota}{\partial x^\nu}+x^\tau\Gamma^\iota_{\tau\nu}\right)\Gamma^\kappa_{\iota\sigma}-\left(\frac{\partial x^\kappa}{\partial x^\iota}+x^\tau\Gamma^\kappa_{\tau\iota}\right)\Gamma^\iota_{\nu\sigma}\right\}\Gamma^\mu_{\kappa\lambda}$$
$$-\left\{\frac{\partial}{\partial x^\sigma}\left(\frac{\partial x^\mu}{\partial x^\kappa}+x^\tau\Gamma^\mu_{\tau\kappa}\right)+\left(\frac{\partial x^\iota}{\partial x^\kappa}+x^\tau\Gamma^\iota_{\tau\kappa}\right)\Gamma^\mu_{\iota\sigma}-\left(\frac{\partial x^\mu}{\partial x^\iota}+x^\tau\Gamma^\mu_{\tau\iota}\right)\Gamma^\iota_{\kappa\sigma}\right\}\Gamma^\kappa_{\nu\lambda}$$
$$-\left\{\frac{\partial}{\partial x^\kappa}\left(\frac{\partial x^\mu}{\partial x^\nu}+x^\tau\Gamma^\mu_{\tau\nu}\right)+\left(\frac{\partial x^\iota}{\partial x^\nu}+x^\tau\Gamma^\iota_{\tau\nu}\right)\Gamma^\mu_{\iota\kappa}-\left(\frac{\partial x^\mu}{\partial x^\iota}+x^\tau\Gamma^\mu_{\tau\iota}\right)\Gamma^\iota_{\nu\kappa}\right\}\Gamma^\kappa_{\sigma\lambda}$$

$$=\frac{\partial^3x^\mu}{\partial x^\nu\partial x^\sigma\partial x^\lambda}+\frac{\partial^2}{\partial x^\sigma\partial x^\lambda}\left(x^\tau\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu}+\frac{\partial g_{\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\right)$$
$$+\frac{\partial}{\partial x^\lambda}\left(\frac{\partial x^\iota}{\partial x^\nu}\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\right)$$
$$+\frac{\partial}{\partial x^\lambda}\left(x^\tau\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu}+\frac{\partial g_{\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\right)$$
$$-\frac{\partial}{\partial x^\lambda}\left(\frac{\partial x^\mu}{\partial x^\iota}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)\right)$$
$$-\frac{\partial}{\partial x^\lambda}\left(x^\tau\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\iota}+\frac{\partial g_{\iota\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\iota}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)\right)$$

$$+\frac{\partial^2x^\kappa}{\partial x^\nu\partial x^\sigma}\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa}-\frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right)$$
$$+\frac{\partial}{\partial x^\sigma}\left(x^\tau\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu}+\frac{\partial g_{\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\right)\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa}-\frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right)$$
$$+\frac{\partial x^\iota}{\partial x^\nu}\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa}-\frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right)$$
$$+x^\tau\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu}+\frac{\partial g_{\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa}-\frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right)$$
$$-\frac{\partial x^\kappa}{\partial x^\iota}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa}-\frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right)$$
$$-x^\tau\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\iota}+\frac{\partial g_{\iota\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\iota}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa}-\frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right)$$

$$-\frac{\partial^2x^\mu}{\partial x^\kappa\partial x^\sigma}\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right)$$
$$-\frac{\partial}{\partial x^\sigma}\left(x^\tau\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\kappa}+\frac{\partial g_{\kappa\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\kappa}}{\partial x^\epsilon}\right)\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right)$$
$$-\frac{\partial x^\iota}{\partial x^\kappa}\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right)$$
$$-x^\tau\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\kappa}+\frac{\partial g_{\kappa\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\kappa}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right)$$
$$+\frac{\partial x^\mu}{\partial x^\iota}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\kappa}-\frac{\partial g_{\kappa\sigma}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right)$$
$$+x^\tau\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\iota}+\frac{\partial g_{\iota\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\iota}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\sigma}+\frac{\partial g_{\sigma\epsilon}}{\partial x^\kappa}-\frac{\partial g_{\kappa\sigma}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right)$$

$$-\frac{\partial^2x^\mu}{\partial x^\nu\partial x^\kappa}\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\sigma}-\frac{\partial g_{\sigma\lambda}}{\partial x^\epsilon}\right)$$
$$-\frac{\partial}{\partial x^\kappa}\left(x^\tau\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu}+\frac{\partial g_{\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\sigma}-\frac{\partial g_{\sigma\lambda}}{\partial x^\epsilon}\right)$$
$$-\frac{\partial x^\iota}{\partial x^\nu}\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\kappa}+\frac{\partial g_{\kappa\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\kappa}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\sigma}-\frac{\partial g_{\sigma\lambda}}{\partial x^\epsilon}\right)$$
$$-x^\tau\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu}+\frac{\partial g_{\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\kappa}+\frac{\partial g_{\kappa\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\kappa}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\sigma}-\frac{\partial g_{\sigma\lambda}}{\partial x^\epsilon}\right)$$

$+\frac{\partial x^\mu}{\partial x^\iota}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\kappa}+\frac{\partial g_{\kappa\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\kappa}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\sigma}-\frac{\partial g_{\sigma\lambda}}{\partial x^\epsilon}\right)$
$+x^\tau\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\iota}+\frac{\partial g_{\iota\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\iota}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\kappa}+\frac{\partial g_{\kappa\epsilon}}{\partial x^\nu}-\frac{\partial g_{\nu\kappa}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^\lambda}+\frac{\partial g_{\lambda\epsilon}}{\partial x^\sigma}-\frac{\partial g_{\sigma\lambda}}{\partial x^\epsilon}\right).$

**Definition 15** *When the next conversion equation is established, $x^\mu_\mu$ is components of a tensor of rank zero.* $x^\mu_\mu=\frac{\partial x^\mu}{\partial x^\nu}\frac{\partial x^\nu}{\partial x^\mu}x^\nu_\nu$

**Definition 16** *When the next conversion equation is established, $x^{\mu\nu}$ is contravariant components of a tensor of the second rank.[4]* $x^{\mu\nu}=\frac{\partial x^\mu}{\partial x^\sigma}\frac{\partial x^\nu}{\partial x^\lambda}x^{\sigma\lambda}$

**Definition 17** *When the next conversion equation is established, $x_{\mu\nu}$ is covariant components of a tensor of the second rank.[4]* $x_{\mu\nu}=\frac{\partial x^\sigma}{\partial x^\mu}\frac{\partial x^\lambda}{\partial x^\nu}x_{\sigma\lambda}$

**Definition 18** *When the next conversion equation is established, $x^\mu_\nu$ is components of the mixed tensor of the second rank.[4]* $x^\mu_\nu=\frac{\partial x^\mu}{\partial x^\sigma}\frac{\partial x^\lambda}{\partial x^\nu}x^\sigma_\lambda$

**Definition 19** *When the next conversion equation is established, $x_{\mu\nu\sigma}$ is covariant components of a tensor of the third rank.[4]* $x_{\mu\nu\sigma}=\frac{\partial x^\lambda}{\partial x^\mu}\frac{\partial x^\iota}{\partial x^\nu}\frac{\partial x^\epsilon}{\partial x^\sigma}x_{\lambda\iota\epsilon}$

**Definition 20** *When the next conversion equation is established, $x^\mu_{\nu\sigma}$ is components of the mixed tensor of the third rank of the second rank covariant in the first rank contravariant.[4]* $x^\mu_{\nu\sigma}=\frac{\partial x^\mu}{\partial x^\lambda}\frac{\partial x^\iota}{\partial x^\nu}\frac{\partial x^\epsilon}{\partial x^\sigma}x^\lambda_{\iota\epsilon}$

**Definition 21** *When the next conversion equation is established, $x^\mu_{\nu\sigma\lambda}$ is components of the mixed tensor of the fourth rank of the third rank covariant in the first rank contravariant.[4]* $x^\mu_{\nu\sigma\lambda}=\frac{\partial x^\mu}{\partial x^\iota}\frac{\partial x^\epsilon}{\partial x^\nu}\frac{\partial x^\alpha}{\partial x^\sigma}\frac{\partial x^\beta}{\partial x^\lambda}x^\iota_{\epsilon\alpha\beta}$

# 3 About Covariant Derivative for the Scalar in Tensor Satisfying Binary Law

**Proposition 1** *When all coordinate systems satisfies Binary Law, $\frac{\partial^2 x_\mu}{\partial x^\nu\partial x^\nu}=0$ is established.*

Proof:I rewrite $\frac{\partial^2 x_\mu}{\partial x^\nu\partial x^\nu}=\frac{\partial^2 x_\mu}{\partial x^\nu\partial x^\nu}$ using Definision2 and get

$$\frac{\partial^2 x_\mu}{\partial x^\nu\partial x^\nu}=\frac{\partial^2 x_\mu}{\partial x_\mu\partial x^\nu}. \tag{1}$$

$\frac{\partial^2 x_\mu}{\partial x^\nu\partial x^\nu}$ is tensor satisfying Binary Law in consideration of Proposition6 here. If a dimensional number is 2, I get

$$\frac{\partial^2 x_1}{\partial x_1\partial \dot{x}^1}+\frac{\partial^2 x_2}{\partial x_2\partial \dot{x}^1}=0,\frac{\partial^2 x_1}{\partial x_1\partial \dot{x}^2}+\frac{\partial^2 x_2}{\partial x_2\partial \dot{x}^2}=0 \tag{2}$$

from $\frac{\partial^2 x_\mu}{\partial x_\mu \partial x^\nu}$. I get

$$\frac{\partial^2 x_\mu}{\partial x_\mu \partial x^\nu} = 0 \tag{3}$$

from (2). I get

$$\frac{\partial^2 x_\mu}{\partial x^\nu \partial x^\nu} = 0 \tag{4}$$

from (1),(3).

– End Proof –

**Proposition 2** *When all coordinate systems satisfies Binary Law,* $\frac{\partial M}{\partial x^\nu} = 0, \frac{\partial^4 x^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu \partial x^\nu} = 0$ *is established for* $\frac{\partial^3 x^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} = M$.

Proof:I get

$$\frac{\partial M}{\partial x^\nu} = 0, \tag{5}$$

$$\frac{\partial^4 x^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu \partial x^\nu} = 0 \tag{6}$$

from (4),(106).

– End Proof –

# 4 About Covariant Derivative for the Vector in Tensor Satisfying Binary Law

**Proposition 3** *When all coordinate systems satisfies Binary Law,* $x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_{\nu(\tau)}\Gamma^{\nu(\tau)}_{\mu\nu}$ *is established for* $x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_\tau \Gamma^\tau_{\mu\nu}$.

