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Cosmology: The Theoretical Possibility of Inverse Gravity as a Cause of Cosmological Inflation in an Isotropic and Homogeneous Universe and its Relationship to Weakly Interacting Massive Particles

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Abstract

It is postulated that cosmological expansion or inflation can be mathematically described by the inverse function of Newton's equation of gravitational force parameterized by a constant coefficient. This theoretical concept is referred to as the inverse gravity inflationary assertion (IGIA). After defining the core concept of the assertion, the theoretical notion is extended to the description of an isotropic and homogeneous universe. This extension is achieved by the derivation of gravitational redshift, the scale factors, Hubble's equations, the Friedman-Lemaitre-Walker-Robertson metric, and the Einstein tensor in terms of the IGIA. Thus, upon establishing the new theoretical assertion in terms of accepted and confirmed concepts that are consistent with astronomical observations in inflationary cosmology and general relativity; the assertion is applied to the widely accepted theoretical notion of weakly interacting massive particles (wimps). Hence, the IGIA is applied to weak interactions requiring the use of particle physics, quantum mechanics, and quantum field theory (QFT). The use of QFT emphatically incorporates the Feynman, Gell-Mann, Marshak, and Sudarshan constructions of the current by current Hamiltonian for the IGIA description of the beta decay process of weak interactions. Lastly, the IGIA's force and energy relationship between weak interactions on the subatomic scale and the expansion of the geometry of the universe as a whole is elucidated.

Keywords: Cosmological expansion, isotropic and homogeneous universe, Robertson-Walker scale factors, Friedman-Lemaitre-Walker-Robertson metric, Dark Energy, Hubble's constant, Einstein tensor, weakly interacting massive particles, beta decay, quantum field theory

Introduction

There are countless theories and assertions in the field of cosmology expressing many ideas on aspects of the nascent and current universe, some of which have been experimentally verified while others remain in the catacombs of speculation. The aspects of cosmology that remains highly speculative are the dynamics and cause of cosmological inflation and the corresponding energy value markedly referred to as dark energy. A new mathematical formulation is introduced where cosmological inflation is described by the inverse function of classical Newtonian gravitational force that is parameterized by a constant which permits the derivation of a unique expression of gravitational force and potential energy [1]. This theoretical notion is stated as the inverse gravity inflationary assertion (or IGIA) [1]. The IGIA accurately describes the general behavior of cosmological expansion in that it expresses that on local scales, the force of expansion is minute and negligible while traditional gravity is dominant [1]. Conversely, on cosmological scales, the IGIA describes an accelerating universe and thus the expansion of the geometry of the universe (i.e. the expansion of space-time) [1]. The aim of this paper is to introduce the notion, concept, and mathematical formulation of the IGIA. Secondly, the IGIA must show consistency with confirmed astronomical observations in the phenomena of cosmological inflation as well as a compatibility with established and accepted mathematical equations and expressions describing cosmological inflation. Therefore, the established aspects of cosmological expansion and inflation that will be defined in terms of the IGIA are gravitational redshift of a photon's wavelength, the Robertson-Walker scale factors, Hubble's law, Friedman-Walker-Robertson metric, and the Einstein tensor in the description of curved space-time thus incorporating general relativity [1]. Additionally, it is important to emphasize that the IGIA is defined within the frame work of an isotropic and homogeneous universe. Logically, the incorporation of the IGIA into established mathematical expressions describing cosmological expansion solidifies the assertion as a theoretical possibility. Lastly, the notion of dark matter in the form of wimps or weakly interacting massive particles is a widely accepted theoretical notion in the field of cosmology. Hence, the relationship of the IGIA to weak interactions between subatomic particles is defined. In the task of defining weakly interacting massive particles as a cause of cosmological inflation in terms of the IGIA, the IGIA is conceptually and mathematically incorporated to the beta decay process of weak interactions. A description of the beta decay process of weak interactions in terms of the IGIA requires the use of particle physics while incorporating quantum mechanics and quantum field theory (QFT). Hence, the relationships of energy, force, time, and the expansion of the geometry of space-time as they relate to both the subatomic and cosmological scale are defined. In reference to the use of QFT in the IGIA description of the beta decay process of weak interactions, there is an emphasis on the Feynman, Gell-Mann, Marshak and Sudarshan constructions of the current by current Hamiltonian formulated to describe weak interactions. Therefore, section 3 gives a complete description of the IGIA relationship to the theoretical wimp as a cause of cosmological expansion (and thus inflation).

1. Inverse gravity and the Newtonian correction

As communicated in the introduction, the validity and theoretical support of the inverse gravity assertion is predicated on the theoretical assertion's ability to be consistent with confirmed astronomical observations in cosmology such as gravitational redshift and other accepted mathematical descriptions which include the equations of general relativity. Therefore, in prudence and prior to introducing the core concept of the inverse gravity inflationary assertion (IGIA), we briefly present the mathematical exposition of the 6 primary aspects and established equations of cosmology to which the IGIA will be applied to in depth in the next section (section 2). As expressed in the introduction, the first phenomenon of cosmological expansion that the IGIA will be applied to is gravitational red shift z (or gravitational redshift of a photon's wavelength) of photons traversing a region of expanding space-time which is mathematically described by Eq.1 shown below [7].

$$z = \frac{\lambda_0}{\lambda_g} - 1 \tag{1}$$

Where λ_0 denotes the photon's initial wavelength prior to propagating through the region of expanding space and λ_g is the photon's wavelength after propagating through the region of expanding space, it follows that gravitational redshift z relates to the scale factors a_0 and $a(t_{em})$ by the known equation of [7][9]:

$$1 + z = \frac{a_0}{a(t_{em})} \tag{2}$$

Where the scale factor in gravitational redshift z of a_0 denotes the universe as it currently is and the scale factor of $a(t_{em})$ denotes the universe at the time of emission or the past. Thus in an isotropic and homogeneous universe, the velocity $V(t)$ of cosmological expansion is expressed in reference to *Hubble's Law* in terms of the scale factors a_0 and $a(t_{em})$ such that [9] [10]:

$$V(t) = \frac{R\dot{a(t)}}{a(t)} = \left[\frac{R}{a(t)}\right]\frac{da(t)}{dt} \tag{3}$$

Where R denotes the distance from the observer and $a(t)$ denotes the corresponding scale factor and $\dot{a(t)}$ denotes the first timed derivative of the scale factor ($\dot{a(t)} = da(t)/dt$); the expression $\dot{a(t)}/a(t)$ denotes *Hubble's constant* ($H_0 = \dot{a(t)}/a(t)$) [9] [10]. The expansion of the universe is then required to be described in terms of the curved geometry of space-time and hence general relativity as it relates to the scale factors. Conclusively, a space-time metric must be defined. The space-time metric corresponding to the geometric description of a homogeneous and isotropic universe is the Friedman-Walker-Robertson metric which is isotropic by a coefficient of the of the scale factor $a(t)$ being applied to (i.e. is a coefficient of) the space-like coordinates as shown below [9].

$$d\Sigma^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right] \tag{4}$$

Therefore expanding this description to Einstein's field equations in terms of the scale factor $a(t)$ which is pivotal to the description of a homogeneous and isotropic universe, the components of the Einstein tensor (G_{tt} and G_{**}) can be expressed in terms of scale factor $a(t)$ such that [9]:

$$G_{tt} = \frac{3(\dot{a(t)})^2}{a(t)^2} = R_{tt} + \frac{1}{2}R = 8\pi\rho \tag{5}$$

$$G_{**} = -2\frac{\ddot{a(t)}}{a(t)} - \frac{\left(\dot{a(t)}\right)^2}{a(t)^2} = R_{**} - \frac{1}{2}R = 8\pi P \tag{6}$$

Where R_{tt} and R_{**} denote the terms of the Ricci tensor, R is the scalar constant, P denotes pressure corresponding to thermal radiation, and ρ is the average mass density [9]. Cosmological expansion is primarily described by Eq.1-6. Conclusively, the inverse gravity inflationary assertion (IGIA) must describe an isotropic and homogeneous universe in the frame work of Eq. 1-6. As previously stated, section 2 will show the IGIA core concept's detailed incorporation into the aforementioned 6 aspects of cosmology and general relativity by deriving Eq. 1-6 and more in terms of the IGIA.

At this juncture, the core concept of the IGIA can be introduced. Thus we begin by defining the IGIA in terms of classical Newtonian Mechanics. Force value $F_g(r)$ denotes classical Newtonian gravitational force as shown by Eq.7 displayed below [10].

$$F_g(r) = \frac{Gm_1 m_2}{r^2} \tag{7}$$

Astronomical observations show that universal inflation or expansion progresses in opposition to gravitational force and thus accelerates cosmological mass and radiation in an inverse direction to gravity. This fact allows one to hypothesize and deduce that the mathematical description of the structure of the universe could inherently incorporate a mathematical term that is inversely proportional to classical Newtonian gravitational force [1]. Due to the fact that cosmological expansion is only detectable at cosmological distances, logically, this implies that the inverse force is parameterized. Thus, a mathematical term that is inversely proportional to gravity must have a parameter that pertains to spatial or astronomical distance [1]. This parameter will permit the inverse term to have substantial effects beyond a given distance and minimal affects below a given distance. Therefore, this gives to the theoretical assertion of an inverse term of gravitational force that is parameterized by a coefficient that is a constant pertaining to an astronomical distance r_0 (to be defined shortly) [1]. More specifically, the inverse gravity term is parameterized by the coefficient which has a value pertaining to the square of cosmological distance r_0 to which universal expansion takes prominence over gravity [1]. Therefore, the IGIA inverse term is denoted $F_g'(r)$ as expressed below [1].

$$F_g'(r) = \left[\frac{1}{r_0^2}\right]\left[\frac{r^2}{Gm_1\, m_2}\right] \tag{8}$$

Resultantly, classical Newtonian gravitational force $F_g(r)$ (of Eq.7) is subtracted from the parameterized inverse force term $F_g'(r)$ (of Eq.8), this difference is equivalent to and denoted as force $F_T(r)$ [1]. Gravitational force $F_T(r)$ combines the variations of classical Newtonian gravitation to the inverse variation of universal expansion [1]. Thus, the gravitational force value $F_T(r)$ below (Eq.9) is stated as the Newtonian correction due to the fact the Newtonian gravitational force is corrected to include the inverse term [1].

$$F_T(r) = F_g'(r) - F_g(r) = \left[\frac{1}{r_0^2}\right]\left[\frac{r^2}{Gm_1\, m_2}\right] - \frac{Gm_1\, m_2}{r^2} \tag{9}$$

Thus, total force $F_T(r)$ of Eq.9 above is composed of the inverse gravity force term denoted $F_g'(r)$ and the classical Newtonian gravitational force term denoted $F_g(r)$[1]. Masses m_1 and m_2 are the cosmological masses (such as galaxies and so on) and G is the gravitational constant. Where the cosmological coefficient of $1/r_0^2$ (or parameter) of inverse term $F_g'(r)$ is proportional to classical gravitational force $(Gm_1\, m_2/r^2)$, the coefficient in the inverse term of force $F_g'(r)$ in Eq. 8-9 is given with a condition such that [1]:

$$1 > \left[\frac{1}{r_0^2}\right] \tag{10}$$

The constant distance r_0 is referred to as the spatial cosmological parameter in reference to the IGIA [1]. The direction (+ or -) of the value of total force $F_T(r)$ has relationships defined by the inequalities of radius r to distance r_0 given by the conditions expressed below [1].

