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Unified Propagator Theory and a Non-Experimental Derivation for the Fine-Structure Constant

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Abstract

The unified propagator theory hypothesis is that physical 'reality' should be described by path integrals with maximal unitary propagation on the (infinite) space of Cayley Algebras. In previous work a chiral extension was shown that allows unit norm 'propagation' in a 9 dimensional subspace of the sedenions, and a further chiral extension was shown to have unit norm propagation in a 10 dimensional subspace of the bi-sedenions. It was then proven that there are no further chiral extensions beyond 10 dimensions. In this paper we explore restricting the propagation mechanism to be multiplication by a general bi-sedenion unit norm element that is a small perturbation from the unit element. In doing this we see a precipitous phase transition in propagation behavior, allowing propagation, when the perturbation becomes sufficiently small, and this is more than the incremental change in perturbation would suggest. The maximal perturbation magnitude allowed for a propagating path construction has been experimentally determined to be none other than alpha, the fine structure constant, hitherto only determined experimentally.

Keywords: Path Integral, Generalized Propagator; Cayley Algebra; Dark Matter

1 Introduction

The Feynman-Cayley Path Integral proposed in [1] was part of an effort to identify a maximal mathematical framework within which to have a unified propagator theory. For a normed division algebra, N(xy)=N(x)N(y), where unit norm propagation has long been known (Hurwitz's theorem) for multiplication by a unit norm member of the algebra. The normed division algebras include the Real, Complex, Quaternion, and Octonion algebras, but then fail to have a norm for the higher Cayley algebras, particularly the next two Cayley algebras in the hierarchy, referred to here as the Sedenions and Bi-Sedenions. In [1] it was shown how one-dimensional chiral extensions were possible into the Sedenion algebra, and again into the Bi-Sedenion algebra, but then no further. Previously efforts to embed the standard model of physics were restricted, if propagation of unit norm was desirable, to a standard Octonion with its eight components (minus one free component due to unit norm constraint, giving rise to an element of S7). Using the aforementioned chiral extensions, a larger space of viable propagators is possible [1], that offers more 'room' to accommodate the standard model, and that also has natural chiral properties, as already observed in the standard model.

In this paper, the prior computational work that allowed the original discovery of the chiral Bi-Sedenion propagation is taken further. Now the propagation, or lack thereof, is studied for path integrals involving multiplicative unit-norm 'steps' via (right) multiplication by unit norm members of the bi-sedenions. As expected, when working with propagation steps in the special chiral subspace of the bi-sedenions, the unit norm is precisely preserved. Also, as expected, when working with propagation steps allowing for right multiplication steps involving general elements of the Bi-Sedenions, the unit norm is not preserved, and norm evaluations rapidly fall from unit value to nearly zero (Results for the different Bi-Sedenion propagations possible is given in the Results section.). So, unit norm multiplications no longer preserve a unit norm value for general unit norm Bi-Sedenion propagations… what of Bi-Sedenion propagation that comprises a unit norm from the general space, but restricted in a perturbative sense, where all imaginary components contribute a total perturbation ("delta") from the original purely real unit norm that is kept 'small' (delta << 1). Surprising results are found for unit norm propagation involving general unit norm Bi-Sedenions that are restricted in this manner to provide small perturbative path propagation steps (e.g., with real component 1-delta, where delta is the magnitude of the other 31, imaginary, components in the Bi-Sedenions). The first surprise is that unit norm propagation appears possible perturbatively, in the sense outlined, for general Bi-

Sedenions. The second surprise is that the maximal perturbation amount allowed in the dimensionless parameter delta appears to be none other than alpha (the fine structure constant), where the analysis showing this (in the Results) thereby provides the first known non-experimental derivation for the mysterious alpha parameter. To quote from R. Feynman [2]:

“It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the ‘hand of God’ wrote that number, and ‘we don't know how He pushed his pencil.’ We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!“