Proof:I get

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_\tau \Gamma^\tau_{\mu\nu} \tag{7}$$

from Definision10. If a dimensional number is 2, I express (7) in components and get

$$\begin{aligned} x_{1;1} &= \frac{\partial x_1}{\partial \dot{x}^1} - \ddot{x}_1\Gamma^1_{11} - \ddot{x}_2\Gamma^2_{11}, x_{1;2} = \frac{\partial x_1}{\partial \dot{x}^2} - \ddot{x}_1\Gamma^1_{12} - \ddot{x}_2\Gamma^2_{12}, \\ x_{2;1} &= \frac{\partial x_2}{\partial \dot{x}^1} - \ddot{x}_1\Gamma^1_{21} - \ddot{x}_2\Gamma^2_{21}, x_{2;2} = \frac{\partial x_2}{\partial \dot{x}^2} - \ddot{x}_1\Gamma^1_{22} - \ddot{x}_2\Gamma^2_{22}. \end{aligned} \tag{8}$$

If all coordinate systems satisfies Binary Law, I get

$$x_{1;1} = \frac{\partial x_1}{\partial \dot{x}^1} - \dot{x}_1\Gamma^1_{11} - \dot{x}_2\Gamma^2_{11}, x_{1;2} = \frac{\partial x_1}{\partial \dot{x}^2} - \dot{x}_1\Gamma^1_{12} - \dot{x}_2\Gamma^2_{12},$$
$$x_{2;1} = \frac{\partial x_2}{\partial \dot{x}^1} - \dot{x}_1\Gamma^1_{21} - \dot{x}_2\Gamma^2_{21}, x_{2;2} = \frac{\partial x_2}{\partial \dot{x}^2} - \dot{x}_1\Gamma^1_{22} - \dot{x}_2\Gamma^2_{22}. \tag{9}$$

from (8). If all coordinate systems satisfies Binary Law, I get

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_\nu\Gamma^\nu_{\mu\nu} \tag{10}$$

from (7). It should accord with (9) if I express (10) in components if a dimensional number is 2 here. If I adopt dummy index of (10) like

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_\nu\Gamma^{\nu(\sigma)}_{\mu\nu(\sigma)}. \tag{11}$$

The components indication of (11) does not accord in (9). On the other hand, if I adopt dummy index of (10) like

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_{\nu(\sigma)}\Gamma^{\nu(\sigma)}_{\mu\nu}. \tag{12}$$

The components indication of (12) accords in (9). Because I don't choose (11), I rewrite (10) in consideration of Definision7 and get

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_{\nu(\tau)}\Gamma^{\nu(\tau)}_{\mu\nu}. \tag{13}$$

– End Proof –

**Proposition 4** $x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu}, x^{;\mu}_\mu = \frac{\partial x_\mu}{\partial x_\mu} - x_\nu \frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^\mu}$ *is established in tensor satisfying Binary Law.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\tau)}\left(\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^\nu} + \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^{\nu(\epsilon)}}\right) \tag{14}$$

from Definision10 in consideration of Proposition3,Definision7. I get

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g_\nu^{\nu(\tau)}}{\partial x^\mu}\right) \tag{15}$$

by establishment of $\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^{\nu(\epsilon)}} = 0$ from (14). If (15) is a tensor equation, (15) must be expressed in

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_\sigma\frac{1}{2}\left(\frac{\partial g_\nu^\sigma}{\partial x^\mu}\right), \tag{16}$$

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} - x_{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right). \tag{17}$$

It is (16) that dummy index accords in Definision10. I decide not to handle (17) when dummy index doesn't accord in Definision10. Three kinds of the index exist in (16). Therefore, I get the conclusion that (16) doesn't satisfy Binary Law in consideration of Definision6. This is a problem. I rewrite (16) using Definision2 and get

$$x_\mu^{;\mu} = \frac{\partial x_\mu}{\partial x_\mu} - x_\sigma\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\mu}\right) = \frac{\partial x_\mu}{\partial x_\mu} - x_\nu\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\mu}\right). \tag{18}$$

The problem of (16) was solved in (18). I rewrite (16) using Definision4 and get

$$-x_{\mu;\mu} = -\frac{\partial x_\mu}{\partial x^\mu} + x_\sigma\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right) = -\frac{\partial x_\mu}{\partial x^\mu} + x_\nu\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\mu}\right). \tag{19}$$

The problem of (16) was solved in (19). I get

$$-x_{\mu;\mu} = -\frac{\partial x_\mu}{\partial x^\mu} \tag{20}$$

in consideration of Definision8 for (19). Because the second term of the right side doesn't exist in (20),

$$x_{\mu;\nu} = \frac{\partial x_\mu}{\partial x^\nu} \tag{21}$$

can rewrite (20) using Definision4. In addition, $x_{\mu;\nu}$ can't rewrite $x_\mu^{;\mu}$ of (18) using Definision2 because the second term of the right side exists in (18).

– End Proof –

**Proposition 5** $x^{\mu}_{;\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}}$ *is established in tensor satisfying Binary Law.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$x^{\mu}_{;\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} + x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}} - \frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}\right) \tag{22}$$

from Definision11 in consideration of Proposition3,Definision7. I get

$$x^{\mu}_{;\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} + x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\tau)}}{\partial x^{\nu}}\right) \tag{23}$$

by establishment of $\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}} - \frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}} = 0$ from (22). I get

$$x^{\mu}_{;\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} + x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu(\tau)}}\right) \tag{24}$$

by establishment of $\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}} - \frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}} = 0$ from (22). If (23),(24) is a tensor equation, (23),(24) must be expressed in

$$x^{\mu}_{;\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} + x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right), \tag{25}$$

$$x^{\mu}_{;\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} + x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right). \tag{26}$$

It is (25),(26) that dummy index accords in Definision11. I decide not to handle it when dummy index doesn't accord in Definision11. Three kinds of the index exist in (25),(26). Therefore, I get the conclusion that (25),(26) doesn't satisfy Binary Law in consideration of Definision6. This is a problem. I rewrite (25),(26) using Definision4 and get

$$-x^{\mu}_{;\mu} = -\frac{\partial x^{\mu}}{\partial x^{\mu}} - x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right) = -\frac{\partial x^{\mu}}{\partial x^{\mu}} - x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right), \tag{27}$$

$$-x^{\mu}_{;\mu} = -\frac{\partial x^{\mu}}{\partial x^{\mu}} - x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right) = -\frac{\partial x^{\mu}}{\partial x^{\mu}} - x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right). \tag{28}$$

The problem of (25),(26) was solved in (27),(28). I get

$$-x^{\mu}_{;\mu} = -\frac{\partial x^{\mu}}{\partial x^{\mu}} \tag{29}$$

in consideration of Definision8 for (27),(28). Because the second term of the right side doesn't exist in (29),

$$x^{\mu}_{;\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} \tag{30}$$

can rewrite (29) using Definision4. I rewrite (25),(26) using Definision2 and get

$$x^{\mu;\mu} = \frac{\partial x^{\mu}}{\partial x_{\mu}} + x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right) = \frac{\partial x^{\mu}}{\partial x_{\mu}} + x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}}\right), \tag{31}$$

$$x^{\mu;\mu} = \frac{\partial x^{\mu}}{\partial x_{\mu}} + x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right) = \frac{\partial x^{\mu}}{\partial x_{\mu}} + x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right). \tag{32}$$

The problem of (25),(26) was solved in (31),(32).

– End Proof –

**Proposition 6** $x_{\mu;\nu:\nu} = \frac{\partial^2 x_\mu}{\partial x^\nu \partial x^\nu}$ *is established in tensor satisfying Binary Law.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$\begin{aligned}
x_{\mu;\nu;\nu(\sigma)} &= \frac{\partial^2 x_\mu}{\partial x^\nu \partial x^{\nu(\sigma)}} \\
&-\frac{\partial}{\partial x^{\nu(\sigma)}}\left(x_{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\tau)}\left(\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\nu(\epsilon)}}\right)\right) \\
&-\frac{\partial x_{\nu(\iota)}}{\partial x^{\nu}}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} + \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right) \\
&+x_{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\tau)}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\iota)}} - \frac{\partial g_{\nu(\iota)\nu}}{\partial x^{\nu(\epsilon)}}\right) \\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} + \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right) \\
&-\frac{\partial x_{\mu}}{\partial x^{\nu(\iota)}}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} + \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu}} - \frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right) \\
&+x_{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\tau)}\left(\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^{\nu(\iota)}} + \frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu(\iota)}}{\partial x^{\nu(\epsilon)}}\right) \\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} + \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu}} - \frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)
\end{aligned} \tag{33}$$

from Definision12 in consideration of Proposition3,Definision7. I get

$$x_{\mu;\nu;\nu(\sigma)} = \frac{\partial^2 x_\mu}{\partial x^\nu \partial x^{\nu(\sigma)}} - \frac{\partial}{\partial x^{\nu(\sigma)}}\left(x_{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g_\nu^{\nu(\tau)}}{\partial x^\mu}\right)\right)$$
$$-\frac{\partial x_{\nu(\iota)}}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^{\nu(\iota)}}{\partial x^\mu}\right) + x_{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g_{\nu(\iota)}^{\nu(\tau)}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^{\nu(\iota)}}{\partial x^\mu}\right)$$
$$-\frac{\partial x_\mu}{\partial x^{\nu(\iota)}}\frac{1}{2}\left(\frac{\partial g_\nu^{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right) + x_{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g_{\nu(\iota)}^{\nu(\tau)}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right) \tag{34}$$

by establishment of $\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^{\nu(\epsilon)}} = 0$, $\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} - \frac{\partial g_{\mu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0$, $\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^{\nu(\iota)}} - \frac{\partial g_{\mu\nu(\iota)}}{\partial x^{\nu(\epsilon)}} = 0$, $\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\iota)}} - \frac{\partial g_{\nu(\iota)\nu}}{\partial x^{\nu(\epsilon)}} = 0$, $\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^\nu} - \frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0$ from (33). I get

$$x_{\mu;\nu;\nu(\sigma)} = \frac{\partial^2 x_\mu}{\partial x^\nu \partial x^{\nu(\sigma)}} - \frac{\partial}{\partial x^{\nu(\sigma)}}\left(x_{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g_\nu^{\nu(\tau)}}{\partial x^\mu}\right)\right)$$
$$-\frac{\partial x_{\nu(\iota)}}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^{\nu(\iota)}}{\partial x^\mu}\right) + x_{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g_\nu^{\nu(\tau)}}{\partial x^{\nu(\iota)}}\right)\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^{\nu(\iota)}}{\partial x^\mu}\right)$$
$$-\frac{\partial x_\mu}{\partial x^{\nu(\iota)}}\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^{\nu(\iota)}}{\partial x^\nu}\right) + x_{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g_{\nu(\iota)}^{\nu(\tau)}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^{\nu(\iota)}}{\partial x^\nu}\right) \tag{35}$$

by establishment of $\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^{\nu(\epsilon)}} = 0$, $\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} - \frac{\partial g_{\mu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0$, $\frac{\partial g_{\mu\nu(\epsilon)}}{\partial x^{\nu(\iota)}} - \frac{\partial g_{\mu\nu(\iota)}}{\partial x^{\nu(\epsilon)}} = 0$, $\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^\nu} - \frac{\partial g_{\nu(\iota)\nu}}{\partial x^{\nu(\epsilon)}} = 0$, $\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} - \frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0$ from (33). If (34),(35) is a tensor equation, (34),(35) must be expressed in

$$x_{\mu;\nu;\nu(\sigma)} = \frac{\partial^2 x_\mu}{\partial x^\nu \partial x^{\nu(\sigma)}} - \frac{\partial}{\partial x^{\nu(\sigma)}}\left(x_\sigma\frac{1}{2}\left(\frac{\partial g_\nu^\sigma}{\partial x^\mu}\right)\right)$$
$$-\frac{\partial x_\sigma}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^\sigma}{\partial x^\mu}\right) + x_\sigma\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^\sigma}{\partial x^\mu}\right)$$
$$-\frac{\partial x_\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\nu^\sigma}{\partial x^{\nu(\sigma)}}\right) + x_\sigma\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\sigma}{\partial x^{\nu(\sigma)}}\right), \tag{36}$$

$$x_{\mu;\nu;\nu(\sigma)} = \frac{\partial^2 x_\mu}{\partial x^\nu \partial x^{\nu(\sigma)}} - \frac{\partial}{\partial x^{\nu(\sigma)}}\left(x_\sigma\frac{1}{2}\left(\frac{\partial g_\nu^\sigma}{\partial x^\mu}\right)\right)$$
$$-\frac{\partial x_\sigma}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^\sigma}{\partial x^\mu}\right) + x_\sigma\frac{1}{2}\left(\frac{\partial g_\nu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^\sigma}{\partial x^\mu}\right)$$
$$-\frac{\partial x_\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^\sigma}{\partial x^\nu}\right) + x_\sigma\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_{\nu(\sigma)}^\sigma}{\partial x^\nu}\right). \tag{37}$$