1. $For\ r > r_0\ ;\ +F_T(r)$ (11)

$$2.\quad For\ r < r_0\ ;\ -F_T(r) \tag{12}$$

Condition (1) (of Eq.11) describes cosmological expansion or force $+F_T(r)$ away from the gravitational force center (i.e. the center of the universe) for distances $r > r_0$ [1]. Conversely, for condition (2) (of Eq. 12), the inverse gravity term $F_g'(r)$ in total force $F_T(r)$ is negligible at distance $r < r_0$ causing force direction $-F_T(r)$ toward the center of gravitational force (i.e. gravity over takes the inverse term $F_g'(r)$ on $r < r_0$ which represents local scales) [1]. Figure 1 below pictorially depicts the model of the IGIA force relationship of cosmological inflation.

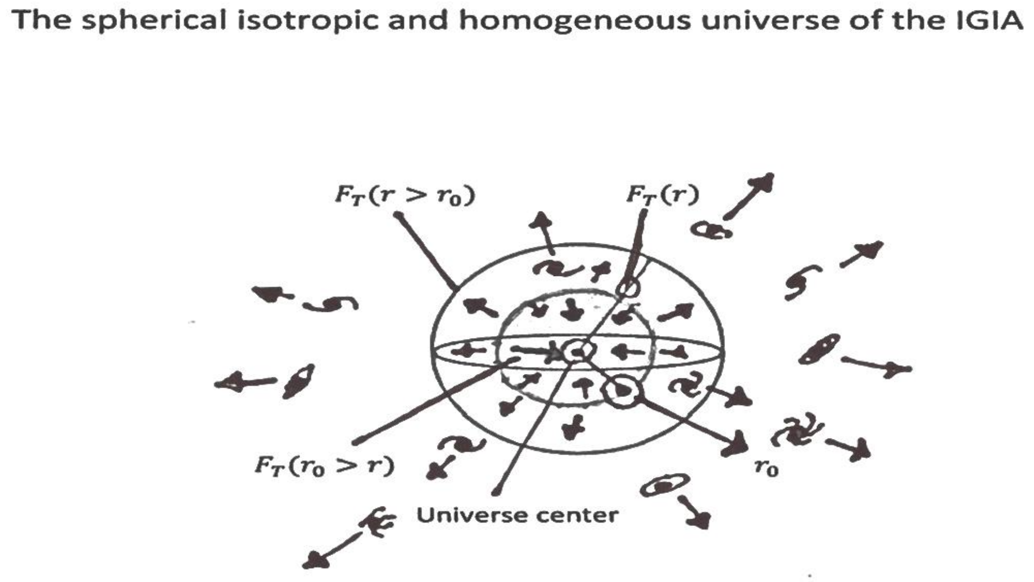

Figure 1

Figure 1 shows that the arrows directed <u>away</u> from the universe center indicate the motion of force value $+F_T(r)$ (or $F_T(r > r_0)$) which is cosmological expansion or inflation and the arrows directed <u>toward</u> the universe center indicate the motion of force value $-F_T(r)$ (or $F_T(r_0 > r)$) which constitutes gravity. Figure 1 clearly conveys that the core concept of the IGIA is consistent with the <u>general description</u> of the phenomenon of universal or cosmological inflation.

The value of the spatial cosmological parameter of distance r_0 is determined where total force value $F_T(r)$ (of Eq.9) equals zero and radius r equals cosmological parameter r_0 (i.e. $F_T(r_0) = 0$ and $r = r_0$) [1]. Furthermore, this implies that for the condition of $F_T(r_0) = 0$, the inverse force terms $F_g'(r_0)$ and the classical gravitational force term $F_g(r_0)$ of total force $F_T(r_0)$ (of Eq.9) at parametric distance r_0 are equal ($F_g'(r_0) = F_g(r_0)$) in [1]. Therefore, total force $F_T(r_0)$ can be presented such that [1]:

$$F_T(r_0) = F_g'(r_0) - F_g(r_0) = \left[\frac{1}{r_0^2}\right]\left[\frac{(r_0)^2}{Gm_1\, m_2}\right] - \frac{Gm_1\, m_2}{(r_0)^2} = 0 \tag{13}$$

Solving Eq. 13 for the spatial cosmological parameter of distance r_0 gives a value such that [1]:

$$r_0 = Gm_1\, m_2 \tag{14}$$

The uniform distribution of cosmological masses m_1 and m_2 over a spherically symmetric volume describing a homogeneous isotropic universe separated by a diameter of distance r_0 mathematically require the mass interaction $m_1\, m_2$ to be expressed as a triple integral such that [1]:

$$r_0 = Gm_1\, m_2 = G\left[\int_0^{m_u}\int_0^{\pi}\int_0^{\pi} m\left(m_{ц}' m_{ц}'\right) dm d\theta d\phi\right] \tag{15}$$

Where mass m_u (highlighted above) which is the upper limit of the integral with respect to variations in mass is the mass of the universe and thus constitutes all matter of the universe. Thus mass m_u is approximately 27% of the critical density of the universe commonly denoted as ρ_c [10].The critical density of matter denoted $(.27)\rho_c$ relates to the mass of the universe m_u and energy E via the relativistic energy equation of $E = m_u c^2 = (.27)\rho_c c^2$, therefore, universal mass m_u has a value such that [10]:

$$m_u = (.27)\rho_c \equiv (.27)\frac{3H_0^2}{8\pi G} \tag{16}$$

The value of critical density ρ_c is given as $3H_0^2/8\pi G$ (e.g. $\rho_c = 3H_0^2/8\pi G$) in terms of Hubble's constant H_0 and gravitational constant G [10]. Variable mass $m_{ц}'$ within Eq.15 is a function of spherical coordinates at $\theta$and ϕ and mass m such that [1]:

$$m_{ц}' = [(mcos\theta sin\phi)^2 + (msin\theta sin\phi)^2 + (mcos\phi)^2]^{1/2} \tag{17}$$

Where variable mass $m_{ц}'$ correspond to the symmetric variations in mass values m_1 and m_2 evenly distributed about the spherically symmetric volume, this implies that mass interaction $m_1\, m_2$ over a spherically symmetric volume is expressed by the triple integral (of Eq. 15) such that [1]:

$$m_1\, m_2 = \int_0^{m_u}\int_0^{\pi}\int_0^{\pi} m^3[(cos\theta sin\phi)^2 + (sin\theta sin\phi)^2 + (cos\phi)^2]\, dm d\theta d\phi \tag{18}$$

As indicated by Eq. 14 and Eq.15, masses m_1 and m_2 are spatially located on opposite sides of (and are separated by) parameter distance r_0, thus the continuous sums progress as a rotation where the mass values on opposite sides of distance r_0 rotate and sum up to universal mass value m_u[1]. Therefore, the rotation of continuous sums over the spherical coordinates at $\theta$and ϕ from zero to π encompass the entire spherical volume due to the fact that the rotation on one side of distance r_0 sum up to half the spherical volume (or π) and the other side of distance r_0 rotates and sum up to the other half (or π) of the spherical volume [1]; which equal the whole. Thus, Eq. 18 above gives the mass interaction $m_1\, m_2$ corresponding to the gravitational interaction of cosmological masses over distance r_0 [1].

2. General relativity and the cosmological expansion of an isotropic and homogeneous universe in terms of the IGIA

Section 1 defined the core concept of the inverse gravity inflationary assertion (IGIA), hence the objective of section 2 is to heuristically define the IGIA in terms Eq.1-6 (and more) which are consistent with actual astronomical observations and the description of an isotropic and homogeneous universe. In beginning the heuristic formulations, we begin with gravitational redshift. The gravitational red shift equation is expressed such that [7]:

$$z = \frac{U_T(r) - E_0}{E_0} = \frac{\left(\frac{Gm_1\, m_2}{r}\right) - E_0}{E_0} \tag{19}$$

Where E_0 is the initial energy of a photon ($E_0 = (hc/\lambda_0)$) propagating through a region of expanding space and $U_T(r)$ is gravitational potential energy [7] [10]. Gravitational potential energy is the integral of gravitational force $F_g(r)$ (of Eq.7) with respect to radial distance r as shown below [10].

$$U_T(r) = \int F_g(r)dr \;\; \equiv \;\; \frac{Gm_1\, m_2}{r} \tag{20}$$

Where force $F_g(r)$ equals Newtonian gravitational force expressed as:

$$F_g(r) = \frac{Gm_1\, m_2}{r^2} \tag{21}$$

Therefore, logically, the value of potential energy pertaining to the IGIA Newtonian correction force $F_T(r)$ (of Eq.9) is the integral of gravitational force $F_T(r)$ with respect to radial distance r as shown below [1] [10]:

$$U_T(r) = \int F_T(r)dr = \int \left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^2}{Gm_1\, m_2}\right] - \frac{Gm_1\, m_2}{r^2}\right] dr \tag{22}$$

Thus after evaluating the integral of Eq. 22 above, one obtains a value of potential energy $U_T(r)$ in terms of the IGIA such that [1]:

$$U_T(r) = \left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] + \frac{Gm_1\, m_2}{r} \tag{23}$$

We briefly express red shift z in terms of wavelength below which will be useful shortly.

$$z = \frac{U_T(r) - E_0}{E_0} = \frac{\left(\frac{hc}{\lambda_g}\right) - E_0}{E_0} \equiv \frac{\lambda_0}{\lambda_g} - 1 \tag{24}$$

