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Effect of Electron Inertia and Permeability on Jeans Instability of Viscous Uniformly Magnetized Gaseous Plasma in the Presence of Suspended Particles

R. K. Pensia

Department of Physics,
Govt. P. G. College, Neemuch(M.P.) India
rkpensia@rediffmail.com

Vikas Prajapat

Department of Physics,
Govt. P. G. College, Neemuch (M.P.) India.
vikasprajapat712@gmail.com

Vishal Kumar

Department of Physics,
Govt. P. G. College, Neemuch (M.P.) India
vishalksurati@gmail.com

Ashok Kumar Patidar

Department of Physics,
Govt. P. G. College, Neemuch (M.P.) India
ashok_patidar341@yahoo.co.in

Abstract

The effect of finite electron inertia and permeability on self-gravitational instability of viscous and uniformly magnetized gaseous plasma in the presence of suspended particles has been investigated. A general dispersion relation is obtained by using the normal mode analysis with the help of relevant linearized perturbation equations of the problem and a modified Jeans criterion on of instability is obtained.

Keywords: Viscosity, Suspended Particles, Permeability, Interstellar medium (ISM) and Jeans criterion

1. Introduction

There has been a rapidly growing interest in understanding various collective process in gaseous plasma, which are ubiquitous in space, including diffuse and dense interstellar media, circum stellar shells, star envelopes, nova eject a, disk, accretion, dark interiors molecule clouds, ionosphere and the out flow of red giant star. It has been found both theoretically and experimentally that the presence of suspended particles, permeability and finite electron inertia modifies the existing plasma wave spectra[1-3]. James Jeans[4] first studied the gravitational instability of infinite homogeneous gaseous plasma and he suggested that an infinite homogeneous self-gravitating fluid is unstable for all wave number which is less than critical Jeans wave number. In this connection, many investigators have discussed the Jeans instability of a homogeneous plasma considering the effects of various parameters[5-6].In addition to this magnetic field can provide pressure support and inhibit the contraction and fragmentation of interstellar clouds. In the interstellar medium (ISM), a large amount of energy is injected by the star, which leads to the formation of shock waves; but when these shock waves weaken, they become large amplitude hydromagnetic Alfven waves. In this connection many investigators have discussed the contribution of the magnetic field in the ISM[11-13].

Along with this, the importance of permeability, electron inertia viscosity, and the presence of suspended particles have been considered widely to study the gravitational instability of a homogeneous plasma. These parameters are important in the problems of magnetic reconnection processes, stability of accelerated plasma and in the plasma confinement problem. The presence of electron inertia gives fundamental knowledge about the wave propagation in the system with a finite plasma frequency. The nature of the coupling of the magnetic field to the neutrals through ion neutral collisions has been studied by Spitzer[14-15]. The effect of suspended particles on gravitational instability has been studied by[16-18].

It is established fact that the viscous force provides the damping effect on the growth rate of the system in astrophysical problems. It is of great importance to incorporate the contribution of the kinematic fluid viscosity in the study of the gravitational instability of a homogeneous plasma. From the above studies, we find that the viscosity, permeability, finite electron inertia and suspended particles are the important parameters to discuss gravitational instability of plasma. Thus in the present problem, we investigate the effects of electron inertia and permeability on gravitational instability of viscous, magnetized gaseous plasma in the presence of suspended particles.

2. Linearized Perturbation Equations of the Problem

We consider an infinite homogeneous viscous, self-gravitating gas particle medium including finite electrical resistivity, finite electron inertia, permeability and suspended particles. It is assumed that the above medium is permeated with a uniform magnetic field $\vec{H}(o,o,H)$.