It is (36),(37) that dummy index accords in Definision12. I decide not to handle it when dummy index doesn't accord in Definision12. Three kinds of the index exist in (36),(37). Therefore, I get the conclusion that (36),(37) doesn't satisfy Binary Law in consideration of Definision6. This is a problem. I rewrite (36),(37) using Definision4 and get

$$\begin{aligned}
x_{\mu;\mu;\mu(\sigma)} &= \frac{\partial^2 x_\mu}{\partial x^\mu \partial x^{\mu(\sigma)}} - \frac{\partial}{\partial x^{\mu(\sigma)}}\left(x_\sigma \frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)\right) \\
&- \frac{\partial x_\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_{\mu(\sigma)}}{\partial x^\mu}\right) + x_\sigma \frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\mu(\sigma)}}{\partial x^\mu}\right) \\
&- \frac{\partial x_\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\sigma)}}\right) + x_\sigma \frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\sigma)}}\right) \\
&= \frac{\partial^2 x_\mu}{\partial x^\mu \partial x^{\mu(\sigma)}} - \frac{\partial}{\partial x^{\mu(\sigma)}}\left(x_\nu \frac{1}{2}\left(\frac{\partial g^\nu_\mu}{\partial x^\mu}\right)\right) \\
&- \frac{\partial x_\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\sigma)}}{\partial x^\mu}\right) + x_\nu \frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\sigma)}}{\partial x^\mu}\right) \\
&- \frac{\partial x_\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\nu_\mu}{\partial x^{\mu(\sigma)}}\right) + x_\nu \frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_\mu}{\partial x^{\mu(\sigma)}}\right), \qquad (38)
\end{aligned}$$

$$\begin{aligned}
x_{\mu;\mu;\mu(\sigma)} &= \frac{\partial^2 x_\mu}{\partial x^\mu \partial x^{\mu(\sigma)}} - \frac{\partial}{\partial x^{\mu(\sigma)}}\left(x_\sigma \frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)\right) \\
&- \frac{\partial x_\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_{\mu(\sigma)}}{\partial x^\mu}\right) + x_\sigma \frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\mu(\sigma)}}{\partial x^\mu}\right) \\
&- \frac{\partial x_\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_{\mu(\sigma)}}{\partial x^\mu}\right) + x_\sigma \frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\mu(\sigma)}}{\partial x^\mu}\right) \\
&= \frac{\partial^2 x_\mu}{\partial x^\mu \partial x^{\mu(\sigma)}} - \frac{\partial}{\partial x^{\mu(\sigma)}}\left(x_\nu \frac{1}{2}\left(\frac{\partial g^\nu_\mu}{\partial x^\mu}\right)\right) \\
&- \frac{\partial x_\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\sigma)}}{\partial x^\mu}\right) + x_\nu \frac{1}{2}\left(\frac{\partial g^\nu_\mu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\sigma)}}{\partial x^\mu}\right) \\
&- \frac{\partial x_\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\sigma)}}{\partial x^\mu}\right) + x_\nu \frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\sigma)}}{\partial x^\mu}\right). \qquad (39)
\end{aligned}$$

The problem of (36),(37) was solved in (38),(39). I get

$$\begin{aligned}
x_{\mu;\mu;\mu(\sigma)} &= \frac{\partial^2 x_\mu}{\partial x^\mu \partial x^{\mu(\sigma)}}, \\
x_{\mu;\mu;\mu} &= \frac{\partial^2 x_\mu}{\partial x^\mu \partial x^\mu} \qquad (40)
\end{aligned}$$

in consideration of Definision8 for (38),(39). I decide not to use the notation of Definision7 in (40) here. Because the second term of the right side doesn't exist in (40),

$$x_{\mu;\nu;\nu} = \frac{\partial^2 x_\mu}{\partial x^\nu \partial x^\nu} \tag{41}$$

can rewrite (40) using Definision4. I rewrite (36),(37) using Definision2 and get

$$\begin{aligned}
x_\mu^{;\mu;\mu(\sigma)} &= \frac{\partial^2 x_\mu}{\partial x_\mu \partial x_{\mu(\sigma)}} - \frac{\partial}{\partial x_{\mu(\sigma)}}\left(x_\sigma \frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\mu}\right)\right) \\
&-\frac{\partial x_\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x^\mu}\right) + x_\sigma \frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x^\mu}\right) \\
&-\frac{\partial x_\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\sigma)}}\right) + x_\sigma \frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\sigma)}}\right) \\
&= \frac{\partial^2 x_\mu}{\partial x_\mu \partial x_{\mu(\sigma)}} - \frac{\partial}{\partial x_{\mu(\sigma)}}\left(x_\nu \frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\mu}\right)\right) \\
&-\frac{\partial x_\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x^\mu}\right) + x_\nu \frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x^\mu}\right) \\
&-\frac{\partial x_\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\sigma)}}\right) + x_\nu \frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\sigma)}}\right),
\end{aligned} \tag{42}$$

$$\begin{aligned}
x_\mu^{;\mu;\mu(\sigma)} &= \frac{\partial^2 x_\mu}{\partial x_\mu \partial x_{\mu(\sigma)}} - \frac{\partial}{\partial x_{\mu(\sigma)}}\left(x_\sigma \frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\mu}\right)\right) \\
&-\frac{\partial x_\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x^\mu}\right) + x_\sigma \frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x^\mu}\right) \\
&-\frac{\partial x_\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_\mu}\right) + x_\sigma \frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_\mu}\right) \\
&= \frac{\partial^2 x_\mu}{\partial x_\mu \partial x_{\mu(\sigma)}} - \frac{\partial}{\partial x_{\mu(\sigma)}}\left(x_\nu \frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\mu}\right)\right) \\
&-\frac{\partial x_\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x^\mu}\right) + x_\nu \frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x^\mu}\right) \\
&-\frac{\partial x_\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_\mu}\right) + x_\nu \frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_\mu}\right).
\end{aligned} \tag{43}$$

The problem of (36),(37) was solved in (42),(43).

– End Proof –

**Proposition 7** $x^\mu_{;\nu:\nu} = \frac{\partial^2 x^\mu}{\partial x^\nu \partial x^\nu}$ *is established in tensor satisfying Binary Law.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$\begin{aligned}
x^{\mu}_{;\nu;\nu(\sigma)} &= \frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\nu(\sigma)}} \\
&+\frac{\partial}{\partial x^{\nu(\sigma)}}x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}\right) \\
&+\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right) \\
&+x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}\right) \\
&\times\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right) \\
&-\frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right) \\
&-x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}+\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu(\iota)}}{\partial x^{\nu(\epsilon)}}\right) \\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)
\end{aligned} \tag{44}$$

from Definision13 in consideration of Proposition3,Definision7. I get

$$\begin{aligned}
x^{\mu}_{;\nu;\nu(\sigma)} &= \frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\nu(\sigma)}}+\frac{\partial}{\partial x^{\nu(\sigma)}}x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\tau)}}{\partial x^{\nu}}\right) \\
&+\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right)+x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\tau)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right) \\
&-\frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\sigma)}}\right)-x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\tau)}}{\partial x^{\nu(\iota)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\sigma)}}\right)
\end{aligned} \tag{45}$$

by establishment of $\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}=0$, $\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}=0$, $\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}=0$, $\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu(\iota)}}{\partial x^{\nu(\epsilon)}}=0$ from (44). I get

$$\begin{aligned}
x^{\mu}_{;\nu;\nu(\sigma)} &= \frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\nu(\sigma)}}+\frac{\partial}{\partial x^{\nu(\sigma)}}x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu(\tau)}}\right) \\
&+\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\sigma)}}{\partial x^{\nu(\iota)}}\right)+x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\tau)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\sigma)}}{\partial x^{\nu(\iota)}}\right) \\
&-\frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\sigma)}}{\partial x^{\nu}}\right)-x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\tau)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\sigma)}}{\partial x^{\nu}}\right)
\end{aligned} \tag{46}$$

by establishment of $\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}} - \frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} - \frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} - \frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\iota)}} - \frac{\partial g_{\nu(\tau)\nu(\iota)}}{\partial x^{\nu(\epsilon)}} = 0$ from (44). If (45),(46) is a tensor equation, (45),(46) must be expressed in

$$
\begin{aligned}
x^{\mu}_{;\nu;\nu(\sigma)} &= \frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\nu(\sigma)}} + \frac{\partial}{\partial x^{\nu(\sigma)}} x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right) \\
&+\frac{\partial x^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu(\sigma)}}\right) + x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu(\sigma)}}\right) \\
&-\frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu(\sigma)}}\right) - x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu(\sigma)}}\right),
\end{aligned}
\tag{47}
$$

$$
\begin{aligned}
x^{\mu}_{;\nu;\nu(\sigma)} &= \frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\nu(\sigma)}} + \frac{\partial}{\partial x^{\nu(\sigma)}} x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right) \\
&+\frac{\partial x^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\sigma)}}{\partial x^{\sigma}}\right) + x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\sigma)}}{\partial x^{\sigma}}\right) \\
&-\frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu(\sigma)}}{\partial x^{\nu}}\right) - x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu(\sigma)}}{\partial x^{\nu}}\right).
\end{aligned}
\tag{48}
$$

It is (47),(48) that dummy index accords in Definision13. I decide not to handle it when dummy index doesn't accord in Definision13. Three kinds of the index exist in (47),(48). Therefore, I get the conclusion that (47),(48) doesn't satisfy Binary Law in consideration of Definision6. This is a problem. I rewrite (47),(48) using Definision4 and get

$$
\begin{aligned}
x^{\mu}_{;\mu;\mu(\sigma)} &= \frac{\partial^2 x^{\mu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}} + \frac{\partial}{\partial x^{\mu(\sigma)}} x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right) \\
&+\frac{\partial x^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu(\sigma)}}\right) + x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu(\sigma)}}\right) \\
&-\frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu(\sigma)}}\right) - x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu(\sigma)}}\right) \\
&= \frac{\partial^2 x^{\mu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}} + \frac{\partial}{\partial x^{\mu(\sigma)}} x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right) \\
&+\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\sigma)}}\right) + x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\sigma)}}\right) \\
&-\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\sigma)}}\right) - x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\sigma)}}\right),
\end{aligned}
\tag{49}
$$

$$
x^{\mu}_{;\mu;\mu(\sigma)} = \frac{\partial^2 x^{\mu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}} + \frac{\partial}{\partial x^{\mu(\sigma)}} x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)
$$

$$+\frac{\partial x^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)+x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)$$
$$-\frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu}}\right)-x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu}}\right)$$
$$=\frac{\partial^2 x^{\mu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}}+\frac{\partial}{\partial x^{\mu(\sigma)}}x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)$$
$$+\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)+x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)$$
$$-\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu}}\right)-x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu}}\right). \tag{50}$$

The problem of (47),(48) was solved in (49),(50). I get

$$x^{\mu}_{;\mu;\mu(\sigma)}=\frac{\partial^2 x^{\mu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}},$$
$$x^{\mu}_{;\mu;\mu}=\frac{\partial^2 x^{\mu}}{\partial x^{\mu}\partial x^{\mu}} \tag{51}$$

in consideration of Definision8 for (49),(50). I decide not to use the notation of Definision7 in (51) here. Because the second term of the right side doesn't exist in (51),

$$x^{\mu}_{;\nu;\nu}=\frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\nu}} \tag{52}$$

can rewrite (51) using Definision4. I rewrite (47),(48) using Definision2 and get

$$x^{\mu;\mu;\mu(\sigma)}=\frac{\partial^2 x^{\mu}}{\partial x_{\mu}\partial x_{\mu(\sigma)}}+\frac{\partial}{\partial x_{\mu(\sigma)}}x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)$$
$$+\frac{\partial x^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu(\sigma)}}\right)+x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu(\sigma)}}\right)$$
$$-\frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\sigma)}}\right)-x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\sigma)}}\right)$$
$$=\frac{\partial^2 x^{\mu}}{\partial x_{\mu}\partial x_{\mu(\sigma)}}+\frac{\partial}{\partial x_{\mu(\sigma)}}x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}}\right)$$
$$+\frac{\partial x^{\nu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu(\sigma)}}\right)+x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu(\sigma)}}\right)$$
$$-\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\sigma)}}\right)-x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\sigma)}}\right), \tag{53}$$

$$x^{\mu;\mu;\mu(\sigma)} = \frac{\partial^2 x^\mu}{\partial x_\mu \partial x_{\mu(\sigma)}} + \frac{\partial}{\partial x_{\mu(\sigma)}} x^\sigma \frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)$$
$$+\frac{\partial x^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\sigma}\right) + x^\sigma \frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\sigma}\right)$$
$$-\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_\mu}\right) - x^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_\mu}\right)$$
$$= \frac{\partial^2 x^\mu}{\partial x_\mu \partial x_{\mu(\sigma)}} + \frac{\partial}{\partial x_{\mu(\sigma)}} x^\nu \frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)$$
$$+\frac{\partial x^\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\nu}\right) + x^\nu \frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\nu}\right)$$
$$-\frac{\partial x^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_\mu}\right) - x^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_\mu}\right). \quad (54)$$

The problem of (47),(48) was solved in (53),(54).