Now substituting the value of Eq.23 into the gravitational red shift equation of Eq. 19 gives [7] [9]:

$$z = \frac{1}{E_0}\left(\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\,m_2}\right] + \frac{Gm_1\,m_2}{r}\right) - 1 \tag{25}$$

Prior to continuing the derivation, an **important clarification** must be expressed. It is paramount to justify the use of gravitational redshift z of Eq. 19 and Eq. 24-25 as opposed to using the *Schwarzchild expression of gravitational redshift* shown below [9].

$$\frac{\lambda_R}{\lambda_E} - 1 = \sqrt{\frac{1-2GM/r_R}{1-2GM/r_E}} - 1 \tag{i.}$$

Where λ_E and λ_R denote the initial and final wavelength values of a photon propagating through a gravitational potential; there are two purposes for not using the Schwarzchild equation of gravitational redshift above (i.) in the IGIA. The first is that in an FRW universe describing an isotropic and homogeneous model, cosmological expansion and redshift (z) is described in relation to the scale factors $a(t)$ (e.g. a_0 and $a(t_{em})$) (see Eq.29). Thus, a photon's wavelength, frequency, and energy are affected by the expansion of space-time itself. A relationship to the scale factors is not defined by the Schwarzchild equation of gravitational redshift as seen above. However, the IGIA postulates and describes the expansion of space-time or cosmological inflation as a result of a parameterized inversion of Newtonian gravitational force. Thus, the theoretical assertion must relate to the scale factors in a non-conventional manner that is not mathematically compatible with the Schwarzchild equation of gravitational redshift. Hence, the expansion of space-time in an FRW universe and the variations of the scale factors will be expressed in terms of theoretical potential energy value $U_T(r)$ of Eq. 23 (as will be shown in Eq.30). This leads to the second purpose for not using the Schwarzchild expression of gravitational redshift in the IGIA. The mass product of $m_1\,m_2$ of Eq.18 expressed in IGIA value of potential energy $U_T(r)$ (of Eq.23) are cosmological masses that are homogeneously and uniformly distributed about a spherical volume where the mass product $m_1\,m_2$ is expressed as triple integral as shown in section 1. In accordance to the IGIA, the expansion of space-time has a theoretical dependence on the evenly distributed cosmological masses of $m_1\,m_2$ which are separated by diametric distances within the spherical universe. Therefore, a photon propagating within the spherical volume of expanding space-time is affected by all of the cosmological mass combined which are evenly distributed through the spherical volume. Whereas the Schwarzchild equation of gravitational redshift of expression (i.) describes the redshift of a photon (of relativistic energy m_0c^2) traversing through a region of gravitational potential of a mass M of a single massive body where the test mass m_0 of the photon typically cancels out of the equations [9]. Additionally, the shift in the photon's wavelength due to a gravitational field in the frame work of a Schwarzchild space-time is described relative to or as a ratio of the distances r_E from the gravitational body's center of mass M and distance r_R from the gravitational center of the body to the propagating photon. Conversely, the IGIA distance r of potential energy $U_T(r)$ (and ladder expressions) is the diametric distance between cosmological masses as previously expressed in section 1.

Thus in continuing the formulation, the theoretical assertion must relate to the scale factors and thus the expression of gravitational redshift of Eq.29 (e.g. $a(t) = 1/(1+z)$) via an expression of gravitational potential energy (in order to be mathematically compatible to the description of an F-R-W universe). Therefore, as expressed in Eq.24, observe that IGIA Gravitational potential energy $U_T(r)$ is equal to the photon's energy hc/λ_g after propagating through expanding space ($U_T(r) = hc/\lambda_g$)[1]. This can be directly expressed such that:

$$\frac{hc}{\lambda_g} = U_T(r) \tag{26}$$

Recalling that E_0 is the initial energy of the photon ($E_0 = (hc/\lambda_0)$), Eq.24 relates the theoretical potential energy $U_T(r)$ to redshift z which relates to the scale factors of Eq.29. Conclusively, the value of the photonic energy affected by the IGIA potential energy field of $U_T(r)$ can be alternatively expressed such that [1]:

$$\frac{hc}{\lambda_g} = \left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] + \frac{Gm_1\, m_2}{r} \tag{27}$$

Where λ_g is the photon's wavelength influenced (or affected) by IGIA potential energy $U_T(r)$, wavelength λ_g has a value such that [1]:

$$\lambda_g = hc\left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] + \frac{Gm_1\, m_2}{r}\right]^{-1} \tag{28}$$

A full description of the IGIA's effects on a photon's energy and wavelength permits the assertion's incorporation to the scale factors. Thus, the next objective is to show the IGIA correlation to the scale factors which is a prelude to the description of the motion of space-time. As previously cited, red shift value z is related to the scale factor of the past (or the time of emission) denoted $a(t_{em})$ and the present denoted a_0 such that [7] [9]:

$$1 + z = \frac{a_0}{a(t_{em})} \tag{29}$$

Substituting the IGIA value of redshift z of Eq. 25 into Eq. 29 above gives [1]:

$$1+\left(\left(\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{r}\right]\right)-1\right)=\frac{a_0}{a(t_{em})} \tag{30}$$

This reduces to [1]:

$$\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{r}\right]=\frac{a_0}{a(t_{em})} \tag{31}$$

If follows that the value of the scale factor $a(t_{em})$ at the time t_{em} of the photon's emission is expressed such that [1]:

$$a(t_{em})=a_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{r}\right]\right]^{-1} \tag{32}$$

Where Eq.32 is of the form $a(t)=1/(1+z)$ [7] which implies that scale factor a_0 equals 1 ($a_0=1$). Recall that a_0 is the scale factor of the universe as it is presently and $a(t_{em})$ is the scale factor at the emission time t_{em} of the photon (or a scale factor of the universe as it was in the past as some authors state it)[1][7]. Eq.32 adequately shows the relationship of the Robertson-Walker scale factors and gravitational redshift to the IGIA [9]. In an isotropic model, the scale factors influence variations in the space-like components of space-time. Therefore, at this juncture, the IGIA mathematical connection to Friedman-Lemaitre-Walker-Robertson metric $d\Sigma^2$ can be conveyed. The Friedman-Lemaitre-Walker-Robertson metric is shown below [9].

$$d\Sigma^2=-dt^2+a^2(t)\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2sin^2\theta d\phi^2\right] \tag{33}$$

Where k in Eq.33 above is equal to +1 ($k=+1$) for the positive spherical curvature of space-time for the description of expanding space-time, the scale factor $a(t)$ in Eq.33 above is set equal to the scale factor $a(t_{em})$ of Eq.32 (i.e. $a(t)=a(t_{em})$) giving Eq.34 such that (note: for our purposes $t=t_{em}$) [1] [9]:

$$d\Sigma^2=-dt^2+a^2(t_{em})\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2sin^2\theta d\phi^2\right] \tag{34}$$

Substituting the value of scale factor $a(t_{em})$ (Eq.32) into Eq. 34 above gives the Friedman-Walker-Robertson metric in terms of the IGIA such that [1]:

$$d\Sigma^2=-dt^2+\left[a_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{r}\right]\right]^{-1}\right]^2\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2sin^2\theta d\phi^2\right] \tag{35}$$

Observe that the space-like coordinates of the isotropic metric of $d\Sigma^2$ with the inclusion of the scale factor $a(t)$ are dependent on variations of radial distance r. The scale factor at time t denoted $a(t)$ is equal to the value of scale factor $a(t_{em})$ of Eq.32 ($a(t) = a(t_{em})$) in terms of the IGIA [1]. The metric of Eq.35 is the first step towards describing the IGIA in terms of curved space-time geometry. Therefore as a clarification, this equivalence can be stated such that [1]:

$$a(t) = a(t_{em}) = a_0 \left[\frac{1}{E_0} \left[\left[\frac{1}{r_0^2} \right] \left[\frac{r^3}{3Gm_1\, m_2} \right] + \frac{Gm_1\, m_2}{r} \right] \right]^{-1} \tag{36}$$

Observe that Eq. 36 or the IGIA scale factor is a function of radius r, which relates to the Minkowski coordinates (t, x, y, z) for signature (- + + +)such that [9]:

$$r = [-t^2 + x^2 + y^2 + z^2]^{1/2} \qquad \rightarrow \qquad t^2 \leq x^2 + y^2 + z^2 \tag{37}$$

Substituting the value of Eq. 37 into Eq. 36 (or Eq. 32) gives the IGIA scale factor as a function of the Minkowski coordinates (denoted $a(t, x, y, z)$) such that [1][9]:

$$a(t_{em}) = a(t, x, y, z) = a_0 \left[\frac{1}{E_0} \left[\left[\frac{1}{r_0^2} \right] \left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\, m_2} \right] + \frac{Gm_1\, m_2}{[-t^2+x^2+y^2+z^2]^{1/2}} \right] \right]^{-1} \tag{38}$$

The time derivative of scale factor $a(t_{em})$ is denoted $a(t, \dot{x}, y, z)$ (where $a(t, x, y, z)$ is simply scale factor (t_{em})) in respect to the time coordinate t (keep in mind that $t = t_{em}$) of the Minkowski coordinates and can be expressed such that [1][9]:

$$a(t, \dot{x}, y, z) = \frac{da(t,x,y,z)}{dt} = \frac{\partial}{\partial t} \left[a_0 \left[\frac{1}{E_0} \left[\left[\frac{1}{r_0^2} \right] \left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\, m_2} \right] + \frac{Gm_1\, m_2}{[-t^2+x^2+y^2+z^2]^{1/2}} \right] \right]^{-1} \right] \tag{39}$$

In implementing <u>the chain rule</u>, the first time derivative $(da(t, x, y, z))/dt)$ of the IGIA scale factor denoted $a(t, \dot{x}, y, z)$ gives a value such that:

$$a(t, \dot{x}, y, z) = 2ta_0 \left[\frac{1}{E_0} \left[\left[\frac{1}{r_0^2} \right] \left[-\frac{(-t^2 + x^2 + y^2 + z^2)^{1/2}}{2Gm_1\, m_2} \right] + \frac{Gm_1\, m_2}{2[-t^2 + x^2 + y^2 + z^2]^{3/2}} \right] \right] \left[\frac{1}{E_0} \left[\left[\frac{1}{r_0^2} \right] \left[\frac{(-t^2 + x^2 + y^2 + z^2)^{3/2}}{3Gm_1\, m_2} \right] + \frac{Gm_1\, m_2}{[-t^2 + x^2 + y^2 + z^2]^{1/2}} \right] \right]^{-2} \tag{40}$$