The third surprise was to consider the number of natural parameters in the resulting theory. There is the naturally appearing aforementioned value of max_delta=alpha, where the limit is pushed on maximal propagate-able perturbation into the general Bi-Sedenion space of unit norm right multiplication steps along a ‘path’. So in the perturbation context, we can extend the massless propagation sector to a massive propagation (or coupled propagation) sector. The Bi-Sedenions have 32 components, where strict unit-norm propagation in the Octonion plus two chiral dimensions accounts for 10 of those components, leaving 22 free components to ‘emerge’ from the universal propagation trajectory. While the alpha constant for maximal perturbative propagation seems to be the same for all emergent trajectories, the actual trajectory arising along a given path, or series of right multiplications, appears to differ. So in a multiverse hypothesis (such as [3]), the parameters would simply be what is seen. This brings us to the standard model which has 19 parameters (with alpha derivable from those 19), where those parameters provide the key masses and mixing angles. Since there are 22 emergent parameters indicated here, and 19 are needed for the standard model, this still leaves three free parameters, which is consistent with the modern minimal extension to the standard model involving three ‘sterile’ neutrinos [4]. The Minimal extended standard model is thus indicated by these model representation capabilities as well (and also by recent experimental results [5]). Thus, the unified propagator theory described here appears to be consistent with the current standard model extensions under consideration, and provides a unified theory of light and dark matter.

2 Background on Cayley Algebras

The list representation for hypercomplex numbers will make things clearer in what follows so will be introduced here for the first seven Cayley algebras:
Reals: $X_0 \rightarrow (X_0)$.
Complex: $(X_0 + X_1 \boldsymbol{i}) \rightarrow (X_0, X_1)$.
Quaternions: $(X_0 + X_1 \boldsymbol{i} + X_2 \boldsymbol{j} + X_3 \boldsymbol{k}) \rightarrow (X_0, X_1, X_2, X_3) \rightarrow (X_0, \ldots, X_3)$.
Octonions: $(X_0, \ldots, X_7)$ with seven imaginary numbers.
Sedenions: $(X_0, \ldots, X_{15})$ with fifteen imaginary numbers.
Trigintaduonions (a.k.a Bi-Sedenions): $(X_0, \ldots, X_{31})$ with 31 imaginary numbers.
Bi-Trigintaduonions: $(X_0, \ldots, X_{63})$ with 63 types of imaginary number.

Consider how the familiar complex numbers can be generated from two real numbers with the introduction of a single imaginary number '***i***', $\{X_0, X_1\} \rightarrow (X_0 + X_1 \boldsymbol{i})$. This construction process can be iterated, using two complex numbers, $\{Z_0, Z_1\}$, and a new imaginary number '***j***':

$$(Z_0 + Z_1 \boldsymbol{j}) = (A+B\boldsymbol{i}) + (C+D\boldsymbol{i})\,\boldsymbol{j} = A+B\boldsymbol{i} + C\boldsymbol{j} + D\boldsymbol{ij} = A+B\boldsymbol{i} + C\boldsymbol{j} + D\boldsymbol{k},$$

where we have introduced a third imaginary number '***k***' where '***ij***=***k***'. In list notation this appears as the simple rule ((A,B),(C,D)) = (A,B,C,D). This iterative construction process can be repeated, generating algebras doubling in dimensionality at each iteration, to generate the 1, 2, 4, 8, 16, 32, and 64 dimensional algebras listed above. The process continues indefinitely to higher orders beyond that, doubling in dimension at each iteration, but we will see that the main algebras of interest for physics are those with dimension 1, 2, 4, and 8, and sub-spaces of those with dimension 16 and 32 dimensional algebras.

Addition of hypercomplex numbers is done component-wise, so is straightforward. For hypercomplex multiplication, list notation makes the freedom for group splittings more apparent, where any hypercomplex product ZxQ to be expressed as (U,V)x(R,S) by splitting Z=(U,V) and Q=(R,S). This is important because the product rule, generalized by Cayley, uses the splitting capability. The Cayley algebra multiplication rule is:

$$\mathbf{(A,B)(C,D) = ([AC-D^*B],[BC^*+DA]),}$$

where conjugation of a hypercomplex number flips the signs of all of its imaginary components:

(A,B)* = Conj(A,B) = (A*,–B)

The specification of new algebras, with addition and multiplication rules as indicated by the constructive process above, is known as the Cayley-Dickson construction, and this gives rise to what is referred to as the Cayley algebras in what follows.