– End Proof –

**Proposition 8** $x^\mu_{;\nu;\nu;\nu} = \frac{\partial^3 x^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}$ *is established in tensor satisfying Binary Law.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$x^\mu_{;\nu;\nu(\sigma);\nu(\lambda)} = \frac{\partial^3 x^\mu}{\partial x^\nu \partial x^{\nu(\sigma)} \partial x^{\nu(\lambda)}}$$
$$+\frac{\partial^2}{\partial x^{\nu(\sigma)}\partial x^{\nu(\lambda)}} x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^\nu} + \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}} - \frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}\right)$$
$$+\frac{\partial}{\partial x^{\nu(\lambda)}}\frac{\partial x^{\nu(\iota)}}{\partial x^\nu}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} + \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}} - \frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)$$
$$+\frac{\partial}{\partial x^{\nu(\lambda)}} x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^\nu} + \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}} - \frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}\right)$$
$$\times\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} + \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}} - \frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)$$
$$-\frac{\partial}{\partial x^{\nu(\lambda)}}\frac{\partial x^\mu}{\partial x^{\nu(\iota)}}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} + \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^\nu} - \frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)$$
$$-\frac{\partial}{\partial x^{\nu(\lambda)}} x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\iota)}} + \frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\tau)}} - \frac{\partial g_{\nu(\tau)\nu(\iota)}}{\partial x^{\nu(\epsilon)}}\right)$$
$$\times\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} + \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^\nu} - \frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)$$
$$+\frac{\partial^2 x^{\nu(\kappa)}}{\partial x^\nu \partial x^{\nu(\sigma)}}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}} + \frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}} - \frac{\partial g_{\nu(\kappa)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)$$

$$
\begin{aligned}
&+\frac{\partial}{\partial x^{\nu(\sigma)}}\left(x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}\right)\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}-\frac{\partial g_{\nu(\kappa)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&+\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}-\frac{\partial g_{\nu(\kappa)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&+x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}-\frac{\partial g_{\nu(\kappa)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-\frac{\partial x^{\nu(\kappa)}}{\partial x^{\nu(\iota)}}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}-\frac{\partial g_{\nu(\kappa)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}+\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu(\iota)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}-\frac{\partial g_{\nu(\kappa)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-\frac{\partial^2 x^{\mu}}{\partial x^{\nu(\kappa)}\partial x^{\nu(\sigma)}}\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-\frac{\partial}{\partial x^{\nu(\sigma)}}\left(x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}+\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu(\kappa)}}{\partial x^{\nu(\epsilon)}}\right)\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu(\kappa)}}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}+\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu(\kappa)}}{\partial x^{\nu(\epsilon)}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\times\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&+\frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}-\frac{\partial g_{\nu(\kappa)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&+x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}+\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu(\iota)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}+\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}-\frac{\partial g_{\nu(\kappa)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-\frac{\partial^{2} x^{\mu}}{\partial x^{\nu}\partial x^{\nu(\kappa)}}\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}-\frac{\partial g_{\nu(\sigma)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-\frac{\partial}{\partial x^{\nu(\kappa)}}\left(x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}\right)\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}-\frac{\partial g_{\nu(\sigma)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}+\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\kappa)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}-\frac{\partial g_{\nu(\sigma)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&-x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}+\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\kappa)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}-\frac{\partial g_{\nu(\sigma)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&+\frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}}\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}+\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\kappa)}}{\partial x^{\nu(\epsilon)}}\right)\\
&\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}-\frac{\partial g_{\nu(\sigma)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right)\\
&+x^{\nu(\tau)}\frac{1}{2}g^{\nu(\epsilon)\mu}\left(\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}+\frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu(\iota)}}{\partial x^{\nu(\epsilon)}}\right)
\end{aligned}
$$

$$\times\frac{1}{2}g^{\nu(\epsilon)\nu(\iota)}\left(\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}+\frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\kappa)}}{\partial x^{\nu(\epsilon)}}\right)$$
$$\times\frac{1}{2}g^{\nu(\epsilon)\nu(\kappa)}\left(\frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}}+\frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}-\frac{\partial g_{\nu(\sigma)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}\right) \quad (55)$$

from Definision14 in consideration of Proposition3,Definision7. I get

$$x^{\mu}_{;\nu;\nu(\sigma);\nu(\lambda)}=\frac{\partial^3 x^{\mu}}{\partial x^{\nu}\partial x^{\nu(\sigma)}\partial x^{\nu(\lambda)}}+\frac{\partial^2}{\partial x^{\nu(\sigma)}\partial x^{\nu(\lambda)}}x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\tau)}}{\partial x^{\nu}}\right)$$
$$+\frac{\partial}{\partial x^{\nu(\lambda)}}\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right)+\frac{\partial}{\partial x^{\nu(\lambda)}}x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\tau)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right)$$
$$-\frac{\partial}{\partial x^{\nu(\lambda)}}\frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\sigma)}}\right)-\frac{\partial}{\partial x^{\nu(\lambda)}}x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\tau)}}{\partial x^{\nu(\iota)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\sigma)}}\right)$$
$$+\frac{\partial^2 x^{\nu(\kappa)}}{\partial x^{\nu}\partial x^{\nu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\kappa)}}{\partial x^{\nu(\lambda)}}\right)+\frac{\partial}{\partial x^{\nu(\sigma)}}\left(x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\tau)}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\kappa)}}{\partial x^{\nu(\lambda)}}\right)$$
$$+\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\kappa)}}{\partial x^{\nu(\lambda)}}\right)+x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\tau)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\kappa)}}{\partial x^{\nu(\lambda)}}\right)$$
$$-\frac{\partial x^{\nu(\kappa)}}{\partial x^{\nu(\iota)}}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\kappa)}}{\partial x^{\nu(\lambda)}}\right)-x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\tau)}}{\partial x^{\nu(\iota)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\kappa)}}{\partial x^{\nu(\lambda)}}\right)$$
$$-\frac{\partial^2 x^{\mu}}{\partial x^{\nu(\kappa)}\partial x^{\nu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu}}{\partial x^{\nu(\lambda)}}\right)-\frac{\partial}{\partial x^{\nu(\sigma)}}\left(x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\tau)}}{\partial x^{\nu(\kappa)}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu}}{\partial x^{\nu(\lambda)}}\right)$$
$$-\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu(\kappa)}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu}}{\partial x^{\nu(\lambda)}}\right)-x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\tau)}}{\partial x^{\nu(\kappa)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu}}{\partial x^{\nu(\lambda)}}\right)$$
$$+\frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\kappa)}}{\partial x^{\nu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu}}{\partial x^{\nu(\lambda)}}\right)+x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\tau)}}{\partial x^{\nu(\iota)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\kappa)}}{\partial x^{\nu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu}}{\partial x^{\nu(\lambda)}}\right)$$
$$-\frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\nu(\kappa)}}\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}}\right)-\frac{\partial}{\partial x^{\nu(\kappa)}}\left(x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\tau)}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}}\right)$$
$$-\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\kappa)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}}\right)-x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\tau)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\kappa)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}}\right)$$
$$+\frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}}\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\kappa)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}}\right)+x^{\nu(\tau)}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\tau)}}{\partial x^{\nu(\iota)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\kappa)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}}\right) \quad (56)$$

by establishment of $\frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}}=0, \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\iota)}}-\frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}=0, \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}}=0, \frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\tau)}}-\frac{\partial g_{\nu(\tau)\nu(\iota)}}{\partial x^{\nu(\epsilon)}}=0, \frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}}-\frac{\partial g_{\nu(\kappa)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}=0, \frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu(\lambda)}}{\partial x^{\nu(\epsilon)}}=$

$0, \frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\tau)}} - \frac{\partial g_{\nu(\tau)\nu(\kappa)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}} - \frac{\partial g_{\nu(\kappa)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\lambda)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} - \frac{\partial g_{\nu(\sigma)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\iota)}} - \frac{\partial g_{\nu(\iota)\nu(\kappa)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu}} - \frac{\partial g_{\nu\nu(\kappa)}}{\partial x^{\nu(\epsilon)}} = 0$ from (55). I get

$$
\begin{aligned}
x^{\mu}_{;\nu;\nu(\sigma);\nu(\lambda)} &= \frac{\partial^3 x^\mu}{\partial x^\nu \partial x^{\nu(\sigma)} \partial x^{\nu(\lambda)}} + \frac{\partial^2}{\partial x^{\nu(\sigma)}\partial x^{\nu(\lambda)}} x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu(\tau)}}\right) \\
&+ \frac{\partial}{\partial x^{\nu(\lambda)}} \frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\sigma)}}{\partial x^{\nu(\iota)}}\right) + \frac{\partial}{\partial x^{\nu(\lambda)}} x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\tau)}}\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\sigma)}}{\partial x^{\nu(\iota)}}\right) \\
&- \frac{\partial}{\partial x^{\nu(\lambda)}} \frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}} \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\sigma)}}{\partial x^{\nu}}\right) - \frac{\partial}{\partial x^{\nu(\lambda)}} x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\tau)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\sigma)}}{\partial x^{\nu}}\right) \\
&+ \frac{\partial^2 x^{\nu(\kappa)}}{\partial x^\nu \partial x^{\nu(\sigma)}} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\lambda)}}{\partial x^{\nu(\kappa)}}\right) + \frac{\partial}{\partial x^{\nu(\sigma)}}\left(x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu}}{\partial x^{\nu(\tau)}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\lambda)}}{\partial x^{\nu(\kappa)}}\right) \\
&+ \frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}} \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\sigma)}}{\partial x^{\nu(\iota)}}\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\lambda)}}{\partial x^{\nu(\kappa)}}\right) + x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\tau)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\sigma)}}{\partial x^{\nu(\iota)}}\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\lambda)}}{\partial x^{\nu(\kappa)}}\right) \\
&- \frac{\partial x^{\nu(\kappa)}}{\partial x^{\nu(\iota)}} \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\sigma)}}{\partial x^{\nu}}\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\lambda)}}{\partial x^{\nu(\kappa)}}\right) - x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\iota)}}{\partial x^{\nu(\tau)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\sigma)}}{\partial x^{\nu}}\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\lambda)}}{\partial x^{\nu(\kappa)}}\right) \\
&- \frac{\partial^2 x^{\mu}}{\partial x^{\nu(\kappa)} \partial x^{\nu(\sigma)}} \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu}}\right) - \frac{\partial}{\partial x^{\nu(\sigma)}}\left(x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\kappa)}}{\partial x^{\nu(\tau)}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu}}\right) \\
&- \frac{\partial x^{\nu(\iota)}}{\partial x^{\nu(\kappa)}} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\sigma)}}{\partial x^{\nu(\iota)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu}}\right) - x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\kappa)}}{\partial x^{\nu(\tau)}}\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\sigma)}}{\partial x^{\nu(\iota)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu}}\right) \\
&+ \frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}} \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\sigma)}}{\partial x^{\nu(\kappa)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu}}\right) + x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\tau)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\sigma)}}{\partial x^{\nu(\kappa)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu}}\right) \\
&- \frac{\partial^2 x^{\mu}}{\partial x^{\nu} \partial x^{\nu(\kappa)}} \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right) - \frac{\partial}{\partial x^{\nu(\kappa)}}\left(x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu(\tau)}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right) \\
&- \frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\kappa)}}{\partial x^{\nu(\iota)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right) - x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu}}{\partial x^{\nu(\tau)}}\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\kappa)}}{\partial x^{\nu(\iota)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right) \\
&+ \frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}} \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\kappa)}}{\partial x^{\nu}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right) + x^{\nu(\tau)} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu(\iota)}}{\partial x^{\nu(\tau)}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\iota)}_{\nu(\kappa)}}{\partial x^{\nu}}\right) \frac{1}{2}\left(\frac{\partial g^{\nu(\kappa)}_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right)
\end{aligned} \tag{57}
$$

