A *G*-function Form of Radial States

in Carbon and Hydrogen Atoms:

Absorption or Emission of Radiation

Amir Pishkoo

Physics and Accelerators Research School
NSTRI, Tehran, Iran, P.O. Box 14395-836

Abstract

The Meijer's *G*-functions (MGFs) are an admirable family of functions which each of them is determined by finitely many indices. Analytic manipulations and numerical computations involving Meijer's G-functions have been provided by the software packages such as *Mathematica, Maple,* and *Matlab.* Recently, MGFs are obtained as the solution of Schrödinger equation as well as diffusion equation, and reaction-diffusion equation by using modified separation of variables method. It was also shown that MGFs are the radial functions of the Hydrogen atom. The radial functions such as R_{20} and R_{21} have crucial role in making superposition of orbitals and the formation of chemical bonds such as SP^2 hybridization. In this paper we deduce some useful relations upon which the radial functions can be represented by summation of a number of Meijer's *G*-functions. As a result, MGFs may be utilized in describing carbon nanotubes, absorption and emission of radiation such as X-rays and γ-rays.

Keywords: Meijer's *G*-function, Schrödinger equation, Bohr operator, Radial functions.

1   INTRODUCTION

The Meijer's G-functions are a remarkable family of functions of one variable,
each of them determined by finitely many indices. They play a crucial role in
a certain useful mathematical enterprise. when look at conceptually, they are
both attractive and attractive, and have remarkable closure properties. The
implementation of Meijer's G-function in Mathematica constitutes an addi-
tional utility for analytic manipulations and numerical computations involving
this special function [4]. Most of the software packages such as *Maple* (Maple
2011), *Mathematica* (Wolfram 2011), and *Matlab* (Matlab 2011) provide com-
putations involving Meijer's G-functions. Additionally, Maple can provide a
means to understand the relationships among Meijer's G-functions and other
functions. Considering the well-known applications of MGF, it is obvious that
MGF is a useful tool in mathematical physics because of its analytical prop-
erties, and more because it can be expressed as a finite sum of generalized
hypergeometric functions[7,10,22]. In [16-18] MGFs are obtained as solution
of physical problems by introducing modified separation of variables (MSV)
method. Being MGFs as the solution form of schrödinger equation [16,19] es-
pecially Hydrogen atom, a new door has been opened for obtaining new results
in the present paper. We aim that MGFs may be utilized in describing car-
bon nanotubes, absorption and emission of radiation [1,8,15]. Because many
deduced results from atomic physics can be generalized to nuclear physics, we
may expect MGFs to be even the wave functions of parent and daughter nu-
clei. As a result, we may properly describe α, β, andγ-decays for which the
first steps has been done in [20,21]. Meijer's G-functions are the solution of
Schrödinger equation, depending on dimensionality, it is opened a new way to
describe quantum dots or clusters 0D, quantum wire 1D, quantum well, thin
film or layer 2D, and bulk material 3D in terms of G-functions [5,9,13,23].

Regarding to radial functions we now refer to their role in the formation of
chemical bonds. The ability of a single carbon atom to form up to four strong
covalent bonds makes it a key player in organic chemistry and hence a corner
stone in the formulation of biological tissue. It is well known fact that atoms
are most stable when the s, p orbital of the outer shell are occupied (octet
rule). This is known as hybridization which is done most efficiently by making
superposition of the symmetric $2s, 2p_1, 2p_0$, and $2p_{-1}$ orbital as follows [6].

$$\mathbf{s}(\mathbf{r}) \equiv 2s = R_{20}Y_0^0,$$

$$p_x(\mathbf{r}) \equiv 2p_1 - 2p_{-1} = R_{21}Y_1^1 - R_{21}Y_{-1}^1,$$

$$p_y(\mathbf{r}) \equiv 2p_1 + 2p_{-1} = R_{21}Y_1^1 + R_{21}Y_{-1}^1,$$

$$p_z(\mathbf{r}) \equiv 2p_0 = R_{21}Y_1^0.$$

The sp^2 hybridization consists of the following superpositions of $s(\mathbf{r}), p_x(\mathbf{r})$, and $p_y(\mathbf{r})$. This type of hybridization is relevant to the formation of graphene and carbon nanotubes [6].