Eq. 38 and 40 afford the opportunity to briefly present Hubble's constant in terms of the IGIA such that [1][9][10]:

$$H(t) = \frac{\dot{a}}{a} = \left[\frac{1}{a(t,x,y,z)}\right]\frac{da(t,x,y,z)}{dt} \tag{41}$$

Therefore, Hubble's constant takes on a value in terms of the IGIA such that:

$$H(t) = 2ta_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[-\frac{(-t^2+x^2+y^2+z^2)^{1/2}}{2Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{2[-t^2+x^2+y^2+z^2]^{3/2}}\right]\right]\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{[-t^2+x^2+y^2+z^2]^{1/2}}\right]\right]^{-1} \tag{42}$$

Thus in an isotropic and homogeneous universe, the $V(t)$ velocity of cosmological expansion is expressed in reference to *Hubble's Law* such that [9] [10]:

$$V(t) = \frac{R\dot{a}(t)}{a(t)} = \left[\frac{R}{a(t)}\right]\frac{da(t)}{dt} \tag{43}$$

Where R is the distance from the observer, the velocity of expansion $V(t)$ in terms of the IGIA is given such that [10]:

$$V(t) = 2tRa_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[-\frac{(-t^2+x^2+y^2+z^2)^{1/2}}{2Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{2[-t^2+x^2+y^2+z^2]^{3/2}}\right]\right]\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{[-t^2+x^2+y^2+z^2]^{1/2}}\right]\right]^{-1} \tag{44}$$

Where the second time derivative of the IGIA scale factor is expressed such that $\ddot{a}(t,x,y,z) = d^2(a(t,x,y,z))/dt^2$ [1] [9], the IGIA scale factor (Eq. 38) and its time valued differential terms affords the expression of the mathematical relationship to Einstein's field equations; specifically the Einstein tensor for describing an isotropic and homogeneous universe as presented by Wald [1] [9]. This permits the IGIA to be described in reference to the curved geometry of space-time and thus General relativity. Therefore, in commencing the IGIA's incorporation to Einstein's field equation; according to Wald [9], the scale factors $a(t,x,y,z)$ (keep in mind that $a(t) = a(t_{em}) = a(t,x,y,z)$ of Eq.38) relate to the symmetric Christoffel symbols (and thus a geodesic) such that [1] [9]:

$$\Gamma^t_{xx} = \Gamma^t_{yy} = \Gamma^t_{zz} = a(t,x,y,z)\dot{a}(t,x,y,z) \tag{45}$$

$$\Gamma^x_{xt} = \Gamma^x_{tx} = \Gamma^y_{ty} = \Gamma^y_{yt} = \Gamma^z_{zt} = \Gamma^z_{tz} = \dot{a}(t,x,y,z)/a(t,x,y,z) \tag{46}$$

Thus we acknowledge that the value of the symmetric Christoffel symbols Γ^a_{bc} are of the form [1] [9]:

$$\Gamma^a_{bc} = \frac{1}{2}\sum_d g^{ad}\left\{\frac{\partial g_{cb}}{\partial x^b} + \frac{\partial g_{ca}}{\partial x^c} - \frac{\partial g_{bc}}{\partial x^c}\right\} \tag{47}$$

Where $g_{\text{ц}v}$ is the metric tensor and $g^{\text{ц}v}$the inverse metric tensor, the components of the Ricci tensor are calculated according to the equation of [1] [9]:

$$R_{ab} = \sum_c R_{acb}{}^{c} \tag{48}$$

Resultantly, as expressed by Wald [9], the Ricci tensor can be expressed in terms of the Christoffel symbols Γ^a_{bc} such that [1] [9]:

$$R_{ab} = \sum_c \frac{\partial y}{\partial x^c}\Gamma^c_{ab} - \frac{\partial}{\partial x^a}\left(\sum_c \Gamma^c_{cb}\right) + \sum_{d,c}\left(\Gamma^d_{ab}\Gamma^c_{dc} - \Gamma^d_{cb}\Gamma^c_{da}\right) \tag{49}$$

According to Wald [9], the Ricci tensor values of R_{tt} and R_{**} are then related to the scale factor $a(t,x,y,z)$ in <u>terms of the IGIA</u> by the equations of (where $\ddot{a} = d^2(a(t,x,y,z))/dt^2$) [1][9]:

$$R_{tt} = -3a(t,\ddot{x},y,z)/a(t,x,y,z) \tag{50}$$

$$R_{**} = a(t,x,y,z)^{-2}R_{xx} = \frac{a(t,\ddot{x},y,z)}{a(t,x,y,z)} + 2\frac{(a(t,\dot{x},y,z))^2}{a(t,x,y,z)^2} \tag{51}$$

As stated by Wald, the value of Ricci tensor R_{xx} in Eq.51 above relates to the Christoffel symbol via the form [1] [9]:

$$R_{xx} = \sum_c \frac{\partial y}{\partial x^c}\Gamma^c_{xx} - \frac{\partial}{\partial x^x}\left(\sum_c \Gamma^c_{cx}\right) + \sum_{d,c}\left(\Gamma^d_{xx}\Gamma^c_{dc} - \Gamma^d_{cx}\Gamma^c_{dx}\right) \tag{52}$$

Which correspond to the expressions of the Ricci tensor R_{tt} and R_{**}. It is important to state that the value of the scalar curvature *R* is given such that [9] [10]:

$$R = -R_{tt} + 3R_{**} \tag{53}$$

Substituting the value of Eq.50 and Eq.51 into Eq.53 give a value such that [1] [9]:

$$R = -R_{tt} + 3R_{**} = 6\left(\frac{a(t,\ddot{x},y,z)}{a(t,x,y,z)} + \frac{(a(t,\dot{x},y,z))^2}{a(t,x,y,z)^2}\right) \tag{54}$$

Conclusively as stated by Wald [9], the values of the Einstein tensor denoted G_{tt} and G_{**} are given such that [1] [9]:

$$G_{tt} = \frac{3(a(t,\dot{x},y,z))^2}{a(t,x,y,z)^2} = R_{tt} + \frac{1}{2}R = 8\pi\rho \tag{55}$$

$$G_{**} = -2\frac{a(t,\ddot{x},y,z)}{a(t,x,y,z)} - \frac{\left(a(t,\dot{x},y,z)\right)^2}{a(t,x,y,z)^2} = R_{**} - \frac{1}{2}R = 8\pi P \tag{56}$$

Due to the fact that the description of the IGIA is defined in reference to a homogeneous and isotropic universe, the general evolutions for a isotropic and homogeneous universe as defined by Wald [9] and in respect to the IGIA scale factors are given such that [1] [9]:

$$\frac{3(a(t,\dot{x},y,z))^2}{a(t,x,y,z)^2} = 8\pi\rho - \frac{3k}{a(t,x,y,z)^2} \quad (57)$$

$$\frac{3a(t,\ddot{x},y,z)}{a(t,x,y,z)} = -4\pi(\rho + 3P\ ) \quad (58)$$

Where P denotes pressure corresponding to thermal radiation pressure and ρ is the average mass density [9], the scale factors $a(t,x,y,z)$ and their corresponding time derivatives ($a(t,\dot{x},y,z)$ and $a(t,\ddot{x},y,z)$) can be defined in terms of the IGIA scale factor of Eq. 38); constant k is equal to +1 ($k = +1$) and $r > r_0$ for positive spherical curvature describing the expansion of the cosmological fluid in a homogeneous isotropic universe [1][9]. This shows the complete mathematical incorporation of the IGIA to the mathematical description of a homogeneous and isotropic universe and achieves the goal of defining the IGIA in terms of curved space-time which is consistent with General relativity.

3. The IGIA relationship to dark energy and Weakly interacting massive particles (wimps)

A highly accepted theoretical notion to explain cosmological inflation and thus the nature of dark matter and dark energy is that of a “wimp” or weakly interacting massive particle [10]. A wimp which composes dark matter, is required to have minute and weak interactions on localized or subatomic scales and conversely substantial effects on a cosmological scale [10]. Currently, there are many candidates that can potentially satisfy the requirement of the elusive theoretical fermions that are virtually undetectable and miniscule but however profoundly defy gravity on a cosmological scale resultantly accelerating and expanding the universe [10]. Theoretically, the description of the IGIA coalesces with the assertion of weakly interacting massive particles (wimps). In showing the correspondence of the IGIA to the theoretical notion of weakly interacting massive particles (and thus dark energy and dark matter) in reference to cosmological expansion, two descriptions must be given. The first description is the IGIA correlation to weak interactions between elementary particles and the expansion of space-time on the subatomic scale and thus the quantum realm which require the use of quantum field theory. The second description is the IGIA correlation between the subatomic scale and the cosmological scale. Thus we commence with the first description below.

I. The subatomic and quantum description of the IGIA in weak interactions

Hitherto, we begin with the first description which is the relationship between the IGIA and weakly interacting elementary particles and hence weak interactions. Weak interactions are governed by the beta decay of elementary particles. Therefore, in demonstrating the effects of the IGIA on the subatomic scale, the effects of the IGIA will be localized to a weak interaction within a nucleus of an atom. The theoretical weak interactions that are responsible for cosmological expansion implicitly permeate the universe and therefore encompass the nucleus of an atom. Hence, as an example of the nuclear process of beta decay in terms of the IGIA, the scenario of when a neutron is released from the nucleus is expressed. As the neutron exits the nucleus, the neutron (n) transitions into a proton (p), an electron (or beta minus particle) (β^-), and an antineutrino ($\bar{v}_e$) as shown by the process below [3] [10].

$$n \rightarrow p + \beta^- + \bar{v}_e \tag{59}$$

The mediating particles for the weak interaction are the W^-, W^+ and Z^0 bosons which are spin 1 particles and have mass values of 80.4GeV/c^2 and 91.2.4GeV/c^2 [10]. Figure 1 below gives the corresponding Feynman diagram describing the fundamental beta decay process of Eq. 59.