by establishment of $\frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu}} - \frac{\partial g_{\nu(\tau)\nu}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} - \frac{\partial g_{\nu(\iota)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} - \frac{\partial g_{\nu\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\iota)}} - \frac{\partial g_{\nu(\tau)\nu(\iota)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}} - \frac{\partial g_{\nu(\kappa)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\lambda)}} - \frac{\partial g_{\nu\nu(\lambda)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\tau)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}} - \frac{\partial g_{\nu(\tau)\nu(\kappa)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\kappa)\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} - \frac{\partial g_{\nu(\kappa)\nu(\sigma)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\sigma)\nu(\epsilon)}}{\partial x^{\nu(\lambda)}} - \frac{\partial g_{\nu(\sigma)\nu(\lambda)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu(\iota)\nu(\epsilon)}}{\partial x^{\nu(\kappa)}} - \frac{\partial g_{\nu(\iota)\nu(\kappa)}}{\partial x^{\nu(\epsilon)}} = 0, \frac{\partial g_{\nu\nu(\epsilon)}}{\partial x^{\nu(\kappa)}} - \frac{\partial g_{\nu\nu(\kappa)}}{\partial x^{\nu(\epsilon)}} = 0$ from (55). If (56),(57) is a tensor equation, (56),(57) must be expressed in

$$
\begin{aligned}
x^{\mu}_{;\nu;\nu(\sigma);\nu(\lambda)} &= \frac{\partial^3 x^\mu}{\partial x^\nu \partial x^{\nu(\sigma)} \partial x^{\nu(\lambda)}} + \frac{\partial^2}{\partial x^{\nu(\sigma)} \partial x^{\nu(\lambda)}} x^\sigma \frac{1}{2} \frac{\partial g^\mu_\sigma}{\partial x^\nu} \\
&+ \frac{\partial}{\partial x^{\nu(\lambda)}} \frac{\partial x^\sigma}{\partial x^\nu} \frac{1}{2} \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\sigma)}} + \frac{\partial}{\partial x^{\nu(\lambda)}} x^\sigma \frac{1}{2} \frac{\partial g^\sigma_\sigma}{\partial x^\nu} \frac{1}{2} \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\sigma)}} \\
&- \frac{\partial}{\partial x^{\nu(\lambda)}} \frac{\partial x^\mu}{\partial x^\sigma} \frac{1}{2} \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\sigma)}} - \frac{\partial}{\partial x^{\nu(\lambda)}} x^\sigma \frac{1}{2} \frac{\partial g^\mu_\sigma}{\partial x^\sigma} \frac{1}{2} \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\sigma)}} \\
&+ \frac{\partial^2 x^\sigma}{\partial x^\nu \partial x^{\nu(\sigma)}} \frac{1}{2} \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\lambda)}} + \frac{\partial}{\partial x^{\nu(\sigma)}} \left( x^\sigma \frac{1}{2} \frac{\partial g^\sigma_\sigma}{\partial x^\nu} \right) \frac{1}{2} \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\lambda)}} \\
&+ \frac{\partial x^\sigma}{\partial x^\nu} \frac{1}{2} \left( \frac{\partial g^\sigma_\sigma}{\partial x^{\nu(\sigma)}} \right) \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\lambda)}} \right) + x^\sigma \frac{1}{2} \left( \frac{\partial g^\sigma_\sigma}{\partial x^\nu} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_\sigma}{\partial x^{\nu(\sigma)}} \right) \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\lambda)}} \right) \\
&- \frac{\partial x^\sigma}{\partial x^\sigma} \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\sigma)}} \right) \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\lambda)}} \right) - x^\sigma \frac{1}{2} \left( \frac{\partial g^\sigma_\sigma}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\sigma)}} \right) \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\lambda)}} \right) \\
&- \frac{\partial^2 x^\mu}{\partial x^\sigma \partial x^{\nu(\sigma)}} \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\lambda)}} \right) - \frac{\partial}{\partial x^{\nu(\sigma)}} \left( x^\sigma \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^\sigma} \right) \right) \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\lambda)}} \right) \\
&- \frac{\partial x^\sigma}{\partial x^\sigma} \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\sigma)}} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\lambda)}} \right) - x^\sigma \frac{1}{2} \left( \frac{\partial g^\sigma_\sigma}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^{\nu(\sigma)}} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\lambda)}} \right) \\
&+ \frac{\partial x^\mu}{\partial x^\sigma} \frac{1}{2} \left( \frac{\partial g^\sigma_\sigma}{\partial x^{\nu(\sigma)}} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\lambda)}} \right) + x^\sigma \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_\sigma}{\partial x^{\nu(\sigma)}} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^{\nu(\lambda)}} \right) \\
&- \frac{\partial^2 x^\mu}{\partial x^\nu \partial x^\sigma} \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}} \right) - \frac{\partial}{\partial x^\sigma} \left( x^\sigma \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^\nu} \right) \right) \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}} \right) \\
&- \frac{\partial x^\sigma}{\partial x^\nu} \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}} \right) - x^\sigma \frac{1}{2} \left( \frac{\partial g^\sigma_\sigma}{\partial x^\nu} \right) \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}} \right) \\
&+ \frac{\partial x^\mu}{\partial x^\sigma} \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}} \right) + x^\sigma \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^{\nu(\lambda)}} \right), \qquad (58)
\end{aligned}
$$

$$
\begin{aligned}
x^{\mu}_{;\nu;\nu(\sigma);\nu(\lambda)} &= \frac{\partial^3 x^\mu}{\partial x^\nu \partial x^{\nu(\sigma)} \partial x^{\nu(\lambda)}} + \frac{\partial^2}{\partial x^{\nu(\sigma)} \partial x^{\nu(\lambda)}} x^\sigma \frac{1}{2} \left( \frac{\partial g^\mu_\nu}{\partial x^\sigma} \right) \\
&+ \frac{\partial}{\partial x^{\nu(\lambda)}} \frac{\partial x^\sigma}{\partial x^\nu} \frac{1}{2} \left( \frac{\partial g^\mu_{\nu(\sigma)}}{\partial x^\sigma} \right) + \frac{\partial}{\partial x^{\nu(\lambda)}} x^\sigma \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\mu_{\nu(\sigma)}}{\partial x^\sigma} \right) \\
&- \frac{\partial}{\partial x^{\nu(\lambda)}} \frac{\partial x^\mu}{\partial x^\sigma} \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^\nu} \right) - \frac{\partial}{\partial x^{\nu(\lambda)}} x^\sigma \frac{1}{2} \left( \frac{\partial g^\mu_\sigma}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^\nu} \right) \\
&+ \frac{\partial^2 x^\sigma}{\partial x^\nu \partial x^{\nu(\sigma)}} \frac{1}{2} \left( \frac{\partial g^\mu_{\nu(\lambda)}}{\partial x^\sigma} \right) + \frac{\partial}{\partial x^{\nu(\sigma)}} \left( x^\sigma \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^\sigma} \right) \right) \frac{1}{2} \left( \frac{\partial g^\mu_{\nu(\lambda)}}{\partial x^\sigma} \right) \\
&+ \frac{\partial x^\sigma}{\partial x^\nu} \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\mu_{\nu(\lambda)}}{\partial x^\sigma} \right) + x^\sigma \frac{1}{2} \left( \frac{\partial g^\sigma_\nu}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\mu_{\nu(\lambda)}}{\partial x^\sigma} \right) \\
&- \frac{\partial x^\sigma}{\partial x^\sigma} \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^\nu} \right) \frac{1}{2} \left( \frac{\partial g^\mu_{\nu(\lambda)}}{\partial x^\sigma} \right) - x^\sigma \frac{1}{2} \left( \frac{\partial g^\sigma_\sigma}{\partial x^\sigma} \right) \frac{1}{2} \left( \frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^\nu} \right) \frac{1}{2} \left( \frac{\partial g^\mu_{\nu(\lambda)}}{\partial x^\sigma} \right)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\partial^2 x^\mu}{\partial x^\sigma \partial x^{\nu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^\nu}\right)-\frac{\partial}{\partial x^{\nu(\sigma)}}\left(x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^\nu}\right)\\
&-\frac{\partial x^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\mu_{\nu(\sigma)}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^\nu}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\mu_{\nu(\sigma)}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^\nu}\right)\\
&+\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^\nu}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\sigma)}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^\nu}\right)\\
&-\frac{\partial^2 x^\mu}{\partial x^\nu \partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right)-\frac{\partial}{\partial x^\sigma}\left(x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right)\\
&-\frac{\partial x^\sigma}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right)\\
&+\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\nu(\lambda)}}{\partial x^{\nu(\sigma)}}\right). \qquad (59)
\end{aligned}
$$

It is (58),(59) that dummy index accords in Definision14. I decide not to handle it when dummy index doesn't accord in Definision14. Three kinds of the index exist in (58),(59). Therefore, I get the conclusion that (58),(59) doesn't satisfy Binary Law in consideration of Definision6. This is a problem. I rewrite (58),(59) using Definision4 and get

$$
\begin{aligned}
-x^\mu_{;\mu;\mu(\sigma);\mu(\lambda)} &= -\frac{\partial^3 x^\mu}{\partial x^\mu \partial x^{\mu(\sigma)} \partial x^{\mu(\lambda)}}-\frac{\partial^2}{\partial x^{\mu(\sigma)}\partial x^{\mu(\lambda)}}x^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\mu}\\
&-\frac{\partial}{\partial x^{\mu(\lambda)}}\frac{\partial x^\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\sigma)}}-\frac{\partial}{\partial x^{\mu(\lambda)}}x^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\sigma)}}\\
&+\frac{\partial}{\partial x^{\mu(\lambda)}}\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\sigma)}}+\frac{\partial}{\partial x^{\mu(\lambda)}}x^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\sigma)}}\\
&-\frac{\partial^2 x^\sigma}{\partial x^\mu \partial x^{\mu(\sigma)}}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\lambda)}}-\frac{\partial}{\partial x^{\mu(\sigma)}}\left(x^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\lambda)}}\\
&-\frac{\partial x^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\lambda)}}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\lambda)}}\right)\\
&+\frac{\partial x^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\lambda)}}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\lambda)}}\right)\\
&+\frac{\partial^2 x^\mu}{\partial x^\sigma \partial x^{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\lambda)}}\right)+\frac{\partial}{\partial x^{\mu(\sigma)}}\left(x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\lambda)}}\right)\\
&+\frac{\partial x^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\lambda)}}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\lambda)}}\right)\\
&-\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\lambda)}}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^{\mu(\lambda)}}\right)\\
&+\frac{\partial^2 x^\mu}{\partial x^\mu \partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)+\frac{\partial}{\partial x^\sigma}\left(x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\partial x^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)+x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)\\
&-\frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)-x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)\\
&=-\frac{\partial^3 x^{\mu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}\partial x^{\mu(\lambda)}}-\frac{\partial^2}{\partial x^{\mu(\sigma)}\partial x^{\mu(\lambda)}}x^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\\
&-\frac{\partial}{\partial x^{\mu(\lambda)}}\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\sigma)}}-\frac{\partial}{\partial x^{\mu(\lambda)}}x^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\sigma)}}\\
&+\frac{\partial}{\partial x^{\mu(\lambda)}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\sigma)}}+\frac{\partial}{\partial x^{\mu(\lambda)}}x^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\sigma)}}\\
&-\frac{\partial^2 x^{\nu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\lambda)}}-\frac{\partial}{\partial x^{\mu(\sigma)}}\left(x^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\lambda)}}\\
&-\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\lambda)}}\right)-x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\lambda)}}\right)\\
&+\frac{\partial x^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\lambda)}}\right)+x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\lambda)}}\right)\\
&+\frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\lambda)}}\right)+\frac{\partial}{\partial x^{\mu(\sigma)}}\left(x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\lambda)}}\right)\\
&+\frac{\partial x^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\lambda)}}\right)+x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\lambda)}}\right)\\
&-\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\lambda)}}\right)-x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu(\lambda)}}\right)\\
&+\frac{\partial^2 x^{\mu}}{\partial x^{\mu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)+\frac{\partial}{\partial x^{\nu}}\left(x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)\\
&+\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)+x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)\\
&-\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right)-x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu(\lambda)}}\right),
\end{aligned}
\tag{60}
$$