$$sp_a^2 \equiv \sqrt{\frac{4}{6}} s(\mathbf{r}) + \sqrt{\frac{2}{6}} p_x(\mathbf{r}),$$

$$sp_b^2 \equiv \sqrt{\frac{2}{6}} s(\mathbf{r}) - \sqrt{\frac{1}{6}} p_x(\mathbf{r}) + \sqrt{\frac{3}{6}} p_y(\mathbf{r}),$$

$$sp_c^2 \equiv \sqrt{\frac{2}{6}} s(\mathbf{r}) - \sqrt{\frac{1}{6}} p_x(\mathbf{r}) - \sqrt{\frac{3}{6}} p_y(\mathbf{r}),$$

It is seen that radial functions such as $R_{20}(r); R_{21}(r)$ have crucial role in making superposition of orbitals. The last part of the Schrödinger equation to be solved concerns the radial coordinate r. The size of the atomic orbitals is not well defined due to the statistical nature of the wavefunction. It is only characterized fully by calculating all the expectation values $< r^k >$ for any power k. Since the potential behaves as $\frac{1}{r}$ it is natural to estimate the size of the orbitals by calculating $< \frac{1}{r} >$.

$$< \frac{1}{r} >_{nlm} = \int_0^\infty dr r^2 \frac{1}{r} |R_{nl}|^2 = \frac{1}{n^2} \frac{1}{r_B}.$$

For the ground state ψ_{100} of the Hydrogen atom we find (with $Z = 1$) the ground state energy E_0 and the size r_B to be

$$r_B = r_{Z=1} = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 0.053nm \ \ \& \ \ E_0 = E_{Z=1} = -\frac{m_e e^4}{32\pi^2\epsilon_0^2\hbar^2} = -13.6ev,$$

where r_B is Bohr radius.

The content of this paper is divided into two main sections. In the first section, the definition of MGFs, some of their properties are recalled. Second section gives our results by introducing Bohr operator and then in the format of two theorems we represent Radial functions of the Hydrogen atom as MGFs by deducing Eq. (3.11). In fact, Bohr operator acts as photon which gives energy to electron and changing Radial functions with regarding selection rules (absorption), and vise versa in the process of emission.

2 MEIJER'S *G*-FUNCTIONS

The *G*-function was introduced by Cornelis Simon Meijer (1936) as a very general function intended to include all elementary functions and most of the known special functions, for instance:

- $\sin x = \sqrt{\pi} G_{0,2}^{1,0}(\frac{x^2}{4} \vert_{\frac{1}{2},0}^{-}); \quad \cos x = \sqrt{\pi} G_{0,2}^{1,0}(\frac{x^2}{4} \vert_{0,\frac{1}{2}}^{-})$

- $\ln x = G_{2,2}^{1,2}(x - 1 \vert_{1,0}^{1,1}); \quad J_\nu(x) = G_{0,2}^{1,0}(\frac{x^2}{4} \vert_{\frac{\nu}{2},\frac{-\nu}{2}}^{-})$

We begin with the definition of Meijer G-function.

Definition 2.1 *A definition of the Meijer G-function is given by the following path integral in the complex plane, called Mellin-Barnes type integral [2,3,11,12,14]:*

$$G_{p,q}^{m,n}(\substack{a_1,\ldots,a_p \\ b_1,\ldots,b_q} \vert z) = \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^m \Gamma(b_j - s) \ \prod_{j=1}^n \Gamma(1 - a_j + s)}{\prod_{j=m+1}^q \Gamma(1 - b_j + s) \ \prod_{j=n+1}^p \Gamma(a_j - s)} z^s ds. \quad (2.1)$$

Here, L is a suitable closed contour in the complex plane; the integers $m; n; p; q$ are called "orders" of the G-function, or the components of the order $(m; n; p; q)$; a_j and b_j are called "parameters" and in general, they are complex numbers. The definition holds under the following assumptions: $0 \leq m \leq q$ and $0 \leq n \leq p$, where $m, n, p,$ and q are integer numbers. Subtracting parameters $a_j - b_k \neq 1, 2, 3, \ldots$ for $k = 1, \ldots, n$ and $j = 1, 2, \ldots, m$ imply that no pole of any $\Gamma(b_j - s), j = 1, \ldots, m$ coincides with any pole of any $\Gamma(1 - a_k + s), k = 1, \ldots, n$. We follow usual convention that the empty product equals 1.