Let $\vec{u},\vec{v},\rho$ and N be the gas velocity, the particle velocity, the density of gas and the number density of particles. If we assume uniform particle size, spherical shape and small relative velocities between the two phases, than the net effect of particles on the gas in equivalent to extra body force term per unit volume $\kappa = 6\pi\rho\upsilon r,$ where r being the particle radius and υ is the kinetic viscosity of clean gas. Self-gravitational attraction U is added with kinetic viscosity term in equation of motion for gas.In writing the equation of motion of particles, we neglect the Buoyancy force as its stabilizing effect for the case of two free boundaries is extremely small. Inter particles reactions also ignored by assuming the distance between particles to be too large compared with their diameters. The stability of the system is investigated by writing the solutions to the fall equations as initial state plus a perturbation. The initial state of the system is taken to be a quiescent layer with uniform particles distribution. The equations thus obtained are liberalized by neglecting the product of two perturbed quantities. Thus the linearzed perturbations equations with finite electron inertia, permeability suspended particles governing the motion of hydromagetic fluid plasmas are given by

$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho}\vec{\nabla}\delta p + \vec{\nabla}\delta U + \frac{\kappa N}{\rho}(\vec{v}-\vec{u}) + \upsilon\left(\nabla^2\vec{u} - \frac{\vec{u}}{k_1}\right) + \frac{1}{4\pi\rho}(\vec{\nabla}\times\vec{h})\times\vec{H} \tag{1}$$

$$\frac{\partial \delta\rho}{\partial t} = -\rho\vec{\nabla}.\vec{u} \tag{2}$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\vec{v} = \vec{u} \tag{3}$$

$$\delta p = C^2 \delta\rho \tag{4}$$

$$\nabla^2 \delta U = -4\pi G \delta\rho \tag{5}$$

$$\vec{\nabla}.\vec{h} = 0 \tag{6}$$

$$\frac{\partial \vec{h}}{\partial t} = \vec{\nabla} \times \left(\vec{u} \times \vec{H}\right) + \frac{C^2}{4\pi\omega_{pe}^2}\frac{\partial}{\partial t}\nabla^2 \vec{h} \tag{7}$$

Where $\delta\rho, \delta p, u(u_x, u_y, u_z), \delta U$ and h denote respectively the perturbation in density ρ pressure p, velocity of the gas, gravitational potential U and magnetic field. G Is the gravitational constant, C is the velocity of sound. $\tau = \frac{m}{\kappa}$ and mN is the mass of particles per unit volumes and ω_{pe} is the electron plasma frequency.

3. Dispersion Relation

Let us assume the perturbation of all the quantities vary as

$$\text{Exp } \left[i\left(K_x x + K_z z + \omega t\right)\right] \tag{8}$$

Where ω is the growth rate of the perturbation and K_x, K_z are the wave number of the perturbation along the X-axis and Z-axis respectively such that

$$K_x^2 + K_z^2 = K^2 \tag{9}$$

Using expression (8) and (9), Equations (1)-(7) give

$$-\tau\omega^3 + i\omega^2\xi_1 + \left(\Omega_v + \frac{\tau K^2 V^2}{\alpha}\right)\omega - \frac{iK^2V^2}{\alpha}\Big]u_x = -\omega\left(\frac{iK_x}{K^2}\right)\Omega_\rho^2 \xi_2 s \tag{10}$$

$$-\tau\omega^3 + i\omega^2\xi_1 + \left(\Omega_v + \frac{\tau K_z^2 V^2}{\alpha}\right)\omega - i\frac{K_z^2V^2}{\alpha}\Big]u_y = 0 \tag{11}$$

$$\left[-\tau\omega^2 + i\omega\xi_1 + \Omega_v\right]u_z = \frac{-iK_z\Omega_j^2\xi_2 s}{K^2} \tag{12}$$

Where

$$\Omega_j^2 = C^2K^2 - 4\pi G\rho$$

$V^2 = \frac{H^2}{4\pi\rho}$, V is the Alfven velocity.