$$
\begin{aligned}
-x^{\mu}_{;\mu;\mu(\sigma);\mu(\lambda)}&=-\frac{\partial^3 x^{\mu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}\partial x^{\mu(\lambda)}}-\frac{\partial^2}{\partial x^{\mu(\sigma)}\partial x^{\mu(\lambda)}}x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\\
&-\frac{\partial}{\partial x^{\mu(\lambda)}}\frac{\partial x^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)-\frac{\partial}{\partial x^{\mu(\lambda)}}x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)\\
&+\frac{\partial}{\partial x^{\mu(\lambda)}}\frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu}}\right)+\frac{\partial}{\partial x^{\mu(\lambda)}}x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu}}\right)\\
&-\frac{\partial^2 x^{\sigma}}{\partial x^{\mu}\partial x^{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\sigma}}\right)-\frac{\partial}{\partial x^{\mu(\sigma)}}\left(x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\sigma}}\right)
\end{aligned}
$$

$$-\frac{\partial x^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\sigma}}\right)-x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\sigma}}\right)$$

$$+\frac{\partial x^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\sigma}}\right)+x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\sigma}}\right)$$

$$+\frac{\partial^{2} x^{\mu}}{\partial x^{\sigma}\partial x^{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu}}\right)+\frac{\partial}{\partial x^{\mu(\sigma)}}\left(x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu}}\right)$$

$$+\frac{\partial x^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu}}\right)+x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu}}\right)$$

$$-\frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu}}\right)-x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\sigma)}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu}}\right)$$

$$+\frac{\partial^{2} x^{\mu}}{\partial x^{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)+\frac{\partial}{\partial x^{\sigma}}\left(x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)$$

$$+\frac{\partial x^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)+x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)$$

$$-\frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)-x^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)$$

$$=-\frac{\partial^{3} x^{\mu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}\partial x^{\mu(\lambda)}}-\frac{\partial^{2}}{\partial x^{\mu(\sigma)}\partial x^{\mu(\lambda)}}x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)$$

$$-\frac{\partial}{\partial x^{\mu(\lambda)}}\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)-\frac{\partial}{\partial x^{\mu(\lambda)}}x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)$$

$$+\frac{\partial}{\partial x^{\mu(\lambda)}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu}}\right)+\frac{\partial}{\partial x^{\mu(\lambda)}}x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu}}\right)$$

$$-\frac{\partial^{2} x^{\nu}}{\partial x^{\mu}\partial x^{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\nu}}\right)-\frac{\partial}{\partial x^{\mu(\sigma)}}\left(x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\nu}}\right)$$

$$-\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\nu}}\right)-x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\nu}}\right)$$

$$+\frac{\partial x^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\nu}}\right)+x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\lambda)}}{\partial x^{\nu}}\right)$$

$$+\frac{\partial^{2} x^{\mu}}{\partial x^{\nu}\partial x^{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\lambda)}}{\partial x^{\mu}}\right)+\frac{\partial}{\partial x^{\mu(\sigma)}}\left(x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\lambda)}}{\partial x^{\mu}}\right)$$

$$+\frac{\partial x^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\lambda)}}{\partial x^{\mu}}\right)+x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\lambda)}}{\partial x^{\mu}}\right)$$

$$-\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\lambda)}}{\partial x^{\mu}}\right)-x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\sigma)}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\lambda)}}{\partial x^{\mu}}\right)$$

$$+\frac{\partial^{2} x^{\mu}}{\partial x^{\mu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)+\frac{\partial}{\partial x^{\nu}}\left(x^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)$$

$$+\frac{\partial x^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)+x^\nu\frac{1}{2}\left(\frac{\partial g^\nu_\mu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)$$
$$-\frac{\partial x^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right)-x^\nu\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_{\mu(\lambda)}}{\partial x^{\mu(\sigma)}}\right). \quad (61)$$

The problem of (58),(59) was solved in (60),(61). I get

$$-x^\mu_{;\mu;\mu(\sigma);\mu(\lambda)}=-\frac{\partial^3 x^\mu}{\partial x^\mu\partial x^{\mu(\sigma)}\partial x^{\mu(\lambda)}},$$
$$-x^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 x^\mu}{\partial x^\mu\partial x^\mu\partial x^\mu} \quad (62)$$

in consideration of Definision8 for (60),(61). I decide not to use the notation of Definision7 in (62) here. Because the second term of the right side doesn't exist in (62),

$$x^\mu_{;\nu;\nu;\nu}=\frac{\partial^3 x^\mu}{\partial x^\nu\partial x^\nu\partial x^\nu} \quad (63)$$

can rewrite (62) using Definision4. I rewrite (58),(59) using Definision2 and get

$$x^{\mu;\mu;\mu(\sigma);\mu(\lambda)}=\frac{\partial^3 x^\mu}{\partial x_\mu\partial x_{\mu(\sigma)}\partial x_{\mu(\lambda)}}+\frac{\partial^2}{\partial x_{\mu(\sigma)}\partial x_{\mu(\lambda)}}x^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu}$$
$$+\frac{\partial}{\partial x_{\mu(\lambda)}}\frac{\partial x^\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\sigma)}}+\frac{\partial}{\partial x_{\mu(\lambda)}}x^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\sigma)}}$$
$$-\frac{\partial}{\partial x_{\mu(\lambda)}}\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\sigma)}}-\frac{\partial}{\partial x_{\mu(\lambda)}}x^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\sigma)}}$$
$$+\frac{\partial^2 x^\sigma}{\partial x_\mu\partial x_{\mu(\sigma)}}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\lambda)}}+\frac{\partial}{\partial x_{\mu(\sigma)}}\left(x^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\lambda)}}$$
$$+\frac{\partial x^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\lambda)}}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\lambda)}}\right)$$
$$-\frac{\partial x^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\lambda)}}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\lambda)}}\right)$$
$$-\frac{\partial^2 x^\mu}{\partial x^\sigma\partial x_{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\lambda)}}\right)-\frac{\partial}{\partial x_{\mu(\sigma)}}\left(x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\lambda)}}\right)$$
$$-\frac{\partial x^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\lambda)}}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\lambda)}}\right)$$
$$+\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\lambda)}}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x_{\mu(\lambda)}}\right)$$

$$-\frac{\partial^2 x^\mu}{\partial x_\mu \partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_{\mu(\lambda)}}\right)-\frac{\partial}{\partial x^\sigma}\left(x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_{\mu(\lambda)}}\right)$$
$$-\frac{\partial x^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_{\mu(\lambda)}}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_{\mu(\lambda)}}\right)$$
$$+\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_{\mu(\lambda)}}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_{\mu(\lambda)}}\right)$$
$$=\frac{\partial^3 x^\mu}{\partial x_\mu \partial x_{\mu(\sigma)}\partial x_{\mu(\lambda)}}+\frac{\partial^2}{\partial x_{\mu(\sigma)}\partial x_{\mu(\lambda)}}x^\nu\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_\mu}$$
$$+\frac{\partial}{\partial x_{\mu(\lambda)}}\frac{\partial x^\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_{\mu(\sigma)}}+\frac{\partial}{\partial x_{\mu(\lambda)}}x^\nu\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_{\mu(\sigma)}}$$
$$-\frac{\partial}{\partial x_{\mu(\lambda)}}\frac{\partial x^\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\sigma)}}-\frac{\partial}{\partial x_{\mu(\lambda)}}x^\nu\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\sigma)}}$$
$$+\frac{\partial^2 x^\nu}{\partial x_\mu \partial x_{\mu(\sigma)}}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_{\mu(\lambda)}}+\frac{\partial}{\partial x_{\mu(\sigma)}}\left(x^\nu\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_{\mu(\lambda)}}$$
$$+\frac{\partial x^\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x_{\mu(\lambda)}}\right)+x^\nu\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x_{\mu(\lambda)}}\right)$$
$$-\frac{\partial x^\nu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x_{\mu(\lambda)}}\right)-x^\nu\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x_{\mu(\lambda)}}\right)$$
$$-\frac{\partial^2 x^\mu}{\partial x^\nu \partial x_{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\lambda)}}\right)-\frac{\partial}{\partial x_{\mu(\sigma)}}\left(x^\nu\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\lambda)}}\right)$$
$$-\frac{\partial x^\nu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\lambda)}}\right)-x^\nu\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\lambda)}}\right)$$
$$+\frac{\partial x^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\lambda)}}\right)+x^\nu\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_{\mu(\sigma)}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x_{\mu(\lambda)}}\right)$$
$$-\frac{\partial^2 x^\mu}{\partial x_\mu \partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_{\mu(\lambda)}}\right)-\frac{\partial}{\partial x^\nu}\left(x^\nu\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_{\mu(\lambda)}}\right)$$
$$-\frac{\partial x^\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_{\mu(\lambda)}}\right)-x^\nu\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_{\mu(\lambda)}}\right)$$
$$+\frac{\partial x^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_{\mu(\lambda)}}\right)+x^\nu\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_{\mu(\lambda)}}\right), \quad (64)$$

$$x^{\mu;\mu;\mu(\sigma);\mu(\lambda)}=\frac{\partial^3 x^\mu}{\partial x_\mu \partial x_{\mu(\sigma)}\partial x_{\mu(\lambda)}}+\frac{\partial^2}{\partial x_{\mu(\sigma)}\partial x_{\mu(\lambda)}}x^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)$$
$$+\frac{\partial}{\partial x_{\mu(\lambda)}}\frac{\partial x^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\sigma}\right)+\frac{\partial}{\partial x_{\mu(\lambda)}}x^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\sigma}\right)$$
$$-\frac{\partial}{\partial x_{\mu(\lambda)}}\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_\mu}\right)-\frac{\partial}{\partial x_{\mu(\lambda)}}x^\sigma\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_\mu}\right)$$

$$+\frac{\partial^2 x^\sigma}{\partial x_\mu \partial x_{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\sigma}\right)+\frac{\partial}{\partial x_{\mu(\sigma)}}\left(x^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\sigma}\right)$$

$$+\frac{\partial x^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\sigma}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\sigma}\right)$$

$$-\frac{\partial x^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\sigma}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\sigma}\right)$$

$$-\frac{\partial^2 x^\mu}{\partial x^\sigma \partial x_{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_\mu}\right)-\frac{\partial}{\partial x_{\mu(\sigma)}}\left(x^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_\mu}\right)$$

$$-\frac{\partial x^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_\mu}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_\mu}\right)$$

$$+\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_\mu}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_\mu}\right)$$

$$-\frac{\partial^2 x^\mu}{\partial x_\mu \partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_{\mu(\sigma)}}\right)-\frac{\partial}{\partial x^\sigma}\left(x^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_{\mu(\sigma)}}\right)$$

$$-\frac{\partial x^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_{\mu(\sigma)}}\right)-x^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_{\mu(\sigma)}}\right)$$

$$+\frac{\partial x^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_{\mu(\sigma)}}\right)+x^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\sigma}}{\partial x_{\mu(\sigma)}}\right)$$

$$=\frac{\partial^3 x^\mu}{\partial x_\mu \partial x_{\mu(\sigma)}\partial x_{\mu(\lambda)}}+\frac{\partial^2}{\partial x_{\mu(\sigma)}\partial x_{\mu(\lambda)}}x^\nu\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)$$

$$+\frac{\partial}{\partial x_{\mu(\lambda)}}\frac{\partial x^\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\nu}\right)+\frac{\partial}{\partial x_{\mu(\lambda)}}x^\nu\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\nu}\right)$$

$$-\frac{\partial}{\partial x_{\mu(\lambda)}}\frac{\partial x^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_\mu}\right)-\frac{\partial}{\partial x_{\mu(\lambda)}}x^\nu\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_\mu}\right)$$

$$+\frac{\partial^2 x^\nu}{\partial x_\mu \partial x_{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\nu}\right)+\frac{\partial}{\partial x_{\mu(\sigma)}}\left(x^\nu\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\nu}\right)$$

$$+\frac{\partial x^\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\nu}\right)+x^\nu\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\nu}\right)$$

$$-\frac{\partial x^\nu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\nu}\right)-x^\nu\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\mu}}{\partial x^\nu}\right)$$

$$-\frac{\partial^2 x^\mu}{\partial x^\nu \partial x_{\mu(\sigma)}}\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_\mu}\right)-\frac{\partial}{\partial x_{\mu(\sigma)}}\left(x^\nu\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_\mu}\right)$$

$$-\frac{\partial x^\nu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_\mu}\right)-x^\nu\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\mu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_\mu}\right)$$

$$+\frac{\partial x^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_\mu}\right)+x^\nu\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\sigma)\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_\mu}\right)$$

$$-\frac{\partial^2 x^\mu}{\partial x_\mu \partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_{\mu(\sigma)}}\right)-\frac{\partial}{\partial x^\nu}\left(x^\nu\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_{\mu(\sigma)}}\right)$$
$$-\frac{\partial x^\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_{\mu(\sigma)}}\right)-x^\nu\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_{\mu(\sigma)}}\right)$$
$$+\frac{\partial x^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_{\mu(\sigma)}}\right)+x^\nu\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu(\lambda)\nu}}{\partial x_{\mu(\sigma)}}\right). \quad (65)$$

