

The Bhatia–Thornton Structure Factor

“Number Density – Number Density” for the Hard-Core Fluid in Random Phase Approximation

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Abstract

The expression for the Bhatia-Thornton partial structure factor “number density – number density” for arbitrary two-component hard-core fluid in the random phase approximation is obtained.

Keywords: Hard-core mixture, random phase approximation, Bhatia-Thornton partial structure factors

The Bhatia-Thornton [1] structure factor “number density – number density”, $S_{NN}(q)$, is connected with the Ashcroft-Langreth (AL) partial structure factors [2], $S_{ij}(q)$ (where $i, j=1,2$), by the following way:

$$S_{NN}(q) = c_1 S_{11}(q) + c_2 S_{22}(q) + 2\sqrt{c_1 c_2} S_{12}(q) \quad . \quad (1)$$

Here, c_i is the concentration of the i -th component.

The random phase approximation (RPA) for a classical binary fluid is formulated as follows:

$$c_{ij}^{\text{RPA}}(r) = c_{0ij}(r) - \beta \varphi_{1ij}(r) \quad , \quad (2)$$

where $c_{ij}(r)$ is the partial direct correlation function, $\varphi_{ij}(r)$ - partial pair interatomic potential, $\beta = (k_B T)^{-1}$, k_B - Boltzmann constant, T - temperature, symbols “0” and “1” are attributes of a reference system and perturbation, respectively.

Arbitrary hard-core (HC) model potential, $\varphi_{ij}^{\text{HC}}(r)$, can be written as follows:

$$\varphi_{ij}^{\text{HC}}(r) = \begin{cases} \infty, & r < \sigma_{ij} \\ \phi_{ij}(r), & r \geq \sigma_{ij} \end{cases}, \quad (3)$$

where σ_{ij} are the partial HC diameters. For this case, the hard-sphere (HS) model is the reference system and Eq. (2) is being rewritten as follows:

$$c_{ij}^{\text{RPA-HC}}(r) = c_{ij}^{\text{HS}}(r) - \beta\phi_{ij}(r). \quad (4)$$

In the wave space Eq. (4) is

$$c_{ij}^{\text{RPA-HC}}(q) = c_{ij}^{\text{HS}}(q) - \beta\phi_{ij}(q). \quad (5)$$

Recently, for the potential (3) within the RPA, the expressions for AL partial structure factors were obtained [3]:

$$S_{ii}^{\text{RPA-HC}}(q) = \frac{1 - c_j \rho c_{jj}^{\text{HS}}(q) + c_j \rho \beta \phi_{jj}(q)}{Z(q)}, \quad (6)$$

$$S_{ij}^{\text{RPA-HC}}(q) = \frac{\sqrt{c_i c_j} \rho c_{ij}^{\text{HS}}(q) - \sqrt{c_i c_j} \rho \beta \phi_{ij}(q)}{Z(q)}, \quad (7)$$

where ρ - mean atomic density of a mixture, $i \neq j$,

$$Z(q) = \prod_{k=1,2} (1 - c_k \rho c_{kk}^{\text{HS}}(q)) - c_i c_j \rho^2 c_{ij}^{\text{HS}2}(q) + \prod_{k=1,2} (1 + c_k \rho \beta \phi_{kk}(q)) - c_i c_j \rho^2 \beta^2 \phi_{ij}^2(q) - \\ - 1 - c_i c_j \rho^2 \beta \left(\sum_{i,j} c_{ii}^{\text{HS}}(q) \phi_{jj}(q) - 2 c_{ij}^{\text{HS}}(q) \phi_{ij}(q) \right). \quad (8)$$

Here, we combine Eq. (1) with Eqs. (6)-(8) to obtain the RPA-HC expression for $S_{NN}(q)$:

$$S_{NN}^{\text{RPA-HC}}(q) = \frac{1 + c_1 c_2 \rho \{ 2 c_{12}^{\text{HS}}(q) - c_{11}^{\text{HS}}(q) - c_{22}^{\text{HS}}(q) + \beta \nu(q) \}}{Z(q)}, \quad (9)$$

where $\nu(q)$ is the Fourier transform of the ordering potential,

$$\nu(r) = \phi_{11}(r) + \phi_{22}(r) - 2\phi_{12}(r). \quad (10)$$

References

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