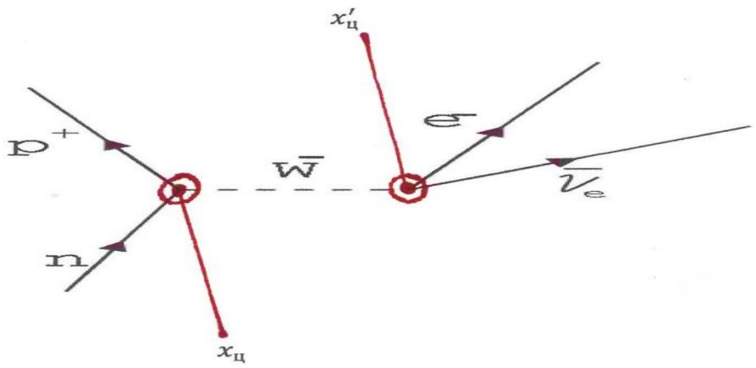

Figure 2

(Image credit to http://pfnicholls.com/physics/particles4.html)

The range of interactions are approximately .001 femto-meters (fm) which is the limit of distance between spatial position values $x'_ц$ and $x_ц$ ($x_ц$, $x'_ц \in R^3$) displayed in figure 2 [10]. The position values $x'_ц$ and $x_ц$ are the positions of transition of the particles in three dimensional space and signify where the particles are created and destroyed (or annihilated). The interaction energy between the particles is denoted E_{med}(pertaining to the energy of mediating particles, i.e. the W^-, W^+ and Z^0 bosons) which is equal to relativistic energy value $E = pc$ (where p denotes the momentum of the mediating particle and c denotes the velocity of light) [10]. Therefore, the energy value of E_{med} pertaining to the mediating particles can be expressed such that:

$$E_{med} = pc \tag{60}$$

Recall the equation of gravitational potential energy $U_T(r)$ of Eq.23, the inverse term of IGIA potential energy $U_T(r)$ Highlighted below essentially describes the energy of expansion.

$$U_T(r) = \left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] + \frac{Gm_1\, m_2}{r} \tag{61}$$

The inverse portion of Eq.61 highlighted above is the IGIA's interpretation of dark energy as expressed below [1].

$$E_{dark} = \left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] \tag{62}$$

Where energy E_{dark} is a function of cosmological level distance r, energy E_{dark} is a close approximation to the j ($j \in Z^+$ where Z^+ denotes the set of positive integers)sums of interaction energies E_{medj} of the mediating particles (or the W^-, W^+, and Z^0 bosons); where j is the number of beta decay interactions within a given region. Thus, energy E_{dark} relates to the j sums of interaction energies E_{medj} by Eq. 63 (see Eq.132) below.

$$E_{dark} \approx \sum_j E_{medj} \tag{63}$$

Thus a single or individual value of interaction energy E_{med} (or the energy of the mediating W^-, W^+, and Z^0 bosons) is generalized as equal to the inverse term of IGIA potential energy at subatomic lengths (or $|x'_ц - x_ц|$) which is equivalent to relativistic energy pc as expressed by Eq. 60 and shown below.

$$E_{med} = \left[\frac{1}{r_0^2}\right]\left[\frac{|x'_ц - x_ц|^3}{3Gm_1\, m_2}\right] \equiv \; pc \tag{64}$$

Observe that energy E_{med} is a function of length $|x'_ц - x_ц|$ pertaining to weak interactions (as will be elucidated shortly) and the gravitational mass interaction $m_1\, m_2$ between cosmological mass of the universe is defined by the previously derived integral of Eq.18 given such that:

$$m_1\, m_2 = \int_0^{m_u}\int_0^{\pi}\int_0^{\pi} m^3[(cos\theta sin\phi)^2 + (sin\theta sin\phi)^2 + (cos\phi)^2]\, dm d\theta d\phi \tag{65}$$

Eq.64 suggest that the cosmological masses m_1 and m_2 theoretically factor into the weak interactions on the subatomic level, and thus implies that the mediating interaction energy E_{med} can be expressed as a surjective map from domain of cosmological masses m_1, m_2, and distances $r_0$and r to the codomain of mediating energy values of E_{med} of the decay process of $n \rightarrow p + \beta^- + \bar{\nu}_e$ within the subatomic realm. For a map g, this function can be expressed such that:

$$g: m_1, m_2, r_0, r \quad \rightarrow \quad E_{med} \in R \tag{66}$$

Or alternatively,

$$g(m_1, m_2, r_0, r) = E_{med} \tag{67}$$

The profound implication of the assertion is that the mass of the entire universe maps or corresponds to energy values of the mediating particle of weak interactions on the subatomic or quantum scale. **A substantial assertion** of the IGIA's application to the subatomic scale and weak interactions is that energy E_{med} is theoretically the energy of the elementary particle or boson which governs the repulsive force of inverse gravity on the subatomic level which drives cosmological expansion. The distance between the two coordinates of $x'_ц$ and $x_ц$ ($|x'_ц - x_ц|$) shown in figure 2, energy E_{med} of Eq.64.The distance $|x'_ц - x_ц|$ is required to equal the distance or range of the interaction of beta decay. As depicted in figure 2, the transitions of the proton (p) and the neutrino (n) transpires at position $x_ц$ producing a mediating particle (W^-, W^+ or Z^0bososns) of energy E_{med}[10]. Thus, over a distance $|x'_ц - x_ц|$, the particles β^-(or e^-) and $\bar{\nu}_e$ at position $x'_ц$ are created; this implies that mediating energy E_{med} ($(E_{med})_{x_ц}$) at position $x_ц$ is equal to the mediating energy E_{med} (E_{med} ($(E_{med})_{x'_ц}$) at position $x'_ц$ (e.g. $(E_{med})_{x_ц} = (E_{med})_{x'_ц}$). Hence, the conservation of energy of the mediating particle (the W^-, W^+ or Z^0 bosons) over length$|x'_ц - x_ц|$. The spatial range of interaction will be denoted l_{med} as a shorthand notation, thus distance $|x'_ц - x_ц|$ is set equal to a range of length l_{med} ($|x'_ц - x_ц| = l_{med}$). This can be mathematically expressed such that:

$$l_{med} = |x'_ц - x_ц| = \left[\sum_{ц=1}^{ц=3} (x'_ц - x_ц)^2\right]^{1/2} \tag{68}$$

IGIA mediation energy E_{med} of weak interactions is expressed in terms of the notation of length l_{med} such that:

$$E_{med} = \left[\frac{1}{r_0^2}\right]\left[\frac{(l_{med})^3}{3Gm_1\, m_2}\right] \tag{69}$$

As previously expressed, the interaction range corresponding to weak interactions is .001 femto-meters. Therefore, the variations of length l_{med} adhere to the inequality of [10]:

$$.001fm \geq l_{med} \tag{70}$$

Thus we are prepared to express that the mediating energy E_{med} of the W^-, W^+ and Z^0 bosons of the beta decay process in terms of the IGIA obey to the Heisenberg uncertainty principle for energy (ΔE) and time (Δt). The Heisenberg uncertainty principle is expressed such that $\Delta E \Delta t \geq \hbar$ [10]. Thus the uncertainty in energy ΔE can be set equal to IGIA mediating energy E_{med} as expressed below [10].

$$\Delta E = E_{med} \tag{71}$$

Alternatively this can be expressed in terms of the IGIA energy of weak interactions such that,

$$\Delta E = \left[\frac{1}{r_0^2}\right]\left[\frac{(l_{med})^3}{3Gm_1\, m_2}\right] \tag{72}$$

The maximum value of lengthl_{med} is $.001fm$ in accordance to the range of weak interactions; therefore we set length l_{med} equal to $.001fm$ ($l_{med} = .001fm$), thus energy ΔE can be expressed such that:

$$\Delta E = \left[\frac{1}{r_0^2}\right]\left[\frac{(.001fm)^3}{3Gm_1\, m_2}\right] \tag{73}$$

The product of energy (ΔE) and time (Δt) ($\Delta E \Delta t$) can be expressed such that:

$$\Delta E \Delta t = \left[\frac{1}{r_0^2}\right]\left[\frac{(.001fm)^3}{3Gm_1\, m_2}\right]\Delta t \tag{74}$$

The uncertainty principle ($\Delta E \Delta t \geq \hbar$) can be expressed in terms of the IGIA such that:

$$\left[\frac{1}{r_0^2}\right]\left[\frac{(.001fm)^3}{3Gm_1\, m_2}\right]\Delta t \geq \hbar \tag{75}$$

In the inequality above, time span Δt must be sufficiently large to satisfy the inequality of Eq.75. **This suggest the profound possibility** that the larger the cosmological masses $m_1\, m_2$ in the denominator, the larger time span Δt to which the mediating particle of the beta decay process in terms of the IGIA can exist in accordance to the Heisenberg uncertainty principle. Moreover, the larger the cosmological masses $m_1\, m_2$, the smaller the energy E_{med} of the W^-, W^+ and Z^0 bosons of the weak interaction in terms of the IGIA. Hence, the force and energy values of an IGIA weak interaction is very minute and very difficult to detect however the time span to which they can exist may be longer than that of typical weak interactions.