The problem of (58),(59) was solved in (64),(65).

– End Proof –

# 5 About a Coordinate Transformations Equation in Tensor Satisfying Binary Law

**Proposition 9** *When all coordinate systems satisfies Binary Law, $x^\mu_\mu = x^\nu_\nu$ is established for $x^\mu_\mu$ components of a tensor satisfying Binary law of rank zero.*

Proof:When all coordinate systems satisfies Binary Law, I get

$$x^\mu_\mu = \frac{\partial x^\mu}{\partial x^\nu}\frac{\partial x^\nu}{\partial x^\mu}x^\nu_\nu = x^\nu_\nu. \quad (66)$$

from Definision15. Because (66) accords in Definision15, the components of a tensor of rank zero is equivalent with components of a tensor satisfying Binary law of rank zero. I adopt $x^\mu_\mu \to x^{;\mu}_\mu, x^\nu_\nu \to x^{;\nu}_\nu$,(18), $\mu,\nu$-inversion form of (18) for (66) and get

$$\frac{\partial x_\mu}{\partial x_\mu} - x_\nu\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\mu}\right) = \frac{\partial x_\nu}{\partial x_\nu} - x_\mu\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\nu}\right). \quad (67)$$

– End Proof –

**Proposition 10** *When all coordinate systems satisfies Binary Law, $x^{\mu\nu} = x^{\nu\mu}$ is established for $x^{\mu\nu}$ contravariant components of a tensor satisfying Binary law of the second rank.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$x^{\mu\nu} = \frac{\partial x^\mu}{\partial x^{\nu(\sigma)}}\frac{\partial x^\nu}{\partial x^{\nu(\lambda)}}x^{\nu(\sigma)\nu(\lambda)} \quad (68)$$

from Definision16 in consideration of Proposition3,Definision7. Because $\frac{\partial x^{\nu}}{\partial x^{\nu(\lambda)}}$ in (68) expresses conversion to the same coordinate systems, it is a problem. Therefore, I rewrite dummy index $\nu$ of (68) in $\mu$ and get

$$x^{\mu\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu(\sigma)}} \frac{\partial x^{\nu}}{\partial x^{\mu(\lambda)}} x^{\nu(\sigma)\mu(\lambda)},$$
$$x^{\mu\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\nu}}{\partial x^{\mu}} x^{\nu\mu} = x^{\nu\mu}. \tag{69}$$

I decide not to use the notation of Definision7 in (69) here. The problem mentioned above is solved in (69). I rewrite (69) using Definision2,Definision3 and get

$$x^{\mu}_{\mu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x_{\mu}}{\partial x_{\nu}} x^{\nu}_{\nu} = x^{\nu}_{\nu}. \tag{70}$$

I rewrite $x^{\mu\nu} = x^{\mu\nu}, x^{\nu\mu} = x^{\nu\mu}$ using Definision2,Definision3 and get

$$x^{\mu\nu} = x^{\mu}_{\mu}, x^{\nu\mu} = x^{\nu}_{\nu}. \tag{71}$$

I get (69) from (70),(71).

– End Proof –

**Proposition 11** *When all coordinate systems satisfies Binary Law, $x_{\mu\nu} = x_{\nu\mu}$ is established for $x_{\mu\nu}$ covariant components of a tensor satisfying Binary law of the second rank.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$x_{\mu\nu} = \frac{\partial x^{\nu(\sigma)}}{\partial x^{\mu}} \frac{\partial x^{\nu(\lambda)}}{\partial x^{\nu}} x_{\nu(\sigma)\nu(\lambda)} \tag{72}$$

from Definision17 in consideration of Proposition3,Definision7. Because $\frac{\partial x^{\nu(\lambda)}}{\partial x^{\nu}}$ in (72) expresses conversion to the same coordinate systems, it is a problem. Therefore, I rewrite dummy index $\nu$ of (72) in $\mu$ and get

$$x_{\mu\nu} = \frac{\partial x^{\nu(\sigma)}}{\partial x^{\mu}} \frac{\partial x^{\mu(\lambda)}}{\partial x^{\nu}} x_{\nu(\sigma)\mu(\lambda)},$$
$$x_{\mu\nu} = \frac{\partial x^{\nu}}{\partial x^{\mu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} x_{\nu\mu} = x_{\nu\mu}. \tag{73}$$

I decide not to use the notation of Definision7 in (73) here. The problem mentioned above is solved in (73). I rewrite (73) using Definision2,Definision3 and get

$$x^{\nu}_{\nu} = \frac{\partial x_{\mu}}{\partial x_{\nu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} x^{\mu}_{\mu} = x^{\mu}_{\mu}. \tag{74}$$

I rewrite $x_{\mu\nu} = x_{\mu\nu}, x_{\nu\mu} = x_{\nu\mu}$ using Definision2,Definision3 and get

$$x_{\mu\nu} = x^{\nu}_{\nu}, x_{\nu\mu} = x^{\mu}_{\mu}. \tag{75}$$

I get (73) from (74),(75). I adopt $x_{\mu\nu}{\to}x_{\mu;\nu}, x_{\nu\mu}{\to}x_{\nu;\mu}, x^{\mu}_{\mu}{\to}x^{;\mu}_{\mu}, x^{\nu}_{\nu}{\to}x^{;\nu}_{\nu}$,(18),(21), $\mu,\nu$-inversion form of (18),(21) for (73),(75) and get

$$\frac{\partial x_{\mu}}{\partial x^{\nu}} = \frac{\partial x_{\nu}}{\partial x^{\mu}}, \tag{76}$$

$$\frac{\partial x_{\mu}}{\partial x^{\nu}} = \frac{\partial x_{\nu}}{\partial x_{\nu}} - x_{\mu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right), \frac{\partial x_{\nu}}{\partial x^{\mu}} = \frac{\partial x_{\mu}}{\partial x_{\mu}} - x_{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^{\mu}}\right). \tag{77}$$

– End Proof –

**Proposition 12** *When all coordinate systems satisfies Binary Law,* $x^{\mu}_{\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\mu}$ *is established for* $x^{\mu}_{\nu}$ *components of the mixed tensor satisfying Binary law of the second rank.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$x^{\mu}_{\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu(\sigma)}}\frac{\partial x^{\nu(\lambda)}}{\partial x^{\nu}}x^{\nu(\sigma)}_{\nu(\lambda)} \tag{78}$$

from Definision18 in consideration of Proposition3,Definision7. Because $\frac{\partial x^{\nu(\lambda)}}{\partial x^{\nu}}$ in (78) expresses conversion to the same coordinate systems, it is a problem. Therefore, I rewrite dummy index $\nu$ of (78) in $\mu$ and get

$$x^{\mu}_{\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu(\sigma)}}\frac{\partial x^{\mu(\lambda)}}{\partial x^{\nu}}x^{\nu(\sigma)}_{\mu(\lambda)},$$

$$x^{\mu}_{\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\mu}. \tag{79}$$

I decide not to use the notation of Definision7 in (79) here. The problem mentioned above is solved in (79). I adopt $x^{\mu}_{\nu}{\to}x^{\mu}_{;\nu}, x^{\nu}_{\mu}{\to}x^{\nu}_{;\mu}$,(30), $\mu,\nu$-inversion form of (30) for (79) and get

$$\frac{\partial x^{\mu}}{\partial x^{\nu}} = \left(\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\right)\frac{\partial x^{\nu}}{\partial x^{\mu}}. \tag{80}$$

I get components indication

$$x^{1}_{1} = \frac{\partial x^{1}}{\partial \dot{x}^{1}}\frac{\partial x^{1}}{\partial \dot{x}^{1}}x^{1}_{1}, x^{1}_{2} = \frac{\partial x^{1}}{\partial \dot{x}^{2}}\frac{\partial x^{1}}{\partial \dot{x}^{2}}x^{2}_{1}.$$

$$x^{2}_{1} = \frac{\partial x^{2}}{\partial \dot{x}^{1}}\frac{\partial x^{2}}{\partial \dot{x}^{1}}x^{1}_{2}, x^{2}_{2} = \frac{\partial x^{2}}{\partial \dot{x}^{2}}\frac{\partial x^{2}}{\partial \dot{x}^{2}}x^{2}_{2} \tag{81}$$

for (79) if I assume the number of the dimensions 2.

– End Proof –

**Proposition 13** *When all coordinate systems satisfies Binary Law,* $x_{\mu\nu\nu} = \frac{\partial x^\mu}{\partial x^\nu} x_{\nu\mu\mu}$ *is established for* $x_{\mu\nu\nu}$ *covariant components of a tensor satisfying Binary law of the third rank..*

Proof:If all coordinate systems satisfies Binary Law, I get

$$x_{\mu\nu\nu(\sigma)} = \frac{\partial x^{\nu(\lambda)}}{\partial x^\mu} \frac{\partial x^{\nu(\iota)}}{\partial x^\nu} \frac{\partial x^{\nu(\epsilon)}}{\partial x^{\nu(\sigma)}} x_{\nu(\lambda)\nu(\iota)\nu(\epsilon)} \tag{82}$$

from Definision19 in consideration of Proposition3,Definision7. Because $\frac{\partial x^{\nu(\iota)}}{\partial x^\nu} \frac{\partial x^{\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}$ in (82) expresses conversion to the same coordinate systems, it is a problem. Therefore, I rewrite dummy index $\nu$ of (82) in $\mu$ and get

$$x_{\mu\nu\nu(\sigma)} = \frac{\partial x^{\nu(\lambda)}}{\partial x^\mu} \frac{\partial x^{\mu(\iota)}}{\partial x^\nu} \frac{\partial x^{\mu(\epsilon)}}{\partial x^{\nu(\sigma)}} x_{\nu(\lambda)\mu(\iota)\mu(\epsilon)},$$

$$x_{\mu\nu\nu} = \frac{\partial x^\nu}{\partial x^\mu} \frac{\partial x^\mu}{\partial x^\nu} \frac{\partial x^\mu}{\partial x^\nu} x_{\nu\mu\mu} = \frac{\partial x^\mu}{\partial x^\nu} x_{\nu\mu\mu}. \tag{83}$$