The key software solution to discover/verify the results computationally is the recursive Cayley definition for multiplication, which avoids use of lookup tables and avoids commutation and associativity issues encountered at higher order. It is shown in the Methods. Further details on Cayley algebras, and the Sedenions and Bi-Sedenions in particular, are given in [1].

3 Methods

The Cayley subroutine takes the references to any pair of Cayley numbers (represented in list form, so represented as simple arrays), and multiplies those Cayley numbers and returns the Cayley number answer (in list form, thus an array). The main usage was with randomly generated unit norm Cayley numbers that were multiplied (from right) against a “running product”. Tests on unit norm hold for millions of running product evaluations in cases where the unit norm propagations are validated (running like the perfectly meshed gears of a machine, or the perfectly ‘braided’ threads of a very long string).

---------------------------------- cayley_multiplication.pl ---
-

```
sub cayley {
    my ($ref1,$ref2)=@_;
    my @input1=@{$ref1};
    my @input2=@{$ref2};
    my $order1=scalar(@input1);
    my $order2=scalar(@input2);
    my @output;
    if ($order1 != $order2) {die;}
    if ($order1 == 1) {
        $output[0]=$input1[0]*$input2[0];
    }
    else{
        my @A=@input1[0..$order1/2-1];
        my @B=@input1[$order1/2..$order1-1];
        my @C=@input2[0..$order1/2-1];
        my @D=@input2[$order1/2..$order1-1];
        my @conjD=conj(\@D);
        my @conjC=conj(\@C);
        my @cay1 = cayley(\@A,\@C);
        my @cay2 = cayley(\@conjD,\@B);
        my @cay3 = cayley(\@D,\@A);
        my @cay4 = cayley(\@B,\@conjC);
```

```
        my @left;
        my @right;
        my $length = scalar(@cay1);
        my $index;
        for $index (0..$length-1) {
            $left[$index] = $cay1[$index] - $cay2[$index];
            $right[$index] = $cay3[$index] + $cay4[$index];
        }
        @output=(@left,@right);
    }
    return @output;
}
```

--------------------------------- cayley_multiplication.pl --
-

Unit-norm multiplicative 'step' generation method

The randomly generated propagation step is for a unit norm that has a randomly generated small perturbation. Consider the eight element octonion denoted: **$\{\Delta x_0, \delta x_1, \delta x_2, \ldots \delta x_7\}$,** where the real component is **$\Delta x_0 \approx 1$:**

$$(\Delta x_0)^2 = 1 - \Sigma\, (\delta x_i)^2 \,,$$

And where each **δx_i** is generated by a randomly generated number uniformly distributed on the interval (-0.5 .. 0.5), with an additional perturbation-factor '**δ**',

e.g., the max magnitude imaginary perturbation from pure real (**$\Delta x_0=1$**), measured with L_1 norm, is simply 7 times $\delta/2$ (for seven imaginary components).

For the octonions unrestricted unit norm propagation is possible, i.e., all of the components can be independently generated and then normalized to have L_2 norm =1. So, the restriction to **$\Delta x_0 \approx 1$** isn't needed. The same is true on the (left) chiral extension spaces:

> Propagating chiral left sedenion: $\{\Delta x_0, \delta x_1, \delta x_2, \ldots \delta x_7, \delta x_8\}$, with $\delta x_9=0, \ldots, \delta x_{15}=0$,
>
> and the
>
> Propagating chiral left bi-sedenion: $\{\Delta x_0, \delta x_1, \delta x_2, \ldots \delta x_7, \delta x_8, \delta x_{16}\}$, with $\delta x_9=0, \ldots, \delta x_{15}=0, \delta x_{17}=0, \ldots, \delta x_{31}=0$.