An important question one must ask is how the repulsive force of inverse gravity and it's corresponding energy value within weak interactions between particles on the subatomic level relate to the expansion of the geometry of space-time. This question also gives way to the question of how the repulsive force of inverse gravity within weak interactions affects the energy and wavelength of a photon propagating through expanding space as described in Eq.19-29 in section 2. In theory, the answer to this question is that the expansion of space-time affects the energy and thus wavelength of a photon propagating through the region, however, the energy and force of expansion itself is carried by the mediating particle of weak interactions which is energy E_{med}. Thus energy E_{med} is the energy to which directly affects the energy of the passing photon. Hence mathematically, energy E_{med} can be added or subtracted from the initial energy E_0 of the passing photon ($E_{med} \pm E_0$). In elucidating this notion and the answer to this query mathematically, we observe the gravitational redshift equation of Eq.19 in terms of the IGIA expressed below [7].

$$z = \frac{U_T(r) - E_0}{E_0} = \frac{U_T(l_{med}) - E_0}{E_0} \tag{76}$$

Where E_0 is the initial energy of a photon traversing the subatomic region, $U_T(r)$ is the gravitational potential energy affecting the photon's energy. On a subatomic scale, conventional gravity is negligible, therefore, potential energy $U_T(r)$ is set equal to inverse gravity potential energy $U_T(l_{med})$ ($U_T(r) = U_T(l_{med})$) at interaction distance l_{med} which is equal to interaction energy $E_{med}(U_T(l_{med}) = E_{med})$. Gravitational redshift z can be expressed in terms of interaction energy E_{med} such that:

$$z = \frac{U_T(l_{med})}{E_0} - 1 \equiv \frac{1}{E_0}\left[\frac{1}{r_0^2}\right]\left[\frac{(l_{med})^3}{3Gm_1\, m_2}\right] - 1 \tag{77}$$

As expressed in section 2, redshift z relates to the scale factors a_0 and $a(t_{em})$ via the equation $1 + z = a_0/a(t_{em})$ [7][9]. Resultantly, the scale factors a_0 and $a(t_{em})$ relate to the value of the IGIA energy $U_T(l_{med})$ such that:

$$\frac{1}{E_0}\left[\frac{1}{r_0^2}\right]\left[\frac{(l_{med})^3}{3Gm_1\, m_2}\right] = \frac{a_0}{a(t_{em})} \tag{78}$$

Thus, scale factor $a(t_{em})$ corresponding to the weak interaction (WI)energy value E_{med} is denoted a_{WI}(i.e. $a(t_{em}) = a_{WI}$). Thus solving Eq.78 for scale factor $a(t_{em})$ which is scale factor a_{WI} on the subatomic level give the value of scale factor a_{WI} such that:

$$a_{WI} = a_0\left[\frac{1}{E_0}\left[\frac{1}{r_0^2}\right]\left[\frac{(l_{med})^3}{3Gm_1\, m_2}\right]\right]^{-1} \tag{79}$$

Therefore, scale factor a_{WI} is the factor of expansion of space-time on the subatomic scale and corresponds to the change in energy (and wavelength) of a photon traversing the subatomic region of expansion. The expansion of space-time on the subatomic level can be measured by the isotropic Friedman-Walker-Robertson metric for flat space such that [9]:

$$ds^2 = -dt^2 + (a_{WI})^2[dx^2 + dy^2 + dz^2] \tag{80}$$

The expansion of the space-like components (dx, dy, dz) of the metric above can be mathematically expressed such that:

$$(a_{WI})^2[dx^2 + dy^2 + dz^2] \quad = \quad \left((a_0)^2\left[\frac{1}{E_0}\left[\frac{1}{r_0^2}\right]\left[\frac{(l_{med})^3}{3Gm_1\, m_2}\right]\right]^{-2}\right)[dx^2 + dy^2 + dz^2] \tag{81}$$

At this juncture, the IGIA's fundamental description of weak interactions and its correlation to the expansion of the geometry of space-time has been sufficiently established. However, in giving an IGIA description of weakly interacting massive particles, one must show the IGIA's dynamic in weak interactions within a field of particles. Therefore, we now extend the IGIA's application to weak interactions to quantum field theory. The continuous scalar fields of each elementary particles at spatial coordinate $x_ц$ $(x_ц \in R^3)$ are denoted as the Neutrino field $n(x_ц)$, the positron field $p(x_ц)$, the electron or beta minus particle field $\beta^-(x_ц)$, and the antineutrino field $\bar{v}_e(x_ц)$ corresponding the beta decay process $(n \to p + \beta^- + \bar{v}_e)$. These fields have the form of the scalar field $\phi(x_ц)$. The continuous particle field $\phi(x_ц)$ describing each particle within the field is of the form [4]:

$$\phi(x_ц) = \int \frac{d^3p}{\sqrt{2(2\pi)^3}}\bar{u}_f(p)\hat{a}^\dagger e^{i\frac{p_ц x_ц}{\hbar}} + \int \frac{d^3p}{\sqrt{2(2\pi)^3}}\bar{v}_f(p)\hat{b}e^{-i\frac{p_ц x_ц}{\hbar}} \tag{82}$$

Where the plane wave function ($e^{i\frac{p_ц x_ц}{\hbar}}$) for free particles are at momentum and positions values $p_ц$ and $x_ц$ $(p_ц x_ц)$ in R^3 respectively [8]. Therefore, in continuing the derivation, the volume element d^3p corresponding to the momentum space has the form such that:

$$d^3p = \prod_1^3 dp_ц \tag{83}$$

The spinors of $\bar{u}_f(p)$ and $\bar{v}_f(p)$ corresponds to the ½ spin fermion and the ½ spin antiparticle in the form of scalar field $\phi(x_ц)$ [4]. Thus the values of spinors $\bar{u}_f(p)$ and $\bar{v}_f(p)$ correspond to the solution set of [4]:

$$\bar{u}_1(p) = \sqrt{\frac{E+m_p}{2m_p}}\begin{bmatrix} 1 \\ 0 \\ \frac{p_3}{E+m_p} \\ \frac{p_1+ip_2}{E+m_p} \end{bmatrix} \qquad \bar{u}_2(p) = \sqrt{\frac{E+m_p}{2m_p}}\begin{bmatrix} 0 \\ 1 \\ \frac{p_1-ip_2}{E+m_p} \\ -\frac{p_3}{E+m_p} \end{bmatrix} \tag{84}$$

$$\bar{v}_1(p) = \sqrt{\frac{E+m_p}{2m_p}}\begin{bmatrix} \frac{p_3}{E+m_p} \\ \frac{p_1+ip_2}{E+m_p} \\ 1 \\ 0 \end{bmatrix} \qquad \bar{v}_2(p) = \sqrt{\frac{E+m_p}{2m_p}}\begin{bmatrix} \frac{p_1-ip_2}{E+m_p} \\ -\frac{p_3}{E+m_p} \\ 1 \\ 0 \end{bmatrix} \tag{85}$$

Where p_1, p_2, and p_3 are the momentum values in 3-space (e.g. $p_ц = (p_1, p_2, p_3)$)corresponding to the momentum operator of $p_ц = i\hbar\partial/\partial x_ц$ [8]; m_p is the particle rest mass and E is the corresponding energy value. Keep in mind that momentum $p_ц$ and energy E obey the relativistic energy equation of $E^2 = {p_1}^2c^2 + {p_2}^2c^2 + {p_3}^2c^2 + {m_p}^2c^4$ [8].The products of spinors $\bar{u}_f(p)$ and $\bar{v}_f(p)$ obey the orthogonal relations of [4]:

$$\bar{u}_1(p)\bar{u}_1(p) = \frac{E}{m_p} \qquad \bar{u}_2(p)\bar{u}_2(p) = \frac{E}{m_p} \tag{86}$$

$$\bar{v}_1(p)\bar{v}_1(p) = \frac{E}{m_p} \qquad \bar{v}_2(p)\bar{v}_2(p) = \frac{E}{m_p} \tag{87}$$

$$\bar{u}_1(p)\bar{u}_2(p) = 0 \qquad \bar{v}_1(p)\bar{v}_2(p) = 0 \tag{88}$$

And thus, in general, for the condition of $(f \neq g)$, we have [4]:

$$\bar{v}_f(p)\bar{u}_g(p) = 0 \qquad \bar{u}_f(p)\bar{v}_g(p) = 0 \tag{89}$$

At this juncture, one can introduce the relationship of the energy of the IGIA mediating particle E_{med} to the field function $\phi(x_ц)$. Due to the fact that the particles are created or destroyed (or annihilated) across interaction distance l_{med}, each particle (protons, neutrinos, and so on) exist with part of the combined energy $E_{med}$of the mediating particle. Energy E presented within the solution set of spinors $\bar{u}_f(p)$ and $\bar{v}_f(p)$ of Eq. 84-85 and Eq.86-87 is equal to the energy of the IGIA mediating particle of energy E_{med} as shown below.

$$E = E_{med} = \left[\frac{1}{r_0^2}\right]\left[\frac{(l_{med})^3}{3Gm_1\, m_2}\right] \tag{90}$$

Thus the products of spinors $\bar{u}_f(p)$ and $\bar{v}_f(p)$ (for $= f$) of Eq.86-87 have values in terms of energy E_{med} such that [4]:

$$\bar{u}_f(p)\bar{u}_f(p) = \frac{1}{m_p}\left[\frac{1}{r_0^2}\right]\left[\frac{(l_{med})^3}{3Gm_1\, m_2}\right] \tag{91}$$

$$\bar{v}_f(p)\bar{v}_f(p) = \frac{1}{m_p}\left[\frac{1}{r_0^2}\right]\left[\frac{(l_{med})^3}{3Gm_1\, m_2}\right] \tag{92}$$

Eq.91-92 implies that the variations in field $\phi(x_ц)$ corresponds to the variations of an IGIA energy value of E_{med} at a position value $x_ц$ via the spinors $\bar{v}_f(p)$ and $\bar{u}_f(p)$as demonstrated below.

$$\Delta\phi(x_ц) \quad \rightarrow \quad \Delta E_{med} \tag{93}$$

Therefore, in continuing to define the components of the field function, the creation operators ($\hat{a}^{\dagger}, \hat{b}^{\dagger}$) and the annihilation operators ($\hat{a}, \hat{b}$) of field $\phi(x_{ц})$ obey the anti-symmetric property required for the description of fermions in accordance to the Pauli exclusion principle [5] [8] [10]. The creation and annihilation operators have the properties such that [5]:

$$\{\hat{a}, \hat{a}^{\dagger}\} = \hat{a}\hat{a}^{\dagger} + \hat{a}^{\dagger}\hat{a} = \delta \qquad \{\hat{b}, \hat{b}^{\dagger}\} = \hat{b}\hat{b}^{\dagger} + \hat{b}^{\dagger}\hat{b} = \delta \tag{94}$$

$$\{\hat{a}^{\dagger}, \hat{a}^{\dagger}\} = \{\hat{a}, \hat{a}\} = 0 \qquad \{\hat{b}^{\dagger}, \hat{b}^{\dagger}\} = \{\hat{b}, \hat{b}\} = 0 \tag{95}$$

And [5],

$$\hat{a}^{\dagger}\hat{a} = \hat{b}^{\dagger}\hat{b} = n \tag{96}$$

Where n (as opposed to n denoting a neutron in this case) is the number of particles in the system and δ is the Kronecker delta which has the property of [8]:

$$\delta_j^i = \begin{cases} i \neq j, \delta = 0 \\ i = j, \delta = 1 \end{cases} \tag{97}$$