Choosing $m = 1, n = 0, p = 0$ and $q = 1$, we have

$$G_{0,1}^{1,0}(\substack{- \\ b_1} \vert z) = \frac{1}{2\pi i} \int_L \Gamma(b_1 - s) z^s ds. \quad (2.2)$$

Or in more general in terms of elementary function is (see[11])

$$G_{0,1}^{1,0}(\substack{- \\ \frac{\beta}{\alpha}} \vert \eta z^\alpha) = \eta^{\frac{\beta}{\alpha}} z^\beta e^{-\eta z^\alpha} \quad (2.3)$$

Based on the definition, the following basic properties are easily derived:

$$z^\alpha G_{p,q}^{m,n}(\substack{\mathbf{a_p} \\ \mathbf{b_q}} \vert z) = G_{p,q}^{m,n}(\substack{\mathbf{a_p}+\alpha \\ \mathbf{b_q}+\alpha} \vert z), \quad (2.4)$$

where the multiplying term z^α changes the parameters of the G-function. The key to a conceptual understanding of a G-function is the differential equation that is satisfies.

3 MAIN RESULTS

The Bohr radius, symbolized r_B, is the mean radius of the orbit of an electron around the nucleus of a hydrogen atom at its ground state (lowest-energy level). We define the Bohr operator to change the mean radius of the orbit for electron. (for example $r_B \to 2r_B$)

The Bohr operator
Generally, an operator acting on a function maps it into another function.

Definition 3.1 *The Bohr operator is represented by $\hat{B}$ and defined as follows:*

$$\hat{B}[(\frac{1}{r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(\bar{0}|\frac{r}{r_B})] = [(\frac{1}{2r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(\bar{0}|\frac{r}{2r_B})] \tag{3.1}$$

The meaning of square $\hat{B}^2$ of the Bohr operator $\hat{B}$ is obvious:

$$\hat{B}^2[(\frac{1}{r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(\bar{0}|\frac{r}{r_B})] = \hat{B}[(\frac{1}{2r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(\bar{0}|\frac{r}{2r_B})] = [(\frac{1}{3r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(\bar{0}|\frac{r}{3r_B})]$$

The operator $\hat{B}^n$ is formed by $\hat{B}$ acting n times:

$$\hat{B}^n[(\frac{1}{r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(\bar{0}|\frac{r}{r_B})] = \hat{B}\hat{B}\hat{B}...\hat{B}[(\frac{1}{r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(\bar{0}|\frac{r}{r_B})] = \hat{B}^{n-1}(\hat{B}[(\frac{1}{r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(\bar{0}|\frac{r}{r_B})])$$

with $r_B = \frac{4\pi\epsilon_0\hbar^2}{me^2}$, where r_B is the Bohr radius.

Theorem 3.1 *Let $R_{10} = 2(\frac{1}{r_B})^{\frac{3}{2}}e^{-\frac{r}{r_B}} = 2(\frac{1}{r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(\bar{0}|\frac{r}{r_B})$ be the ground radial state of the Hydrogen atom, then the operators $\hat{B}(\frac{r}{r_B})$ and $\hat{B}^2(\frac{r}{r_B})^2$ map the radial function R_{10} into multiple of R_{21} and of R_{32}, respectively.*

Proof 1 *From (2.3) and (2.4) for $\alpha = 1$ we have*

$$(\frac{r}{r_B})e^{-\frac{r}{r_B}} = (\frac{r}{r_B})G_{0,1}^{1,0}(\bar{0}|\frac{r}{r_B}) = G_{0,1}^{1,0}(\bar{1}|\frac{r}{r_B})$$

so

$$(\frac{r}{r_B})R_{10} = 2(\frac{1}{r_B})^{\frac{3}{2}}(\frac{r}{r_B})e^{-\frac{r}{r_B}} \tag{3.2}$$

Affecting $\hat{B}$ on (3.2) gives

$$\hat{B}[(\frac{r}{r_B})R_{10}] = \hat{B}[2(\frac{1}{r_B})^{\frac{3}{2}}(\frac{r}{r_B})e^{-\frac{r}{r_B}}] = 2(\frac{1}{2r_B})^{\frac{3}{2}}(\frac{r}{2r_B})e^{-\frac{r}{2r_B}} = \sqrt{3}R_{21}$$