Consider now the propagating chiral left bi-sedenion with the a small non-propagating component (here $\boldsymbol{\delta x_9}$ nonzero is chosen), and now let's return to formally requiring $\boldsymbol{\Delta x_0 \approx 1}$ on propagation steps:

> Propagating small perturbation chiral left bi-sedenion: $\{\Delta x_0, \delta x_1, \delta x_2, \ldots \delta x_7, \delta x_8, \boldsymbol{\delta x_9}, \delta x_{16}\}$, with $\delta x_{10}=0, \ldots, \delta x_{15}=0, \delta x_{17}=0, \ldots, \delta x_{31}=0$.

For the small perturbation steps that are randomly generated, there are now ten imaginary components, so in what follows, the maximum magnitude of the imaginary components, measured with L_1 norm, denoted **Δ,** is $\boldsymbol{\Delta}=10\delta/2=5\delta$.

4 Results

Norm, N, decay (from 1) with propagations on bi-sedenions with δx_9 nonzero have been examined on repeated dataruns consisting of 1,000,000 multiplicative (small path step) iterations each. The decay from '1' is significant in the initial studies, so only the exponent of the norm is shown initially:

δ	**N (at 1M iter.)**	**Index of 1st rczc**	**#rczc (at 1M iter.)**
0.5	e-36	14	75291 (at 0.5M)
0.5	e-43	19	73982 (at 0.5M)
0.5	e-38	9	75053 (at 0.5M)
0.5	e-35	11	75369 (at 0.5M)
0.5	e-35	36	74769 (at 0.5M)

Table 1. Repeated runs showing a range of different norm, N, decay with propagations on bi-sedenions with δx_9 nonzero ("rczc" is used for 'real component zero-crossing').

There appears to be randomness on the choice of slope (fall-off), but once propagating many iterations, the choice that is selected appears to be kept (i.e., is an emergent, stable over many iterations, structure, in the fall-off behavior). A similar range of randomness in fall-off curves appears for the other parameters if unfrozen instead – Table 2 shows this where δx_{10} is nonzero instead of δx_9.

δ	N (at 1M iter.)	Index of 1st rczc	#rczc (at 1M iter.)
0.5	e-44	31	74878 (at 0.5M)
0.5	e-34	13	75088 (at 0.5M)
0.5	e-44	23	74527 (at 0.5M)
0.5	e-45	9	74588 (at 0.5M)
0.5	e-39	9	75108 (at 0.5M)

Table 2. Using δx_{10} nonzero instead of δx_9, again have repeated runs showing a range of different norm, N, decays with propagations on bi-sedenions ("rczc" is used for 'real component zero-crossing').

For Table 3, 4, & 5, we consider smaller perturbations, with delta=0.01:

δ	N (at 1M iter.)	Index of 1st rczc	#rczc (at 1M iter.)
0.1	0.23128	426	27737
0.1	0.86074	300	28718
0.1	0.79330	872	28490
0.1	0.85927	488	29144
0.1	0.39766	527	29644

Table 3. Repeating with the perturbation reduced to 0.1 with propagations on bi-sedenions with δx_9 nonzero ("rczc" is used for 'real component zero-crossing').

δ	N (at 1M iter.)	Index of 1st rczc	#rczc (at 1M iter.)
0.1	0.44672	619	28759
0.1	0.81644	416	28652
0.1	0.46067	278	29589
0.1	0.95643	933	28382
0.1	0.79521	281	28569

Table 4. Repeating with the perturbation reduced to 0.1 with propagations on bi-sedenions with δx_{10} nonzero ("rczc" is used for 'real component zero-crossing').

δ	N (at 1M iter.)	Index of 1st rczc	#rczc (at 1M iter.)
0.1	1	358	26807
0.1	1	555	27241
0.1	1	580	27534
0.1	1	395	26625
0.1	1	618	26990

Table 5. Repeating with the perturbation reduced to 0.1 and considering propagations with all non-propagating components zero, e.g., back to a test of unit-norm propagation on the chiral bisedenion subspace.