Hence, the particle field $\phi(x_{ц})$ and its anti-particle field $\phi^{\dagger}\,(x_{ц})$ are expressed by Eq.98-101 such that [4]:

$$\phi(x_{ц}) = \int \frac{d^3p}{\sqrt{2(2\pi)^3}} \bar{u}_f(p)\hat{a}^{\dagger} e^{i\frac{p_{ц}x_{ц}}{\hbar}} + \int \frac{d^3p}{\sqrt{2(2\pi)^3}} \bar{v}_f(p)\hat{b} e^{-i\frac{p_{ц}x_{ц}}{\hbar}} \tag{98}$$

Or Alternatively,

$$\phi \quad = \phi^{+} + \phi^{-} \tag{99}$$

And [4],

$$\phi^{\dagger}\,(x_{ц}) = \int \frac{d^3p}{\sqrt{2(2\pi)^3}} \bar{u}_f(p)\hat{b}^{\dagger} e^{i\frac{p_{ц}x_{ц}}{\hbar}} + \int \frac{d^3p}{\sqrt{2(2\pi)^3}} \bar{v}_f(p)\hat{a} e^{-i\frac{p_{ц}x_{ц}}{\hbar}} \tag{100}$$

Or Alternatively,

$$\phi^{\dagger} \;= \phi^{\dagger -} + \phi^{\dagger +} \tag{101}$$

The IGIA full description of weak interactions within quantum field theory require the use of Hamiltonian's equation, specifically, the Hamiltonian formulated within Fermi's theory in the description of the beta decay process i.e. weak interactions ($n \rightarrow p + \beta^{-} + \bar{\nu}_e$). Therefore we use the Feynman, Gell-Mann, Marshak and Sudarshan constructions of the current by current Hamiltonian $H^{Int}(x'_{ц}, x_{ц})$ of the beta decay process at positions $x'_{ц}$ and $x_{ц}$ (over interaction distance l_{med}) [2] [3]. It is important to specify that the Hamiltonian $H^{Int}(x'_{ц}, x_{ц})$ only pertains to the energy of the weak interaction within the nucleus and thus excludes other energy values pertaining to the processes of the nucleus and structure of the nucleus (e.g. the energy pertaining to nuclear force). Hence, the Hamiltonian $H^{Int}(x'_{ц}, x_{ц})$of the weak interaction is expressed such that [2] [3]:

$$H^{Int}(x'_{ц}, x_{ц}) = \frac{G_F}{\sqrt{2}} J^{ц}(x_{ц}) J_{ц}(x'_{ц}) \tag{102}$$

Where $J^{ц}(x_{ц})$ denotes the current density corresponding to particle fields $\phi^{\dagger}_{AL,R}(x_{ц})$ and $\phi_{BL,R}(x_{ц})$ at position $x_{ц}$ and $J_{ц}(x'_{ц})$ denotes the current density corresponding to fields $\phi^{\dagger}_{CL,R}(x'_{ц})$ and $\phi_{DL,R}(x'_{ц})$ *at position* $x'_{ц}$. Current densities $J^{ц}(x_{ц})$ and $J_{ц}(x'_{ц})$ at positions $x'_{ц}$ and $x_{ц}$ have values such that [2] [3]:

$$J^{ц}(x_{ц}) = \left(\phi^{\dagger}_{AL,R}(x_{ц}) \gamma^{ц} \phi_{BL,R}(x_{ц})\right) \qquad J_{ц}(x'_{ц}) = \left(\phi^{\dagger}_{CL,R}(x'_{ц}) \gamma_{ц} \phi_{DL,R}(x'_{ц})\right) \tag{103}$$

The subscripts L, R denote the left and right handed projections of the particle fields which will be defined with more detail shortly; the subscripts A,B,C, and D denote the type of particle field (e.g. protons, electrons, neutrons, etc.). The particle fields $\phi^{\dagger}_{AL,R}(x_{ц})$ and $\phi_{BL,R}(x_{ц})$ at position $x_{ц}$ of current density $J^{ц}(x_{ц})$ relate to the particle fields $\phi^{\dagger}_{CL,R}(x'_{ц})$ and $\phi_{DL,R}(x'_{ц})$ at position $x'_{ц}$ of current $J_{ц}(x'_{ц})$ via the mediating particle of energy E_{med} over distance l_{med} of the weak interaction[2][3]. Where the current densities $J^{ц}(x_{ц})$ and $J_{ц}(x'_{ц})$, the general expression of the Hamiltonian $H^{Int}(x'_{ц}, x_{ц})$is given such that [2] [3]:

$$H^{Int}(x'_{ц}, x_{ц}) = \frac{G_F}{\sqrt{2}} \left(\phi^{\dagger}_{AL,R}(x_{ц}) \gamma^{ц} \phi_{BL,R}(x_{ц})\right)\left(\phi^{\dagger}_{CL,R}(x'_{ц}) \gamma_{ц} \phi_{DL,R}(x'_{ц})\right) + h.c. \tag{104}$$

Where the notation $h.c.$ denotes "Hermitian conjugation", G_F denotes Fermi's constant which has a value of $1.166 \times 10^{-5} GeV^{-2}$and $\gamma_{ц}$ denotes the gamma matrices [2][3].The components of $\gamma^{ц}$ are matrices such that:

$$\gamma^{ц} \rightarrow \tag{105}$$

$$\gamma^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad \gamma^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$\gamma^2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix} \qquad \gamma^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

Where $\gamma^{ц} = \eta\gamma_{ц}$ and η is the Minkowski metric, the field functions $\phi^{\dagger}_{AL,R}(x_{ц})$, $\phi_{BL,R}(x_{ц})$, $\phi^{\dagger}_{CL,R}(x'_{ц})$, and $\phi_{DL,R}(x'_{ц})$ are the right and left handed projections [2] [3]. It is well known that weak interactions do not obey the parity law of symmetry [2] [3]. Thus the characteristic of chirality within the beta decay process states that the right handed field signifies that the momentum and spin transpire in the same direction while the left handed field signify that the momentum and spin have opposite directions; hence the right and left handed field functions $\phi^{\dagger}_{AL,R}(x_{ц})$, $\phi_{BL,R}(x_{ц})$, $\phi^{\dagger}_{CL,R}(x'_{ц})$, and $\phi_{DL,R}(x'_{ц})$ are of the form [2] [3]:

$$\phi_R(x_{ц}) = \phi(x_{ц})\frac{1+\gamma^5}{2} \qquad \phi_L(x_{ц}) = \phi(x_{ц})\frac{1-\gamma^5}{2} \tag{106}$$

Where $\frac{1+\gamma^5}{2}$ and $\frac{1-\gamma^5}{2}$ are the right and left hand operators, either field function can be presented in the general form [2] [3]:

$$\phi_{R,L}(x_{ц}) = \phi(x_{ц})\frac{1\pm\gamma^5}{2} \tag{107}$$

The value γ^5 (or gamma 5 matrix) has a value such that [2] [3]:

$$\gamma^5 = -\frac{\vec{p}\bullet\vec{\Sigma}}{|\vec{p}|} \tag{108}$$

Where $\vec{p}$ is a momentum vector ($\vec{p} = (mc, p_1, p_2, p_3) \to p_{ц}$) and $\vec{p}/|\vec{p}|$ denotes the corresponding unit vector. The matrices of $\vec{\Sigma}$ have values such that [2] [3]:

$$\vec{\Sigma} = \begin{bmatrix} \bar{\sigma} & \mathbf{0} \\ \mathbf{0} & \bar{\sigma} \end{bmatrix} = \left\{ \begin{bmatrix} \sigma_x & \mathbf{0} \\ \mathbf{0} & \sigma_x \end{bmatrix}, \begin{bmatrix} \sigma_y & \mathbf{0} \\ \mathbf{0} & \sigma_y \end{bmatrix}, \quad \begin{bmatrix} \sigma_z & \mathbf{0} \\ \mathbf{0} & \sigma_z \end{bmatrix} \right\} \tag{109}$$

Where $\mathbf{0}$ denotes the 4 by 4 zero matrices such that:

$$\mathbf{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{110}$$

The symbol $\bar{\sigma}$ conventionally denotes the 4 by 4 Pauli spin matrices $\bar{\sigma}$ such that [8]:

$$\bar{\sigma} \quad \rightarrow \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{111}$$

The matrices $\vec{\Sigma}$ obeys the relation of [2] [3]:

$$\bar{\alpha} = \gamma^5 \vec{\Sigma} \tag{112}$$

And matrices $\bar{\alpha}$ takes on values of [8]:

$$\bar{\alpha} = \begin{bmatrix} \mathbf{0} & \bar{\sigma} \\ \bar{\sigma} & \mathbf{0} \end{bmatrix} = \left\{ \begin{bmatrix} \mathbf{0} & \sigma_x \\ \sigma_x & \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{0} & \sigma_y \\ \sigma_y & \mathbf{0} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{0} & \sigma_z \\ \sigma_z & \mathbf{0} \end{bmatrix} \right\} \tag{113}$$

After a brief definition of the terms composing the weak Hamiltonian $H^{Int}(x'_ц, x_ц)$ of the beta decay process ($n \rightarrow p + \beta^- + \bar{v}_e$), one can express the Hamiltonian in terms of the <u>Neutrino field</u> $n_{L,R}(x_ц)$, the <u>positron field</u> $p_{L,R}(x_ц)$, the <u>electron or beta minus particle field</u> $\beta^-{}_{L,R}(x'_ц)$, and the <u>antineutrino field</u> $\bar{v}_{e_{L,R}}(x'_ц)$ such that [2] [3]:

$$H^{Int}(x'_ц, x_ц) = \frac{G_F}{\sqrt{2}} \left(n_{L,R}(x_ц) \gamma^ц p_{L,R}(x_ц) \right) \left(\beta^-{}_{L,R}(x'_ц) \gamma_ц \bar{v}_{e_{L,R}}(x'_ц) \right) + h.c. \tag{114}$$

Where the particle field functions $n_{L,R}(x_ц)$, $p_{L,R}(x_ц)$, $\beta^-{}_{L,R}(x'_ц)$, and $\bar{v}_{e_{L,R}}(x'_ц)$ are of the form of $\phi_{R,L}(x_ц)$ [2] [3]. The IGIA energy of the weak interactions E_{med} will be denoted $E(x'_ц, x_ц)$ as shown below (here we revert to the notation of $|x'_ц - x_ц|$ for interaction length l_{med}).

$$E(x'_ц, x_ц) = E_{med} = \left[\frac{1}{r_0^2} \right] \left[\frac{|x'_ц - x_ц|^3}{3Gm_1 \, m_2} \right] \tag{115}$$

Due to the orthogonal relationships of spinors $\bar{u}_f(p)$ and $\bar{v}_f(p)$ ($\bar{u}_f(p)\bar{u}_f(p) = \bar{v}_f(p)\bar{v}_f(p) = E(x'_ц, x_ц)/m_p \ \forall f = f$) of Eq. 86-89, variations in energy $E(x'_ц, x_ц)$ (or energy E_{med}) corresponds to variations in the field functions of the form $\phi_{R,L}(x_ц)$ shown below [4].

$$\Delta\phi_{R,L}(x_ц) \quad \rightarrow \quad \Delta E(x'_ц, x_ц) \tag{116}$$

Thus the variations in field functions $n_{L,R}(x_ц)$, $p_{L,R}(x_ц)$, $\beta^-{}_{L,R}(x'_ц)$, and $\bar{v}_{e_{L,R}}(x'_ц)$ are contingent on variations of energy $E(x'_ц, x_ц)$ of the IGIA.