$s = \frac{\delta\rho}{\rho}$ Is the condensation of the medium

$C = \left(\frac{\gamma p}{\rho}\right)^{1/2}$ is the adiabatic velocity of round is the medium

$$\Omega_v = \upsilon\left(K^2 + \frac{1}{k_1}\right) \qquad \xi_1 = 1 + \Omega_v \tau + \frac{KN\tau}{\rho}, \alpha = 1 + \frac{C^2 K^2}{4\pi\omega_{pe}^2}$$

And

$$\xi_2 = 1 + i\tau\omega$$

Now taking the divergence of Equation - (1) with the use of Equations (2)-(7) and expression (8)-(9) we get.

$$\frac{K_x K^2 V^2}{\alpha}\xi_2 u_x + \left[i\tau\omega^4 + \xi_1\omega^3 - \left(\Omega_v + \tau\Omega_j^2\right)i\omega^2 - \Omega_j^2\omega\right]s = 0 \tag{13}$$

Now Equations (10),(11),(12) and (13) can be written in the matrix form. From equation (14) we obtain the general dispersion relation

$$\begin{vmatrix} -\tau\omega^3 + i\omega^2\xi_1 + \left(\Omega_v + \frac{\tau K^2 V^2}{\alpha}\right)\omega - \frac{iK^2V^2}{\alpha} & 0 & 0 & iK_x\Omega_j^2\omega\xi_2 \\ 0 & -\tau\omega^3 + i\omega^2\xi_1 + \omega\left(\Omega_v + \frac{\tau K_z^2 V^2}{\alpha}\right) - \frac{iK_z^2V^2}{\alpha} & 0 & 0 \\ 0 & 0 & -\tau\omega^2 + i\omega\xi_1 + \frac{4}{3}\upsilon K^2 & -\frac{iK_z}{K^2}\Omega_j^2\xi_z \\ \frac{K_x K^2 V^2}{\alpha}\xi_2 & 0 & 0 & i\tau\omega^4 + \omega^3\xi_1 - i\omega^2\left(\Omega_v + \tau\Omega_j^2\right) - \Omega_j^2\omega \end{vmatrix} \begin{vmatrix} u_x \\ u_y \\ u_z \\ s \end{vmatrix} = 0 \tag{14}$$

From Equation (14) we obtain general dispersion relation

$$\omega\left[-\tau\omega^3+i\omega^2\xi_1+\omega\left(\Omega_v+\frac{K_z^2V^2}{\alpha}\right)-\frac{iK_z^2V^2}{\alpha}\right]\times\left[\tau\omega^2-i\omega\xi_1-\Omega_v\right]$$

$$\left[i\tau^2\omega^6+2\tau\xi_1\omega^5-\tau\left\{\left(\tau\Omega_j^2+\frac{\tau K^2V^2}{\alpha}\right)+\Omega_v\right\}+\xi_1\right]i\omega^4$$

$$-\left\{\xi_1\left(\tau\Omega_j^2+\frac{K^2V^2\tau}{\alpha}+2\Omega_v\right)+\tau\left(\Omega_j^2+\frac{K^2V^2}{\alpha}\right)\right\}\omega^3+ \qquad (15)$$

$$\left\{\Omega_v\left(\Omega_v+\tau\Omega_j^2\right)+\frac{\Omega_vK^2V^2}{\alpha}+\frac{\tau^2K_z^2V^2\Omega_j^2}{\alpha}+\left(\frac{\mathrm{K}^2\mathrm{V}^2}{\alpha}+\Omega^2\right)\right\}i\omega^2$$

$$+\left\{\Omega_j^2\left(\Omega_V+2\tau K_z^2V^2\right)+\frac{\Omega_VK^4V^2\nu}{\alpha}\right\}\omega-\frac{iK_z^2V^2\Omega_j^2}{\alpha}\Bigg]=0$$

Equation (15) gives the general dispersion relation for an infinite homogeneous, self-gravitating, viscous gaseous plasmas medium incorporating permeability, finite electron inertia and suspended particles with permeated with uniform magnetic field.

4. Discussion

We now, discuss the dispersion relation (15) into two mode of propagation (1) propagation paralleled to the direction of magnetic field, (ii) Propagation perpendicular to the direction of magnetic field.

4.1 Propagation Parallel to the Direction of Magnetic Field

For this case we take $K_x = 0$ and $K_z = K$ and $i\omega = \sigma$

The perturbation are taken parallel to the magnetic field, the dispersion relation -(15), has four independent factors, each represents the mode of propagation incorporating different parameters.