I decide not to use the notation of Definision7 in (83) here. The problem mentioned above is solved in (83). There is two same index in $x_{\mu\nu\nu}, x_{\nu\mu\mu}$ of (83). This is a problem. I rewrite (83) using Definision2,Definision3 and get

$$x^\nu_{\nu\nu} = \frac{\partial x_\mu}{\partial x_\nu} \frac{\partial x^\mu}{\partial x^\nu} \frac{\partial x^\mu}{\partial x^\nu} x^\mu_{\mu\mu} = \frac{\partial x^\mu}{\partial x^\nu} x^\mu_{\mu\mu}. \tag{84}$$

I rewrite $x_{\mu\nu\nu} = x_{\mu\nu\nu}, x_{\nu\mu\mu} = x_{\nu\mu\mu}$ using Definision2,Definision3 and get

$$x_{\mu\nu\nu} = x^\nu_{\nu\nu} = x_\nu, x_{\nu\mu\mu} = x^\mu_{\mu\mu} = x_\mu. \tag{85}$$

I get (83) from (84),(85). The problem mentioned above is solved in (85). I adopt $x_{\mu\nu\nu} \to x_{\mu;\nu;\nu}, x_{\nu\mu\mu} \to x_{\nu;\mu;\mu}$, (41), $\mu, \nu$-inversion form of (41) for (83),(85) and get

$$\frac{\partial^2 x_\mu}{\partial x^\nu \partial x^\nu} = \frac{\partial x^\mu}{\partial x^\nu} \frac{\partial^2 x_\nu}{\partial x^\mu \partial x^\mu}, \tag{86}$$

$$\frac{\partial^2 x_\mu}{\partial x^\nu \partial x^\nu} = x_\nu, \frac{\partial^2 x_\nu}{\partial x^\mu \partial x^\mu} = x_\mu. \tag{87}$$

– End Proof –

**Proposition 14** *When all coordinate systems satisfies Binary Law,* $x^\mu_{\nu\nu} = \frac{\partial x^\nu}{\partial x^\mu} x^\nu_{\mu\mu}$ *is established for* $x^\mu_{\nu\nu}$ *components of the mixed tensor satisfying Binary law of the third rank of the second rank covariant in the first rank contravariant.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$x^{\mu}_{\nu\nu(\sigma)}=\frac{\partial x^{\mu}}{\partial x^{\nu(\lambda)}}\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{\partial x^{\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}x^{\nu(\lambda)}_{\nu(\iota)\nu(\epsilon)} \tag{88}$$

from Definision20 in consideration of Proposition3,Definision7. Because $\frac{\partial x^{\nu(\iota)}}{\partial x^{\nu}}\frac{\partial x^{\nu(\epsilon)}}{\partial x^{\nu(\sigma)}}$ in (88) expresses conversion to the same coordinate systems, it is a problem. Therefore, I rewrite dummy index $\nu$ of (88) in $\mu$ and get

$$x^{\mu}_{\nu\nu(\sigma)}=\frac{\partial x^{\mu}}{\partial x^{\nu(\lambda)}}\frac{\partial x^{\mu(\iota)}}{\partial x^{\nu}}\frac{\partial x^{\mu(\epsilon)}}{\partial x^{\nu(\sigma)}}x^{\nu(\lambda)}_{\mu(\iota)\mu(\epsilon)},$$

$$x^{\mu}_{\nu\nu}=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\mu\mu}. \tag{89}$$

I decide not to use the notation of Definision7 in (89) here. The problem mentioned above is solved in (89). There is two same index in $x^{\mu}_{\nu\nu},x^{\nu}_{\mu\mu}$ of (89). This is a problem. I rewrite (89) using Definision4,Definision5 and get

$$x^{\mu}_{\mu\mu}=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{\partial x^{\nu}}{\partial x^{\mu}}x^{\nu}_{\nu\nu}=\frac{\partial x^{\nu}}{\partial x^{\mu}}x^{\nu}_{\nu\nu}. \tag{90}$$

I rewrite $x^{\mu}_{\nu\nu}=x^{\mu}_{\nu\nu},x^{\nu}_{\mu\mu}=x^{\nu}_{\mu\mu}$ using Definision4,Definision5 and get

$$x^{\mu}_{\nu\nu}=x^{\mu}_{\mu\mu}=x_{\mu},x^{\nu}_{\mu\mu}=x^{\nu}_{\nu\nu}=x_{\nu}. \tag{91}$$

I get

$$x^{\mu}_{\nu\nu}=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{\partial x^{\nu}}{\partial x^{\mu}}x^{\nu}_{\mu\mu} \tag{92}$$

from (90),(91). I get

$$\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}=\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{\partial x^{\nu}}{\partial x^{\mu}} \tag{93}$$

from (89),(92). If (93) assumes it false, I rewrite (93) using Definision5 and get

$$\frac{\partial x^{\nu}}{\partial x^{\nu}}\frac{\partial x^{\nu}}{\partial x^{\nu}}=\frac{\partial x^{\mu}}{\partial x^{\mu}}\frac{\partial x^{\mu}}{\partial x^{\mu}}\quad (False). \tag{94}$$

Because (94) isn't established, (93) is established. The problem mentioned above is solved in (91). I get

$$x^{\mu}_{\nu\nu}=\frac{\partial x^{\nu}}{\partial x^{\mu}}x^{\nu}_{\mu\mu} \tag{95}$$

from (89),(93). I adopt $x^{\mu}_{\nu\nu}\rightarrow x^{\mu}_{;\nu;\nu},x^{\nu}_{\mu\mu}\rightarrow x^{\nu}_{;\mu;\mu}$, (52), $\mu,\nu$-inversion form of (52) for (91),(95) and get

$$\frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\nu}}=x_{\mu},\frac{\partial^2 x^{\nu}}{\partial x^{\mu}\partial x^{\mu}}=x_{\nu}, \tag{96}$$

$$\frac{\partial^2 x^{\mu}}{\partial x^{\nu}\partial x^{\nu}}=\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{\partial^2 x^{\nu}}{\partial x^{\mu}\partial x^{\mu}}. \tag{97}$$

– End Proof –

**Proposition 15** *When all coordinate systems satisfies Binary Law, $x^{\mu}_{\nu\nu\nu} = x^{\nu}_{\mu\mu\mu}$ is established for $x^{\mu}_{\nu\nu\nu}$ components of the mixed tensor satisfying Binary law of the fourth rank of the third rank covariant in the first rank contravariant.*

Proof:If all coordinate systems satisfies Binary Law, I get

$$x^{\mu}_{\nu\nu(\sigma)\nu(\lambda)} = \frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}} \frac{\partial x^{\nu(\epsilon)}}{\partial x^{\nu}} \frac{\partial x^{\nu(\alpha)}}{\partial x^{\nu(\sigma)}} \frac{\partial x^{\nu(\beta)}}{\partial x^{\nu(\lambda)}} x^{\nu(\iota)}_{\nu(\epsilon)\nu(\alpha)\nu(\beta)} \tag{98}$$

from Definision21 in consideration of Proposition3,Definision7. Because $\frac{\partial x^{\nu(\epsilon)}}{\partial x^{\nu}} \frac{\partial x^{\nu(\alpha)}}{\partial x^{\nu(\sigma)}} \frac{\partial x^{\nu(\beta)}}{\partial x^{\nu(\lambda)}}$ in (98) expresses conversion to the same coordinate systems, it is a problem. Therefore, I rewrite dummy index $\nu$ of (98) in $\mu$ and get

$$x^{\mu}_{\nu\nu(\sigma)\nu(\lambda)} = \frac{\partial x^{\mu}}{\partial x^{\nu(\iota)}} \frac{\partial x^{\mu(\epsilon)}}{\partial x^{\nu}} \frac{\partial x^{\mu(\alpha)}}{\partial x^{\nu(\sigma)}} \frac{\partial x^{\mu(\beta)}}{\partial x^{\nu(\lambda)}} x^{\nu(\iota)}_{\mu(\epsilon)\mu(\alpha)\mu(\beta)},$$

$$x^{\mu}_{\nu\nu\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} x^{\nu}_{\mu\mu\mu} \tag{99}$$

I decide not to use the notation of Definision7 in (99) here. The problem mentioned above is solved in (99). There is three same index in $x^{\mu}_{\nu\nu\nu}, x^{\nu}_{\mu\mu\mu}$ of (99). This is a problem. I rewrite (99) using Definision4,Definision5 and get

$$x^{\mu}_{\mu\mu\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\nu}}{\partial x^{\mu}} \frac{\partial x^{\nu}}{\partial x^{\mu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} x^{\nu}_{\nu\nu\mu}. \tag{100}$$

I rewrite $x^{\mu}_{\nu\nu\nu} = x^{\mu}_{\nu\nu\nu}, x^{\nu}_{\mu\mu\mu} = x^{\nu}_{\mu\mu\mu}$ using Definision4,Definision5 and get

$$x^{\mu}_{\nu\nu\nu} = x^{\mu}_{\mu\mu\nu} = x_{\mu\nu}, x^{\nu}_{\mu\mu\mu} = x^{\nu}_{\nu\nu\mu} = x_{\nu\mu}. \tag{101}$$

I get (99) from (93),(100),(101). The problem mentioned above is solved in (101). I rewrite (101) using Definision2,Definision3 and get

$$x^{\mu}_{\nu\nu\nu} = x_{\mu\nu} = x^{\nu}_{\nu}, x^{\nu}_{\mu\mu\mu} = x_{\nu\mu} = x^{\mu}_{\mu}. \tag{102}$$

I get

$$x^{\mu}_{\nu\nu\nu} = x^{\nu}_{\mu\mu\mu} \tag{103}$$

from (93),(99). I adopt $x^{\mu}_{\nu\nu\nu} \rightarrow x^{\mu}_{;\nu;\nu;\nu}, x_{\mu\nu} \rightarrow x_{\mu;\nu}, x^{\nu}_{\nu} \rightarrow x^{;\nu}_{\nu}, x^{\nu}_{\mu\mu\mu} \rightarrow x^{\nu}_{;\mu;\mu;\mu}, x_{\nu\mu} \rightarrow x_{\nu;\mu}, x^{\mu}_{\mu} \rightarrow x^{;\mu}_{\mu}$,(18),(21),(63), $\mu, \nu$-inversion form of (18),(21),(63) for (102),(103) and get

$$\frac{\partial^3 x^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} = \frac{\partial x_{\mu}}{\partial x^{\nu}} = \frac{\partial x_{\nu}}{\partial x_{\nu}} - x_{\mu} \frac{1}{2} \left( \frac{\partial g^{\nu\mu}}{\partial x^{\nu}} \right),$$

$$\frac{\partial^3 x^{\nu}}{\partial x^{\mu} \partial x^{\mu} \partial x^{\mu}} = \frac{\partial x_{\nu}}{\partial x^{\mu}} = \frac{\partial x_{\mu}}{\partial x_{\mu}} - x_{\nu} \frac{1}{2} \left( \frac{\partial g^{\mu\nu}}{\partial x^{\mu}} \right), \tag{104}$$

$$\frac{\partial^3 x^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} = \frac{\partial^3 x^{\nu}}{\partial x^{\mu} \partial x^{\mu} \partial x^{\mu}}. \tag{105}$$

I get

$$\frac{\partial^3 x^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} = \frac{\partial x_\mu}{\partial x^\nu} = \frac{\partial x_\nu}{\partial x_\nu} - x_\mu \frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\nu}\right) = M, \tag{106}$$

$$\frac{\partial^3 x^\nu}{\partial x^\mu \partial x^\mu \partial x^\mu} = \frac{\partial x_\nu}{\partial x^\mu} = \frac{\partial x_\mu}{\partial x_\mu} - x_\nu \frac{1}{2}\left(\frac{\partial g^{\mu\nu}}{\partial x^\mu}\right) = M \tag{107}$$

from (104),(105),Definision9.

– End Proof –

# 6 Discussion

About Proposition4
The collecting of the dummy index $\sigma$ can consider two of (16) and (17) from (15). It is (16) that dummy index accords in Definision10. I decide not to handle (17) when dummy index doesn't accord in Definision10. A reason of the prohibition of the handling of (17) becomes clear by Proposition3. These coping considered it in the whole Chapter 4,Chapter 5 as well as Proposition4.

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**Received: April 25, 2021; Published: May 17, 2021**