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Path-Integral Quantization in Topologically (2+1) Massive Supergravity Theory. Diagrammatic to One Loop Structure

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Abstract

The path integral quantization for higher derivative Chern-Simons theories in (2+1) topologically massive supergravity is treated. The diagrammatic to one loop and the Feynman rules are constructed and later on, the regularization and renormalization of this higher derivative model is analysed in the framework of the perturbation theory.

1 Introduction

The quantum field theories in (2+1) dimensions have been studied with increasing interest because many interesting problems are present in the (2+1) dimensional physics[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. On the other hand, dynamical systems described by means of singular higher derivative Lagrangians were also investigated by several authors and it is a problem of current research in quantum field theory[13, 14, 15, 16, 17].

In a recent paper[18], we have considered the canonical quantum formalism for constrained Hamiltonian system with a singular higher order Lagrangian which describes the Chern-Simons (Ch-S) theories in (2+1) dimensions for the topological massive supergravity.

When higher derivative terms are added to the Lagrangian, the convergence of the corresponding Feynman diagrams can be improved[16, 19].

Due to the presence of the volume form $\varepsilon^{\mu\nu\rho}$ in the (2+1) Ch-S theories, the dimensional regularization cannot be used. Consequently, another mixed

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regularization method involving higher covariant derivative and Pauli-Villars method is applied [19]. This procedure preserves also the gauge invariance of the theory. In Ref.[19], it was shown how the technique to introduce higher derivative terms in the action, improves the behavior of propagators at large momentum, rendering the theory less divergent.

It is necessary to emphasize that the perturbative formalism developed in Ref.[19] is not a proper formalism for a higher derivative field theory. Really, in a higher derivative field theory the phase space is generated by means of the Ostrogradski transformation[16, 18, 19]. In fact, in Ref.[19] higher derivative terms are added to the action with the unique purpose to render the theory regularized. At the end of the procedure, the multiplicative parameter Λ in front of the higher derivative terms, which has dimensions of mass and acts as a cutoff, is removed by taking the $\Lambda \to \infty$ limit.

The main motivation of the present paper arises from the point of view of the field theory itself. More precisely, on how to treat quantum field theories described by singular higher devivative Lagrangians.

Starting from a suitable definition of propagators and vertices and after a diagrammatic for the higher derivative model is constructed, we must study the number of divergent diagrams given prescriptions about regularization and renormalization . The results must be confronted with those obtained for the corresponding model without higher derivative terms in the Lagrangian. Consequently, at least for this case, a response about the convenience or not to add higher derivative terms in the Lagrangian density can be given.

The paper is organized as follows. In section 2, some definitions and preliminaries are given. Results about Hamiltonian formalism for system with singular Lagrangians containing higher derivative terms for the topologically (2+1) massive supergravity theory are considered. In section 3, by extending the Faddeev-Senjanovic method to higher derivative theory, we perform the path integral quantisation. By defining an effective Lagrangian density, we study the Feynman rules and the diagrammatic corresponding to this system. In section 4, by means of a suitable definition of a "new" bosonic propagator, we analyze the one-loop structure of the theory from a perturbative point of view. In section 5, prescriptions about the ultraviolet behavior of the correction to the boson line, the correction to the fermion line, and the vertex correction can be done. Finally, in section 6, the conclusions are detailed.

2 Previous results

The gravity or supergravity in (2+1) dimensions are described by a Chern-Simons gauge theory of the Poincaré group ISO(2,1) or the de Sitter group

[20]. In particular, the (2+1)-dimensional planar physics was also studied for a long time by several authors [1, 3]. There is also mathematical interest in manifolds that are locally flat but topologically non-trivial [21]. From some time ago it is well known that the Poincaré supergravity in (2+1) dimensions can be described by a Chern-Simons term [22].
Moreover, pure U(1) and SU(N) Chern-Simons theories and topologically massive theory in (2+1) dimensions were quantized by means of the Dirac formalism [5]. The constraint structure and the symmetry properties of the dynamical system were also analyzed. The dynamical unitary and possible renormalizable topologically massive three-dimensional supergravity was also investigated [23]. The counting power is established, and the theory is really 1-loop finite. However, renormalizability requires not only correct power counting, but a concrete regularization to ensure that no anomalies appear.