From (2.3) and (2.4) for $\alpha = 2$ we have

$$(\frac{r}{r_B})^2 e^{-\frac{r}{r_B}} = (\frac{r}{r_B})^2 G_{0,1}^{1,0}(\bar{0}|\frac{r}{r_B}) = (\frac{r}{r_B})G_{0,1}^{1,0}(\bar{1}|\frac{r}{r_B}) = G_{0,1}^{1,0}(\bar{2}|\frac{r}{r_B})$$

$$\left(\frac{r}{r_B}\right)^2 R_{10} = 2\left(\frac{1}{r_B}\right)^{\frac{3}{2}}\left(\frac{r}{r_B}\right)^2 e^{-\frac{r}{r_B}} \tag{3.3}$$

Affecting $\hat{B}$ *on (3.3) gives*

$$\hat{B}\left(\frac{r}{r_B}\right)^2 R_{10} = \left(\frac{1}{2r_B}\right)^{\frac{3}{2}} 2\left(\frac{r}{2r_B}\right)^2 e^{-\frac{r}{2r_B}} \tag{3.4}$$

Affecting $\hat{B}$ *for the second time on (3.3) gives*

$$\hat{B}^2\left[\left(\frac{r}{r_B}\right)^2 R_{10}\right] = \hat{B}\left[\left(\frac{1}{2r_B}\right)^{\frac{3}{2}} 2\left(\frac{r}{2r_B}\right)^2 e^{-\frac{r}{2r_B}}\right] = \left(\frac{1}{3r_B}\right)^{\frac{3}{2}} 2\left(\frac{r}{3r_B}\right)^2 e^{-\frac{r}{3r_B}} = \frac{3\sqrt{10}}{2} R_{32}$$

In more general it can be deduced that $\hat{B}^n\left(\frac{r}{r_B}\right)^n$ maps the ground radial state R_{10} into multiple of radial state $R_{n+1,n}$ such that

$$\hat{B}^n\left(\frac{r}{r_B}\right)^n R_{10} = \hat{B}^n\left[\left(\frac{r}{r_B}\right)^n 2\left(\frac{1}{r_B}\right)^{\frac{3}{2}} e^{-\frac{r}{r_B}}\right] = 2\left[\frac{1}{(n+1)r_B}\right]^{\frac{3}{2}}\left[\frac{r}{(n+1)r_B}\right]^n e^{-\frac{r}{(n+1)r_B}} \tag{3.5}$$

Theorem 3.2 *Let* $R_{10} = 2\left(\frac{1}{r_B}\right)^{\frac{3}{2}} e^{-\frac{r}{r_B}} = 2\left(\frac{1}{r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{0}\,|\frac{r}{r_B}\right)$ *be the ground radial state of the Hydrogen atom, then the radial function* R_{21}, R_{20}, R_{32}, R_{31}, *and* R_{30} *may be written in terms of linear combination of* $G_{0,1}^{1,0}$ *with different value of parameters:*

$$R_{21} = a_1\left(\frac{1}{2r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{1}\,|\frac{r}{2r_B}\right), \tag{3.6}$$

where $a_1 = \frac{2}{\sqrt{3}}$.

$$R_{20} = b_0\left(\frac{1}{2r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{0}\,|\frac{r}{2r_B}\right) + b_1\left(\frac{1}{2r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{1}\,|\frac{r}{2r_B}\right), \tag{3.7}$$

where $b_0 = 2$ *and* $b_1 = 2$.

$$R_{32} = c_1\left(\frac{1}{3r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{2}\,|\frac{r}{3r_B}\right), \tag{3.8}$$

where $c_1 = \frac{4}{3\sqrt{10}}$.

$$R_{31} = d_1\left(\frac{1}{3r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{1}\,|\frac{r}{3r_B}\right) + d_2\left(\frac{1}{3r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{2}\,|\frac{r}{3r_B}\right), \tag{3.9}$$

where $d_1 = \frac{8}{3\sqrt{2}}$ *and* $d_2 = \frac{4}{3\sqrt{2}}$.

$$R_{30} = e_0\left(\frac{1}{3r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{0}\,|\frac{r}{3r_B}\right) + e_1\left(\frac{1}{3r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{1}\,|\frac{r}{3r_B}\right) + e_2\left(\frac{1}{3r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(\bar{2}\,|\frac{r}{3r_B}\right), \tag{3.10}$$

where $e_0 = 2$, $e_1 = -4$ *and* $e_2 = \frac{4}{3}$.