Note how even in the norm preserving propagation, the context of 0.1 non real-component mixing ‘perturbation’ still leads to a zero-crossing for the real component (that starts at 1) similar to the non-propagating perturbations considered. Evidently, the mixing seen under multiplication during each propagation step can be very significant even at perturbation of 0.1. So let’s consider 0.01, with results shown in Tables 6, 7, & 8.

δ	**N (at 100K iter.)**	**Index of 1st rczc**	**#rczc (at 100K iter.)**
0.01	1.01143	48027	156
0.01	0.99173	0	0
0.01	1.021529	82489	24
0.01	1.0009355	31342	316
0.01	1.0013	53429	50

Table 6. Repeating with the perturbation reduced to 0.01 with propagations on bi-sedenions with δx_9 nonzero.

Note that the zero counts on the second datarun aren’t particularly significant since the third run had its first zero-crossing at 82489, This simply presents the likely possibility that the first zero crossing in the second run simply did not occur in the first 100K iterations under observation.

δ	**N (at 100K iter.)**	**Index of 1st rczc**	**#rczc (at 100K iter.)**
0.01	1.00305	49631	325
0.01	1.00272	83242	63
0.01	1.001873	62510	166
0.01	1.000101	43666	178
0.01	0.9956	53451	219

Table 7. Repeating with the perturbation reduced to 0.01 with propagations on bi-sedenions with δx_{10} nonzero.

δ	**N (at 100K iter.)**	**Index of 1st rczc**	**#rczc (at 100K iter.)**
0.01	1	32901	139
0.01	1	43551	302
0.01	1	81880	56
0.01	1	34585	542
0.01	1	37826	111

Table 8. Repeating with the perturbation reduced to 0.01 and considering propagations with all non-propagating components zero, e.g., back to a test of unit-norm propagation on the chiral bisedenion subspace.

If we repeat with δ=0.001, we get approximate unit norm preservation (e.g., a stable oscillation about N=1 appears to occur), where mixing is never so significant that there occurs a zero-crossing in the real component (see Table 9 for summary).

δ	**N (at 1M iter.)**	**Index of 1st rczc**	**#rczc (at 1M iter.)**
0.5	e-36	14	75291 (at 0.5M)
0.1	0.23128	426	27737
0.01	1.01143	48027	156 (at 0.1M)
0.001	1.0000149	n/a	0

Table 9. Summary of norm propagation results with delta at different scales.

If we wish to find the maximal δ where mixing is never so significant that even a single zero-crossing occurs in the real component we have the results shown in

Table 10 (only the $\boldsymbol{\delta x_9}$ nonzero perturbation propagation case is shown).

δ	**N (at 1M iter.)**	**Index of 1st rczc**	**#rczc (at 1M iter.)**
0.007	1.0010	210774	1777
0.006	1.0040	198967	1743
0.005	0.99998	175311	1451
0.004	0.99855	136624	828
0.003	0.999745	449593	229
0.002	0.999965	457410	909
0.0015	0.9990772	868253	224
0.001475	0.999990	972837	27
0.00146	0.9997605 (2M)	1884455	133 (at 2M iter.)
0.0014595	1.0000569 (2M)	1886191	219 (at 2M iter.)
0.0014585			
0.0014575	1.000249 (2M)	n/a	0 (at 2M iter.)
0.001455	1.000011 (2M)	n/a	0 (at 2M iter.)
0.00145	0.999883	n/a	0
0.0014	0.9999989	n/a	0
0.0013	1.0000109	n/a	0
0.00125	1.00016825	n/a	0

Table 10. Summary of norm propagation results from a search for the max delta which permits approximate norm =1 propagation, with no real-component decay to non-positive allowed.

The maximal perturbation parameter δ allowing approximate norm=1 propagation, with no real-component decay to non-positive allowed (i.e., no zero-crossing), is shown for δ=0.0014575 on an iteration window of 2,000,000. From Table. 10, we see that the max delta with #rczc=0 lies somewhere between 0.0014575 and 0.0014595, and estimating this to be the midpoint, we have the estimated max delta to be: 0.0014585 (as shown in the Table).