This implies that the Hamiltonian $H^{Int}(x'_ц, x_ц)$ of the beta decay process corresponds variations of the IGIA mediating energy of $E(x'_ц, x_ц)$ as shown below.

$$\Delta H^{Int}(x'_ц, x_ц) \rightarrow \Delta E(x'_ц, x_ц) \tag{117}$$

The definition of the energy E_{med} pertaining to the mediating particles (i.e. the W^-, W^+ and Z^0 bosons) of the beta decay process and the particle field functions $\phi_{R,L}(x_ц)$ of Eq. 107 corresponding to the Hamiltonian of $H^{Int}(x'_ц, x_ц)$formulated for weak interactions in terms of the IGIA constitute the theoretical and fundamental description of wimps (weakly interacting massive particles). Conclusively, the mathematical description of weak interactions in terms of the IGIA can be generalized to the notion of wimps as a cause of cosmological expansion.

II. The cosmological scale description of the IGIA on a field of weakly interacting massive particles

As previously asserted, the mediating particles between wimps conduct the repulsive force of inverse gravity. The notion of wimps in terms of the IGIA is simply the relationship of force and energy between the subatomic scale and the cosmological scale which will now be elucidated. Consider the closed set $D_c$which represents a given region in $R^ц$ dimensional space. Set D_c is expressed such that:

$$D_c = \{x, y \in R^ц |\ r \geq |x - y|\} \tag{118}$$

Therefore, positions $x'_{цj}$ and $x_{цj}$ are elements of set D_c ($x'_{цj}, x_{цj} \in D_c$), where subscript j (recall that $j \in Z^+$ where Z^+ denotes the set of positive integers) denotes the number of particle interactions at positions $x'_{цj}$ and $x_{цj}$. Hence, positions $x'_{цj}$ and $x_{цj}$ are in the neighborhood of the IGIA distance r (which is cosmological distance).Each weak interaction or process of beta decay within a region D_c at particle positions $x'_{цj}$ and $x_{цj}$ separated by length$|x'_{цj} - x_{цj}|$ and has an IGIA energy value $E(x'_{цj}, x_{цj})$ ($E(x'_{цj}, x_{цj}) = E_{med}$) which can be expressed as equal to relativistic energy pc as shown below.

$$E(x'_{цj}, x_{цj}) = \left[\frac{1}{r_0^2}\right]\left[\frac{|x'_{цj} - x_{цj}|^3}{3Gm_1\, m_2}\right] \equiv pc \tag{119}$$

Where,

$$|x'_{цj} - x_{цj}| = l_{med} \tag{120}$$

And it follows that,

$$.001fm \quad \geq \quad l_{med} \tag{121}$$

Each value of energy $E(x'_{\text{ц}j}, x_{\text{ц}j})$ corresponds to a force value $F(x'_{\text{ц}j}, x_{\text{ц}j})$ which corresponds to individual weak interactions within region D_c . More directly stated, force value $F(x'_{\text{ц}j}, x_{\text{ц}j})$ is the force of each weak interaction. In the task of deriving an expression of force $F\left(x'_{\text{ц}j}, x_{\text{ц}j}\right)$ pertaining to each weak interaction in terms of the IGIA, the IGIA mediating particle energy $E(x'_{\text{ц}j}, x_{\text{ц}j})$ is expressed as equal to relativistic energy such that:

$$pc = \left[\frac{1}{r_0^2}\right]\left[\frac{\left|x'_{\text{ц}j} - x_{\text{ц}j}\right|^3}{3Gm_1\, m_2}\right] \tag{122}$$

The value of momentum p can be expressed in terms of the IGIA such that:

$$p = \left[\frac{1}{cr_0^2}\right]\left[\frac{\left|x'_{\text{ц}j} - x_{\text{ц}j}\right|^3}{3Gm_1\, m_2}\right] \tag{123}$$

Force value $F\left(x'_{\text{ц}j}, x_{\text{ц}j}\right)$ is classically expressed as the timed derivative of momentum p, giving the first order differential equation such that:

$$F\left(x'_{\text{ц}j}, x_{\text{ц}j}\right) = \frac{dp}{dt} \tag{124}$$

This can be algebraically manipulated such that:

$$F\left(x'_{\text{ц}j}, x_{\text{ц}j}\right)dt = dp \tag{125}$$

Thus we integrate in respect to volume elements dt and dp such that:

$$\int_0^{\Delta t} F\left(x'_{\text{ц}j}, x_{\text{ц}j}\right)dt = \int_0^{p} dp \tag{126}$$

After evaluating both integrals, one obtains:

$$F\left(x'_{\text{ц}j}, x_{\text{ц}j}\right)\Delta t = p \tag{127}$$

Force value $F\left(x'_{\text{ц}j}, x_{\text{ц}j}\right)$ can be expressed such that:

$$F\left(x'_{\text{ц}j}, x_{\text{ц}j}\right) = \frac{p}{\Delta t} \tag{128}$$

Substituting the value of Eq.123 into Eq.128 gives the IGIA force value of each weak interaction such that:

$$F\left(x'_{цj}, x_{цj}\right) = \left[\frac{1}{\Delta t c r_0^2}\right]\left[\frac{\left|x'_{цj} - x_{цj}\right|^3}{3Gm_1\, m_2}\right] \tag{129}$$

At this juncture, we briefly revert to the IGIA energy of repulsion on the cosmological level. As previously asserted, the IGIA inverse term of energy on the <u>cosmological level</u> will be denoted E_{dark}, hence energy E_{dark} takes on a value such that [1]:

$$E_{dark} = \left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] \tag{130}$$

Energy E_{dark} is a function of <u>cosmological level distance r</u> while recalling that the IGIA mathematically <u>interprets dark energy</u> as energy E_{dark} of Eq.130 above. As expressed in Eq.63, energy E_{dark} is a close approximate to the sum of j interactions of energies $E(x'_{цj}, x_{цj})$ at all positions $x'_{цj}$ and $x_{цj}$ within region D_c, therefore energy E_{dark} can be expressed as an approximate to the discrete sums of subatomic energies $E(x'_{цj}, x_{цj})$ such that:

$$E_{dark} \approx \sum_j E(x'_{цj}, x_{цj}) \tag{131}$$

Or Alternatively,

$$E_{dark} = \left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] \approx \sum_j \left[\frac{1}{r_0^2}\right]\left[\frac{\left|x'_{цj} - x_{цj}\right|^3}{3Gm_1\, m_2}\right] \tag{132}$$

This implies that cosmological distance r can be expressed as an approximate such that:

$$r \approx \left[\sum_j \left|x'_{цj} - x_{цj}\right|^3\right]^{\frac{1}{3}} \tag{133}$$

The <u>sum</u> of <u>subatomic force</u> values $F(x'_{цj}, x_{цj})$ (e.g. $\sum_j F(x'_{цj}, x_{цj})$) pertaining to <u>weak interactions</u> within region D_c will be denoted F_{ex} which is approximated to the derivative of energy E_{dark} in respect to radial distance r on the cosmological level (which gives the force value corresponding to energy E_{dark}). The IGIA force F_{ex} can be expressed such that:

$$F_{ex} = \sum_j F(x'_{\text{Ц}j}, x_{\text{Ц}j}) \approx \frac{dE_{dark}}{dr} \tag{134}$$

Or alternatively,

$$F_{ex} = \sum_j F(x'_{\text{Ц}j}, x_{\text{Ц}j}) \approx \frac{d}{dr}\left(\left[\frac{1}{r_0^2}\right] \left[\frac{r^3}{3Gm_1\, m_2}\right] \right) \tag{135}$$

This results in the expression of:

$$F_{ex} = \sum_j \left[\frac{1}{\Delta tcr_0^2}\right] \left[\frac{\left|x'_{\text{Ц}j} - x_{\text{Ц}j}\right|^3}{3Gm_1\, m_2}\right] \approx \left[\frac{1}{r_0^2}\right] \left[\frac{r^2}{Gm_1\, m_2}\right] \tag{136}$$

Hence, force F_{ex} is approximately equal to inverse force $F'_g(r)$ of Eq.8 as shown below.

$$F_{ex} \approx F'_g(r) \tag{137}$$

Therefore, the force of cosmological expansion F_{ex} is the combined effect of weak interactions of wimps within region D_c of radial distance r and constitutes the inverse gravity force of $F'_g(r)$ within a margin of error. As previously postulated, the combined force of the weak interactions within region D_c correspond to the stretching of the geometry of space-time. This formulation defines the relationship between the cosmological scale and the subatomic scale of the IGIA as it pertains to the total energy of weak interactions or weakly interacting massive particles over a region of space.

Concluding thoughts

The difficulty with the IGIA and its postulates is the typical skepticism that inherently opposes new assertions. As stated in the introduction, the actual cause of cosmological expansion is still unknown. Due to the horizon problem [6] as well as other factors, the universe does not (at least at this point in time) reveal the entirety of its true nature. Thus, even the most intricate and logical theoretical proposals published are still speculative. If God permits the fateful day when the human race sufficiently develops in both knowledge and in technology to confirm the cause and dynamics of cosmological inflation experimentally, the vast majority of even the most eloquent assertions will be found to be incorrect.

Therefore, the purpose of this paper is to present the theoretical possibility of the inverse gravity inflationary assertion to the physics community and the entire scientific community. Although any given postulate may be incorrect as a whole, aspects of any theoretical assertion may possess fragments of the full truth.

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