2.1 Definitions

The starting point is to consider the (anti) commutation relation of the superalgebra and the curvature two forms in three dimensions. The superalgebra in three dimensions is composed by the Lorentz generators $M_{ab}(a, b = 1, 2, 3)$, the translations P_a, moreover the odd generators (supersymmetry generators) $Q^\alpha(\alpha = 1, 2)$. Hence, it is obtained the superalgebra given in [32].

In the first order geometrical formalism the dynamics is described by the 1-form fields $\mu^B = (V^a, \omega^{ab}, \psi^\alpha)$. The fields V^a, ω^{ab} and ψ_α are the dreibein, the Lorentz spin connection and the gravitino, respectively. The 2-forms $d\mu^B$ play the role of velocities. The curvature 2-forms corresponding to the above fields are called $R^B = (R^a, R^{ab}, R^\alpha)$, and are defined by:

$$R^B = d\mu^B - \frac{1}{2}C^B{}_{CD}\mu^D \wedge \mu^C \tag{1}$$

where $C_{ABC} = \gamma_{AD}C^D_{BC}$ are the graded structure constants and γ_{AB} is the constant symmetric Killing metric.

2.2 Supersymmetric Lagrangian density $\mathcal{L}$

The action to the supersymmetric extension of the three-dimensional topologically massive gravity is defined by means of a Lagrangian density containing higher-derivative terms and it is given by:

$$\mathcal{L} = \mathcal{L}_E + \mathcal{L}_{3/2} + \mathcal{L}_{CS} + \mathcal{L}_{TF}, \tag{2}$$

where $\mathcal{L}_E$ is the usual Einstein-Hilbert term, playing the role of the "mass" term, and it writes:

$$\mathcal{L}_E = -R = \frac{1}{2} L_{\mu a} R_{\nu\rho bc}\, \varepsilon^{abc}\, \varepsilon^{\mu\nu\rho}. \tag{3}$$

In equation (3) R is the scalar curvature, $L_{\mu a}$ are the components of the dreibein in the holonomic basis ($V^a =^3 L^a_\mu dx^\mu$) and $\omega_{\mu ab}$ are the corresponding components of the spin connection.

The gravitational Chern-Simons term is a higher derivative one, and it is viewed as the kinetic term,

$$\mathcal{L}_{CS} = \partial_\mu \omega_{\nu ab}\, \omega_\rho{}^{ab}\, \varepsilon^{\mu\nu\rho} - \frac{2}{3} \omega_\mu{}^{ab}\, \omega_{\nu b}{}^{c}\, \omega_{\rho ca}\, \varepsilon^{\mu\nu\rho} \tag{4}$$

Its fermionic part is the sum of the (dynamically trivial) Rarita-Schwinger action and a gauge-invariant topological term, of second derivative order, analogous to the gravitational one reads:

$$\mathcal{L}_{3/2} = i\, \bar{\psi}_\mu\, \partial_\nu \psi_\rho\ \varepsilon^{\mu\nu\rho} + i\, \bar{\psi}_\mu\, \omega_{\nu ab}\, \tau_c\, \psi_\rho\ \varepsilon^{abc} \varepsilon^{\mu\nu\rho} \tag{5}$$

$$\mathcal{L}_{TF} = \mathcal{D}_\alpha \bar{\psi}_\beta\, \varepsilon^{\mu\alpha\beta} \tau_a \tau_b\, \mathcal{D}_\rho \psi_\sigma\, \varepsilon^{\nu\rho\sigma} L_{\nu a} L_{\mu b} \tag{6}$$

where ψ is a Majorana spinor, and the covariant derivative is defined by:

$$\mathcal{D}_\mu \psi_\nu = \partial_\mu \psi_\nu + \omega_{\mu ab}\ \tau_c\ \psi_\nu\ \varepsilon^{abc} \tag{7}$$