Proof 2 *Choosing $z = r$, $\alpha = 1$, $\beta = 0$, and $\eta = \frac{1}{2r_B}$ in (2.3) gives*

$$G_{0,1}^{1,0}(_0^{-}|\frac{r}{2r_B}) = e^{-\frac{r}{2r_B}}$$

$$R_{21} = (\frac{1}{2r_B})^{\frac{3}{2}}\frac{1}{\sqrt{3}}(\frac{r}{2r_B})e^{-\frac{r}{2r_B}} = (\frac{1}{2r_B})^{\frac{3}{2}}\frac{2}{\sqrt{3}}(\frac{r}{2r_B})G_{0,1}^{1,0}(_0^{-}|\frac{r}{2r_B})$$

Thus we have

$$R_{21} = \frac{2}{\sqrt{3}}(\frac{1}{2r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_1^{-}|\frac{r}{2r_B})$$

$$R_{20} = b_0(\frac{1}{2r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_0^{-}|\frac{r}{2r_B}) + b_1(\frac{1}{2r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_1^{-}|\frac{r}{2r_B})$$

$$R_{32} = c_2(\frac{1}{3r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_2^{-}|\frac{r}{3r_B})$$

$$R_{31} = d_1(\frac{1}{3r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_1^{-}|\frac{r}{3r_B}) + d_2(\frac{1}{3r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_2^{-}|\frac{r}{3r_B})$$

$$R_{30} = e_1(\frac{1}{3r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_2^{-}|\frac{r}{3r_B}) + e_2(\frac{1}{3r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_1^{-}|\frac{r}{3r_B}) + e_3(\frac{1}{3r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_0^{-}|\frac{r}{3r_B})$$

Finally, we can suggest the following general formula

$$R_{nl} = \sum_{k=l}^{n-1} a_k(\frac{1}{nr_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_k^{-}|\frac{r}{nr_B}), \tag{3.11}$$

where n is principal quantum number, l is azimuthal quantum number which its value ranges from 0 to $n-1$.

Example 3.1 *Find the expression for radial function R_{60} in terms of Meijer's G-functions*
Since n=6 and $l = 0$, using (4.8) gives

$$R_{60} = \sum_{k=0}^{5} c_k(\frac{1}{6r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_k^{-}|\frac{r}{6r_B})$$

$$= c_0(\frac{1}{6r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_0^{-}|\frac{r}{6r_B}) + c_1(\frac{1}{6r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_1^{-}|\frac{r}{6r_B}) + c_2(\frac{1}{6r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_2^{-}|\frac{r}{6r_B}) +$$

$$c_3(\frac{1}{6r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_3^{-}|\frac{r}{6r_B}) + c_4(\frac{1}{6r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_4^{-}|\frac{r}{6r_B}) + c_5(\frac{1}{6r_B})^{\frac{3}{2}}G_{0,1}^{1,0}(_5^{-}|\frac{r}{6r_B})$$

Example 3.2 *Find the expression for radial function R_{52} in terms of Mei-jer's G-functions*
Since $n = 5$ and $l = 2$, using (4.8) gives

$$
\begin{aligned}
R_{52} &= \sum_{k=2}^{4} d_k \left(\frac{1}{5r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(_k^- \Big| \frac{r}{5r_B}\right) \\
&= d_2 \left(\frac{1}{5r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(_2^- \Big| \frac{r}{5r_B}\right) + d_3 \left(\frac{1}{5r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(_3^- \Big| \frac{r}{5r_B}\right) + d_4 \left(\frac{1}{5r_B}\right)^{\frac{3}{2}} G_{0,1}^{1,0}\left(_4^- \Big| \frac{r}{5r_B}\right)
\end{aligned}
$$

4   CONCLUSION

The radial functions such as R_{20} and R_{21} have crucial role in making superposition of orbitals and the formation of chemical bonds such as SP^2 hybridization. Using of operator method by introducing Bohr operator, we show that radial functions can be represented with summation of Meijer's G-functions. Absorption or emission of photons can be occurred mathematically through changing in combination of Meijer's G-functions.

References

[1] S. Anchala, P. Purohit, and K. C. Mathur, Two photon absorption processes in silicon Quantum Dot embedded in SiO_2 and Al_2O_3 Matrix, J. Comput. Theor. Nanosci., 11 (2014), 2578-2584. http://dx.doi.org/10.1166/jctn.2014.3675

[2] L. C. Andrews, Special Functions for Engineers and Applied Mathematicians, MacMillan, New York, 1985.

[3] R. A. Askey, Meijer G-function, Cambrige University Press, 2010.