The first factor of equation - (15) for parallel to the direction of magnetic field gives

$$\sigma = 0 \qquad (16)$$

Which is a neutrally stable mode.

The second factor of Equation (15) equating to zero, we get

$$\tau\sigma^3+\sigma^2\xi_1+\sigma\left(\Omega_v+\frac{\tau K^2V^2}{\alpha}\right)+\frac{K^2V^2}{\alpha}=0 \tag{17}$$

Equation (17) represents the dispersion relation for an infinite homogeneous, viscous, magnetized gaseous plasmas medium incorporating suspended particles, permeability and finite electron inertia. This mode of propagation is independent of self-gravitation and represents modified Alfven mode due to viscosity, permeability, finite electron inertia and suspended particles. Equation (17) does not allow a positive real root or a complex root whose real part is positive and so the system is stable.

If $\sigma_1,\sigma_2 and \sigma_3$ are the three mode of the growth rate of perturbation, then we have

$$\sigma_1+\sigma_2+\sigma_3=-\left[\frac{1}{\tau}+\Omega_v+\frac{KN}{\rho}\right] \tag{18}$$

and $\sigma_1.\sigma_2.\sigma_3=\dfrac{K^2V^2}{\tau\alpha}$

(19)

Science for magnetized gaseous plasmas having finite electron inertia the value of $\dfrac{K^2V^2}{\tau\alpha}$ is positive, then all the coefficient will be positive and the three principal diagonal mixers of Hurwitz matrix will also be positive as follows.

$$\Delta_1=\left(\frac{1}{\tau}+\Omega_V+\frac{KN}{\rho}\right)>0$$

$$\Delta_2=\Omega_V\xi_1+\frac{\tau K^2V^2}{\alpha}\left(\Omega_V\tau+\frac{KN\tau}{\rho}\right)>0$$

$$\Delta_3=\frac{K^2V^2}{\alpha}\Delta_2>0$$

All the Δ's are positive, this is the sufficient condition for stability of the gaseous plasmas. The third factor of Equation (15) gives

$$\tau\sigma^2+\xi_1\sigma+\Omega_v=0 \tag{20}$$

Equation (20) does not allow any root whose real part is positive and so that the system is stable. The last factor of Equation (15) gives

$$\tau^2\sigma^6 + 2\tau\sigma^5\xi_1 + \sigma^4\left[\tau\left(\Omega_j^2 + \frac{K^2V^2}{\alpha}\right) + \Omega_v + \xi_1^2\right] + \sigma^3\left[\begin{array}{l}\tau\left(\Omega_j^2 + \frac{K^2V^2}{\alpha}\right)\\ + \xi_1\left(2\Omega_V + \tau\left(2\Omega_j^2 + \frac{K^2V^2}{\alpha}\right)\right.\end{array}\right]$$

$$+\sigma^2\left[\begin{array}{l}\left(\frac{K^2V^2}{\alpha} + \Omega_j^2\right)\xi_1 + \Omega_v\left(\Omega_v + \tau\Omega_j^2\right)\\ + \frac{\tau\Omega_v K^2V^2}{\alpha} + \frac{K^2V^2\Omega_j^2\tau^2}{\alpha}\end{array}\right] + \sigma\left[\Omega_j^2\left(\Omega_v + \frac{2\tau K^2V^2}{\alpha} + \frac{\Omega_v K^2V^2}{\alpha}\right.\right]$$

$$+\frac{K^2V^2\Omega_j^2}{\alpha} = 0 \tag{21}$$

This Equation represents the simultaneous effect of kinematic viscosity, permeability gravitational attraction, finite electron inertia magnetic field and presence of suspended particles.

It follows that when $\Omega_j^2 = C^2K^2 - 4\pi G\rho < 0$ then one of the roots of Equation (21) is positive, that means instability occurs with condition.

$$K < K_j = \sqrt{\frac{4\pi G\rho}{C^2}} \tag{22}$$

Which is Jeans criterion.