The explicit expressions, in components, for the three curvatures are written as:

$$R_{\mu\nu}{}^{ab} = \partial_\mu \omega_\nu{}^{ab} + \omega_\mu{}^{ac} \omega_{\nu c}{}^{b} \tag{8}$$

$$R_{\mu\nu}{}^{a} = \partial_\mu L_\nu{}^{a} + \omega_\mu{}^{ab} L_{\nu b} - \frac{i}{2} \bar{\psi}_\mu\, \tau^a \psi_\nu \tag{9}$$

$$\bar{\rho}^{(\alpha)}_{\mu\nu} = \partial_\mu \psi^{(\alpha)}_\nu - \frac{1}{2} (\bar{\psi}_\mu \tau_{ab})^{(\alpha)} \omega_\nu{}^{ab} \tag{10}$$

So, the resulting Lagrangian density is:

$$\begin{aligned}
\mathcal{L} \;=\;& (\partial_\mu L_{\nu a}\, \omega_{\rho bc} + \omega_{\mu a}{}^{d}\, \omega_{\nu db}\, L_{\rho c})\, \varepsilon^{\mu\nu\rho} \varepsilon^{abc} \\
&+ (\partial_\mu \omega_{\nu ab}\, \omega_\rho{}^{ab} - \frac{2}{3} \omega_\mu{}^{ab}\, \omega_{\nu b}{}^{c}\, \omega_{\rho ca})\, \varepsilon^{\mu\nu\rho} \\
&+ i\, \bar{\psi}_\mu\ \partial_\nu \psi_\rho\ \varepsilon^{\mu\nu\rho} + i\, \bar{\psi}_\mu\, \omega_{\nu ab}\, \tau_c\, \psi_\rho\ \varepsilon^{abc} \varepsilon^{\mu\nu\rho} \\
&+ i\, \mathcal{D}_\mu \bar{\psi}_\nu\, \tau^a \tau^b\, \mathcal{D}_\tau \psi_\sigma\, L_{\gamma a} L_{\rho b}\, \varepsilon^{\gamma\tau\sigma} \varepsilon^{\mu\nu\rho}
\end{aligned} \tag{11}$$

2.3 Second-order Hamiltonian formalism

From the Lagrangian density (11) we are able to construct the second-order formalism, which is obtained by considering the following equation of motion.

$$R^a_{\mu\nu} = 0 \tag{12}$$

By using the equation (8) for this curvature, it can be obtained:

$$\begin{aligned}\omega_{\mu ab}(L,\psi) &= \frac{1}{2}L^\nu_a\left(\partial_\mu L_{\nu b} - \partial_\nu L_{\mu b}\right) - \frac{1}{2}L^\nu_b\left(\partial_\mu L_{\nu a} - \partial_\nu L_{\mu a}\right) \\ &\quad -\frac{1}{2}L^\rho_a L^\sigma_b\left(\partial_\rho L_{\sigma c} - \partial_\sigma L_{\rho c}\right)L^c_\mu \\ &\quad +\frac{1}{4}\left(\bar{\psi}_\mu \tau_a \psi_b - \bar{\psi}_\mu \tau_b \psi_a + \bar{\psi}_a \tau_\mu \psi_b\right).\end{aligned} \tag{13}$$

Therefore, the Lagrangian density only depends on the graviton field ${}^3L_{a\mu}$ and the gravitino field $\bar{\psi}_\mu$. The equation (13) is used to eliminate $\omega_{\mu ab}(L,\psi)$ as independent dynamical variable. The Lagrangian density (11) contains second times derivatives on the dreibein components and because of the form of the term of the Lorentz-Chern-Simons expression it is not possible to eliminate it by partial integration. Consequently, we are in the presence of a constrained Hamiltonian system with a singular higher-order Lagrangian, in the framework of the Dirac formalism. We consider the Ostrogradski transformation to introduce canonical momenta in this higher derivative theory [16, 17]. We will work as close as possible to the Dirac conjectures [25]. We start by defining the following independent dynamical field variables: ${}^3L_{a\mu}$, $B_{a\mu} = \partial_0\, {}^3L_{a\mu}$ and $\bar{\psi}_\mu$. The Ostrogradski transformation [28] respectively introduces the following canonical momenta:

$$\overset{(1)}{\Pi}{}^\mu_c = \frac{\partial \mathcal{L}}{\partial \dot{L}^c_\mu} - \partial_\nu \frac{\partial \mathcal{L}}{\partial(\partial_\nu \dot{L}^c_\mu)} \tag{14}$$

$$\overset{(2)}{\Pi}{}^\mu_c = \frac{\partial \mathcal{L}}{\partial(\partial_0 \dot{L}^c_\mu)} \tag{15}$$

$$\Pi^\mu(\psi) = \frac{\partial \mathcal{L}}{\partial \dot{\bar{\psi}}_\mu}, \tag{16}$$

By computing the explicitly the above expressions, the canonical conjugate momenta (14)-(16) independent of the velocities, and the primary constraints associates, can be found (see Ref. [32]).

The remaining momentum which depends on the velocities is $\overset{(1)}{\Pi}{}^i_c$.

By means of these momenta, the canonical Hamiltonian remains defined by:

$$\mathcal{H}_{can} = B^c_\mu \overset{(1)}{\Pi}{}^\mu_c + \dot{B}^c_\mu \overset{(2)}{\Pi}{}^\mu_c - \dot{\bar{\psi}}_\mu \, \Pi^\mu(\psi) - \mathcal{L}, \tag{17}$$

where $\partial_0({}^3L^a_\mu)$ was replaced by B^a_μ. We note that the canonical Hamiltonian is formed by eliminating only the velocity $\dot{B}^a_\mu$. The field B^a_μ cannot be eliminated from the formalism when we treat with higher derivative Lagrangians [16]. Once the Lagrangian (11) is used and the velocities $\dot{B}^a_\mu$ and $\dot{\bar{\psi}}_\mu$ are eliminated, the final expression for $\mathcal{H}_{can}$ is:

$$\begin{aligned}
\mathcal{H}_{can} \;=\; & 2B^{0b} B_i{}^a \omega_{jab}\, \varepsilon^{0ij} + B_i{}^b \partial_j \Big(L^{ia} \omega_{kab} \Big)\, \varepsilon^{0jk} \\
& - B_i{}^c \partial_j \Big(L^{0a} L^{ib} L_{0c}\, \omega_{kab} \Big)\, \varepsilon^{0jk} \\
& - B_i{}^c\, \partial_0 \Big(L^{0a} L^{ib} L_{jc}\, \varepsilon^{0jk} \Big)\, \omega_{kab} + 2L^{ib} B_i{}^c \partial_j L_k{}^a \varepsilon_{abc}\, \varepsilon^{0jk} \\
& + L_j{}^c \omega_k{}^{bd} \Big(B_i{}^d L^{ia} - B_i{}^a L^i{}_d - B_i{}^e L^{0a} L^i{}_d L_{0e} \Big)\, \varepsilon_{abc}\, \varepsilon^{0jk} \\
& - \Big(\omega_i{}^{ab} \partial_j \omega_{0ab} + \omega_0^{ab} \partial_i \omega_{jab} - 2\omega_0{}^{ab} \omega_{ib}{}^c\, \omega_{jca} \Big)\, \varepsilon^{0ij} \\
& - 4\, L^{ia} B_i{}^b \omega_{jb}{}^c\, \omega_{kca}\, \varepsilon^{0jk} + \omega_{idb} \Big(\omega_{ja}{}^d L_{0c} - 2\omega_{0a}{}^d L_{jc} \Big)\, \varepsilon^{abc} \varepsilon^{0ij} \\
& + \left(\partial_i L_{0a}\, \omega_{jbc} - \partial_i L_{ja}\, \omega_{0bc} \right) \varepsilon^{abc}\, \varepsilon^{0ij} \\
& + L^{kb} \left(\partial_i B_k{}^a - \partial_k B_i{}^a \right) \omega_{jab}\, \varepsilon^{0ij} \\
& - \frac{1}{2} L^{kb} L^{la} \left(\partial_k B_l{}^c - \partial_l B_k{}^c \right) L_{ic}\, \omega_{jab}\, \varepsilon^{0ij} \\
& - i\, \partial_i \bar{\psi}_\rho\, \psi_\mu\; \varepsilon^{\mu i \rho} - 2i\, \bar{\psi}_0\, \tau_c\, \psi_j\; \omega_{iab}\, \varepsilon^{abc} \varepsilon^{0ij} \\
& - i\, \bar{\psi}_j\, \tau_c\, \psi_i\; \omega_{0ab}\, \varepsilon^{abc} \varepsilon^{0ij} - 2i\, \bar{\psi}_j\, \tau^c\, \psi_k L^{ia} B_i{}^b \varepsilon_{abc}\, \varepsilon^{0jk} \\
& - i B_i{}^g \omega_{0cd}\, \bar{\psi}_j\, \tau_e \tau_\gamma \tau_k (\tau^s \varepsilon_{0gs}\, \varepsilon^{\gamma i \sigma} + \tau_s\, L_{lg}\, \varepsilon^{0is} \varepsilon^{\gamma l \sigma}) \psi_\sigma\, \varepsilon^{cde} \varepsilon^{0jk} \\
& - i\, \omega_{0cd}\, \bar{\psi}_i\, \tau_e \tau_\gamma \tau_j\, \mathcal{D}_k \psi_\sigma\, \varepsilon^{\gamma k \sigma} \varepsilon^{cde} \varepsilon^{0ij} + i\, \mathcal{D}_j \bar{\psi}_\mu\, \tau_\gamma \tau_\nu\, \mathcal{D}_k \psi_\sigma\, \varepsilon^{\gamma \sigma k} \varepsilon^{\mu \nu j} \\
& - i B_{ig}\, \bar{\psi}_j\, \tau_e \tau_\mu \tau_k\, \mathcal{D}_l \psi_\nu (L_0{}^g \varepsilon^{0ie} - \varepsilon^{ige}) \varepsilon^{\mu l \nu} \varepsilon^{0jk}
\end{aligned} \tag{18}$$

Finally, we can write the extended Hamiltonian, which is a first class dynamical quantity and so, the generator of time evolutions of generic functionals writes:

$$H_T = \int d^2x\, \mathcal{H}_T. \tag{19}$$

The Hamiltonian density $\mathcal{H}_T$ remains defined by:

$$\mathcal{H}_T = \mathcal{H}_{can} + \overset{(1)}{\lambda}{}^c_0\, \overset{(1)}{\Phi}{}^0_c + \overset{(2)}{\lambda}{}^c_\mu\, \overset{(2)}{\Phi}{}^\mu_c + \bar{\delta}_\mu\, \Phi^\mu(\psi), \tag{20}$$

where $\overset{(1)}{\lambda}{}_0^c$ and $\overset{(2)}{\lambda}{}_\mu^c$ are arbitrary bosonic Lagrange multipliers, and $\bar{\delta}_\mu$ is arbitrary fermionic Lagrange multiplier.

Now, we must go on with the Dirac' s algorithm and impose the consistency conditions on the constraints according to:

$$\Omega^{(k)} = \dot{\Omega}^{(k-1)} = \left[\Omega^{(k-1)}, H_T\right] \approx 0. \tag{21}$$

Hence, for the bosonic constraint $\overset{(2)}{\Phi}{}_c^0$ we find the following secondary constraint:

$$\Omega^{(1)} = \dot{\overset{(2)}{\Phi}}{}_c^0 = \left[\overset{(2)}{\Phi}{}_c^0, H_T\right]_{PB} = -\overset{(1)}{\Pi}{}_c^0 + \partial_i \overset{(2)}{\Pi}{}_c^i \approx 0. \tag{22}$$