[4] R. Beals and J. Szmigielski, Meijer G-functions: A Gentle Introduction, Notices of AMS, 60 (2013), 866-872. http://dx.doi.org/10.1090/noti1016

[5] P. K. Bose, N. Patya, S. Bhattacharya, D. De, S. Saha, K. M. Chatterjee, S. Pahari, and K. P. Ghatak, Influence of light waves on the effective electron mass in Quantum Wells, Wires, Inversion layers and Superlattices, Quantum Matter, 1 (2012), 89-126. http://dx.doi.org/10.1166/qm.2012.1009

[6] H. Bruus, Introduction to Nanotechnology, Department of Micro and Nanotechnology, Technical University of Denmark, 2004.

[7] M. Daoud, Photon-added coherent states for exactly solvable Hamiltonians, Phys. Lett. A, 305 (2002), 135-143. http://dx.doi.org/10.1016/s0375-9601(02)01441-x

[8] I. O. Dogan and Y. Yazicioglu, Structural vibration analysis of single walled carbon nanotubes with atom vacancies, J. Comput. Theor. Nanosci., 11 (2014), 2263-2275. http://dx.doi.org/10.1166/jctn.2014.3635

[9] A. Elsayed, S. Wageh, and A. A. El-Azm, Theoretical Study of Hybrid Light Emitting Device Based on Core-Shell Quantum Dots as an Emission Layer, Quantum Matter, 2 (2013), 109-115. http://dx.doi.org/10.1166/qm.2013.1032

[10] E. E. Fitchard and V. Franco, Differential properties of Meijer's G-function, J. Phys. A: Math. Gen., 13 (1980), 2331-2340. http://dx.doi.org/10.1088/0305-4470/13/7/016

[11] V. Kiryakova, Generalized Fractional Calculus and Applications, Longman, Harlow, UK, 1994.

[12] A. U. Klimyik, Meijer G-function, Springer, 2001.

[13] V. Lozovski and V. Piatnytsia, The Analytical study of electronic and optical properties of Pyramid-like and Cone-like Quantum Dots, J. Comput. Theor. Nanosci., 8 (2011), 2335-2343. http://dx.doi.org/10.1166/jctn.2011.1965

[14] Y. L. Luke, The Special Functions and Their Approximations Complex Variables, Acamedic Press, New York, 1969.

[15] I. H. Nayyar, A. E. Masunov, Two-Photon absorption spectra predicted by semiempirical methods, J. Comput. Theor. Nanosci., 11 (2014), 2208-2220. http://dx.doi.org/10.1166/jctn.2014.3629

[16] A. Pishkoo, M. Darus, Some applications of Meijer G-Functions as solutions of differential equations in physical models, Journal of Mathematical Physics, Analysis, Geometry, 9 (2013), 379-391.

[17] A. Pishkoo and M. Darus, Meijer's G-Functions (MGFs) in Micro- and Nano-structures, J. Comput. Theor. Nanosci., 10 (2013), 2478-2483. http://dx.doi.org/10.1166/jctn.2013.3234

[18] A. Pishkoo, M. Darus, On Meijer's G-Functions (MGFs) and its applications, Rev. Theor. Sci., 3 (2015), 216-223. http://dx.doi.org/10.1166/rits.2015.1034

[19] A. Pishkoo, M. Darus, G-Function solutions for Schr'odinger equation in cylindrical coordinates system, Applied Mathematics, 5 (2014), 342-346. http://dx.doi.org/10.4236/am.2014.53035

[20] A. Pishkoo, The Meijers G-functions convenient for describing β & γ-decays, Journal of Nuclear Physics, Material Sciences, Radiation and Applications, 2 (2014), 25-31.

[21] A. Pishkoo, The simplest Meijers G-function as the radial functions of the Hydrogen atom, Bulletin of Society For Mathematical Services & Standards, 3 (2014), 1-5.

[22] R. K. Saxena, A. M. Mathai and H. J. Haubold, Astrophysical thermonu-
     clear functions for BoltzmannGibbs statistics and Tsallis statistics, Phys-
     ica A: Statistical Mechanics and its Applications, 344 (2004), 649-656.
     http://dx.doi.org/10.1016/j.physa.2004.06.047

[23] M. van den Broek and F. M. Peeters, Confined states in two-
     dimensional flat elliptic quantum dots and elliptic quantum wires, Phys-
     ica E: Low-Dimensional Systems and Nanostructures, 11 (2001), 345-355.
     http://dx.doi.org/10.1016/s1386-9477(01)00169-2