The system is unstable for all wave number when $K < K_j$

Thus we can say that in the case of longitudinal mode of propagation for an infinite homogeneous, uniformly magnetized viscous, self-gravitating gaseous plasmas medium incorporating permeability, finite electron inertia and suspended particle is unstable, when Jeans condition is satisfied.

When Jeans condition is not satisfied i.e.

$K > K_j = \sqrt{\frac{4\pi G\rho}{C^2}}$, the system is stable.

4.2 Propagation Perpendicular to the Direction Magnetic Field

For transverse mode of propagation $K_x = K$, and $K_z = 0$, the Equation (15) will becomes

$$\sigma^3\left[\sigma^2+\sigma\xi_1+\Omega_v\right]\left[\tau\sigma^2+\sigma\xi_1+\Omega_v\right]\begin{bmatrix}\tau^2\sigma^5+2\tau\sigma^4\xi_1+\sigma^3\tau\left(\tau\Omega_j^2+\frac{K^2V^2}{\alpha}+\Omega_4+\xi_1^2\right)+\\ \sigma^2\left\{\xi_1\left(\tau\Omega_j^2+\frac{K^2V^2}{\alpha}+2\Omega_v\right)+\tau\left(\Omega_j^2+\frac{K^2V^2}{\alpha}\right)\right\}+\\ \sigma\left\{\Omega_v(\Omega_v)+\tau\Omega_j^2\right\}+\frac{\Omega_v K^2V^2\tau}{\alpha}+\xi_1\left(\frac{K^2V^2}{\alpha}+\Omega_j^2\right)+\\ \Omega_v\left(\Omega_j^2+\frac{K^2V^2}{\alpha}\right)\end{bmatrix} \tag{24}$$

Equation (24) represents the dispersion relation for an infinite homogeneous, uniformly magnetized, viscous, self-gravitating gaseous plasmas medium incorporating permeability, finite electron inertia and suspended particles in the case of propagation perpendicular to the direction of the magnetic field. This dispersion relation also have four independent factors, each represents the mode of propagation incorporating different parameters.

Three mode of propagation of equation (24) are same as discussed in equation (16),(17) and (20) and all these modes represent stable mode of the system.

The last mode of propagation of Equation (24) gives

$$\begin{aligned}&\tau^2\sigma^5+\sigma^4 2\tau\xi_1+\sigma^3\left[\tau\left\{\tau\left(\Omega_j^2K^2V^2\right)+\Omega_4\right\}+\xi_1^2\right]\\&+\sigma^2\left[\tau\left(\Omega_j^2+\frac{K^2V^2}{\alpha}\right)+\xi_1\left\{2\Omega_v+\tau\left(\Omega_j^2+\frac{K^2V^2}{\alpha}\right)\right\}\right]\\&+\sigma\left[\xi_1\left(\frac{K^2V^2}{\alpha}+\Omega_j^2\right)+\Omega_v\left(\Omega_v+\tau\Omega_j^2\right)+\tau\frac{\Omega_vK^2V^2}{\alpha}\right]+\Omega_v\left(\Omega_j^2+\frac{K^2V^2}{\alpha}\right)=0\end{aligned} \tag{25}$$

The system is unstable when $\left(\Omega_j^2 + \frac{K^2V^2}{\alpha}\right) < 0$ then one the root of Equation (25) is positive that means instability occurs with condition

$$K < K_{j2} = \sqrt{\frac{4\pi G\rho}{C^2 + \frac{V^2}{\alpha}}} \tag{26}$$

It shows that the Jeans criterion of instability is modified by Alfven Velocity and finite electron inertia.

5. Conclusion

To summarize, we have dealt with the effect of electron inertia and permeability on self- gravitating instability of viscous uniformly magnetized gaseous plasma in the presence of suspended particles. The general dispersion relation is obtained, which is modified due to the presence of these parameters. We find that the Jeans condition remains valid but the expression of the critical Jeans wave number is modified. Alfven mode is modified by the presence of finite electron inertia. The viscosity and permeability have a stabilizing effect on the gravitational instability. It is also found that the viscosity, permeability and suspended particles have dissipative effect but do not affect the Jeans expression. In transverse direction it is found that the magnetic field has stabilizing effect.

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