3 Path-integral quantization

The system we are treating has first and second class constraints and so, the path-integral quantization must be accomplished according to the Faddeev-Senjanovic formalism (FSF) (Faddeev, 1970-Senjanovic 1976). We have constructed the partition function for a higher derivative model. That was done by generalizing the expression given by (FSF) for the partition function. For the model containing higher derivative terms, we assume that the partition function in the Hamiltonian formalism is given by:

$$\begin{aligned} Z \;\; = \;\; & \int \mathcal{D}L_{\nu a}\, \mathcal{D}\Pi^{(1)\nu a}\, \mathcal{D}B_{\mu c}\, \mathcal{D}\Pi^{(2)\mu c}\, \mathcal{D}\bar{\psi}_{(\alpha)}\, \mathcal{D}\Pi^{(\alpha)} \delta(\Sigma_1)\, \delta(\Sigma_2)\, \delta(\Sigma_3) \\ & \delta(f_1)\, \delta(f_2)\, \delta(f_3)\, det[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3]_D \delta(\Omega_{(\alpha)})\, \delta(\Omega_{(\beta)}) \\ & det\left[\Omega_{(\alpha)}, \Omega_{(\beta)}\right] exp\; i \left[\int d^3x\, (B_\nu \Pi^{(1)\nu} + \dot{B}_\mu \Pi^{(2)\mu} + \dot{\bar{\psi}}\Pi - H_T\right], \end{aligned} \tag{23}$$

where the functional integration is performed over all the phase space volume corresponding to the independent dynamical variables $L_{\nu a}$, $B_{\mu c}$, and $\bar{\psi}_{(\alpha)}$.

In the equation (23) H_T is the extended Hamiltonian defined in (19). The quantities $f_i \approx 0$ are the gauge fixing conditions, one for each first-class constraints $\Sigma_i(x)$. They are all independent and really restrict the phase space variables to the physical one, and so the true Hilbert space is obtained. The gauge fixing conditions, for all the first-class constraints, must satisfy $\det\left[\Sigma_i, f_j\right]_D \neq 0$. Moreover, they must be compatible with the equations of motion and satisfy the condition $[f_i, f_j] \approx 0$.

Really, when higher derivative terms are present, only under certain assumptions depending on the gauge fixing conditions this matrix can be written as a suitable reversible non-local operator. In such case the determinant of this matrix results a non local functional linearly dependent of the gauge field $L_{\nu a}$.

From the path-integral formalism for gauge theories (Faddeev, 1970) it is well known that in the framework of perturbation theory, the determinant of a nonlocal operator M can be written in the integral representation, by using anticommuting scalar functions $\bar{\eta}$ and η. Thus, a new term is added to the partition function giving rise to an effective action.

So, in the higher derivative model under consideration, to obtain a suitable non-local operator M we must choose three particular gauge fixing conditions. Of course, this requirement does not avoid the arbitrariness to choose the functions $f_i \approx 0$. For instance, a convenient set of such conditions compatible with the equations of motion and verifiying $det[\Sigma_i, f_j]_D \neq 0$ for all first-class constraints Σ_i are, f_1 , f_2 and f_3:

$$f_1^a = B_0^a \approx 0 \;, \tag{24}$$

$$f_2^a = \partial_i B^{ai} \approx 0 \;, \tag{25}$$

$$f_3^a = \partial_i L^{ai} \approx 0 \;. \tag{26}$$

As we will see later on, the gauge fixing conditions allows to go over a general covariant gauge in which the non local operator M appearing in the path-integral quantization of Yang-Mills fields, take the well known covariant expression (se Ref. 27).

On the other hand, the matrix $[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3]_D$ can be written as follows:

$$[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3]_D = \begin{pmatrix} A & 0 & 0 \\ 0 & M^{ab} & B \\ 0 & 0 & M^{ab} \end{pmatrix} , \tag{27}$$

where A and M^{ab} are respectively given by:

$$A = \left[\Sigma_1^a, f_1^b\right] \tag{28}$$

$$M^{ab} = \left[\Sigma_3^a, f_3^b\right] . \tag{29}$$

and $B = \left[\Sigma_3^a, f_2^b\right]$ is a cumbersome functional depending on the fields. Anyway, the $det\,[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3]_D$ result independent of the functional B.