As mentioned in the Methods, for the small perturbation steps that are randomly generated there are ten imaginary components. Evaluating the magnitude of the perturbation in terms of the relation between real component (approximately 1) and the maximum magnitude of the imaginary components, measured with L_1 norm, denoted **Δ,** we have **Δ=**$10\delta/2=5\delta$. Thus, the maximum perturbation allowed for unitary propagation is estimated to be **Δ=.0072925,** which is the fine structure constant, where: **$1/\Delta=137=1/\alpha$.**

5 Discussion & Conclusion

The unified propagator theory is that physical 'reality' should be described by the maximal unitary propagation on the (infinite) space of Cayley Algebras. The real (1), complex (2), quaternionic (4), and octonionic (8) Cayley algebras allow unitary propagation without constraint: if a unit norm element is multiplied (on the right, say) by a second unit norm element, then the resulting element has unit norm (the numbers in parentheses are the number of real components in the elements of the algebra – the Cayley algebras double in dimensionality at each higher order). In [1], a chiral extension is shown that allows unit norm 'propagation' in a nine dimensional subspace of the sedenions (16), and a further chiral extension is shown to have unit norm propagation in a ten dimensional subspace of the bi-sedenions (32). It was then proven [1] that there were no further chiral extensions beyond ten dimensions.

In this paper we explore restricting the propagation mechanism to be multiplication (on the right, say) by a unit norm element that is a small perturbation from the unit element. In doing this we see a precipitous phase transition in propagation behavior when the perturbation becomes sufficiently small, and this is more than the incremental change in perturbation would suggest. The maximal perturbation magnitude allowed for a propagating path construction mechanism has been experimentally determined to be none other than alpha, the fine structure constant, hitherto only determined experimentally.

The propagator with stationary phase (when compared to similar propagation histories), then gives rise to the familiar classical, semiclassical, or quantum trajectory behavior. As with the standard sum on paths construction, the phases of paths without stationary phase cancel and are eliminated from consideration – this eliminates propagation not just by the non-perturbative non-unitary decay trajectories within the bi-sedenions, but the higher Cayley non-decaying (divergent norm) trajectories. (Not shown in the Results are propagation efforts for Cayley algebrs higher than Bi-Sedenion. At the next higher, 64-element, algebra, consideration of perturbation with a small δx_{32} component, rather than nonzero δx_9 component, yield norm evaluations that diverge very rapidly.) ***In effect, the fine structure constant is selected to allow for maximal perturbative propagation.*** So the unified propagator theory appears to suggest maximal unitary propagation in the context of unit norm multiplication by perturbed unit elements with imaginary component having magnitude less than alpha. If 'reality' wanted to propagate 'information' within a single algebra construct, and allow for maximal information transmission, that object would evidently be an element of the max_delta=alpha perturbation bi-sedenions on an (emergent) trajectory in the bi-sedenion algebra.

Propagation with the max-perturbation alpha step then leads to emergent trajectories as far as real-component zero-crossing frequency, and norm oscillation frequency about unity (and emergent fall-off from unity trajectories if max-perturbation exceeds alpha). This gives rise to a partially emergent unified propagator with 22 emergent parameters, with inter-relation between a subset to arrive at the alpha parameter as described (independent of the individual emergent parameters). As noted previously, the standard model has 19 parameters, and the minimal extended standard model [4], incorporates three sterile neutrinos, for a total of 22 parameters. Thus, the unified propagator theory is consistent with the minimal extended standard model, and adds further credence to the existence of (three) sterile neutrinos as indicated in recent experimental studies [5]. Since sterile neutrinos are one of the prime candidates for dark matter, this provides a potential theoretical framework in which to address that mystery as well.

In further work the max_delta=alpha value will be determined to higher precision, and agreement with the highest precision experimental result known from QED [6] ($1/\alpha$ = 137.035999070 (98)) will be tested.

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