Therefore, we can write $det\left[\Sigma_1, \Sigma_2, \Sigma_3, f_1, f_2, f_3\right]_D = detA.detM^{ab}.\ detM^{ab}$ and so, the unique dependence on the field is present in the operator M^{ab}, which is reversible in the framework of perturbation theory. As the quantity $detA$ is independent of the dynamical variables, it is included in the normalization factor appearing in the partition function (23). The same thing can be said for the factor $det\left[\Omega_{(\alpha)}, \Omega_{(\beta)}\right]$ written in equation (23).

Before constructing the Feynman rules and the diagrammatic, we come again to the original gauge field $L_{\nu a}$. To do this, we must add to the partition function, equation (23), a term of the form $\int d^3x \Lambda^\mu \left(B_{\mu c} - \dot{L_{\nu a}}\right)$ with arbitrary multipliers Λ^μ and perform the integration on all their possible values.

Consequently, the partition function results:

$$
\begin{aligned}
Z \;=\; & \int \mathcal{D}L_{\nu a}\ \mathcal{D}\Pi^{(1)\nu a}\ \mathcal{D}B_{\mu c}\ \mathcal{D}\Pi^{(2)\mu c}\ \mathcal{D}\bar{\psi}_{(\alpha)}\ \mathcal{D}\Pi^{(\alpha)}\ \delta(\Sigma_1)\ \delta(\Sigma_2)\ \delta(\Sigma_3) \\
& \delta(f_1)\ \delta(f_2)\ \delta(f_3)\ detM^{ab}\delta(\Omega_{(\alpha)})\ \delta(\Omega_{(\beta)})\delta(B_\mu - \dot{L_{\nu a}}) \\
& exp\ i\left[\int d^3x\ (B_\mu \Pi^{(1)\mu} + \dot{B}_\nu \Pi^{(2)\nu} + \dot{\bar{\psi}}\Pi - H_T\right].
\end{aligned}
\tag{30}
$$

Now, by performing the path integral over the fields $B_{\mu c}$, $\Pi^{(1)\nu a}$, $\Pi^{(2)\mu c}$ and $\Pi^{(\alpha)}$, the partition function results:

$$
Z = \int \mathcal{D}L_{\nu a}\ \mathcal{D}\bar{\psi}_{(\alpha)}\ \delta\left(f_2\right)\delta\left(f_3\right)\left(detM\right)^2\ exp\ i\left[\mathcal{L}_{eff}\right], \tag{31}
$$

where the effective Lagrangian density $\mathcal{L}_{eff}$ is given by the following cumbersome expressions:

$$
\begin{aligned}
\mathcal{L}_{eff} \;=\; & -\frac{1}{2}B^{ic}B^{lb}\left(L_{0b}L_{ia} - L_{0a}\,L_{ib}\right)L_{ld}L_{kc}L_{je}\varepsilon^{0jk}\varepsilon^{ade} \\
& -\frac{1}{2}B_{ic}(\ B_{jb}L_{0a} - B_{ja}L_{0b})\,\varepsilon^{abc}\varepsilon^{0ij} - \frac{1}{2}B_i^c B_{ka}\Big(\partial_j L^{ia} - \partial^i L_j^a\Big)\,L_{0c}\varepsilon^{0jk} \\
& +B_0^c\partial_i L^{0a}(B_{jc}L_{0a} - B_{ja}L_{0c})\,\varepsilon^{0ij} + \frac{1}{2}B^{ic}\partial_j L_i^a(B_{ka}L_{0c} - B_{kc}L_{0a})\,\varepsilon^{0jk} \\
& -B_{ic}B^{kd}L_{0a}L_{kb}L_{jd}\varepsilon^{abc}\varepsilon^{0ij} + \frac{1}{2}B_i^c B_{kc}\Big(\partial_j L^{ia} - \partial^i L_j^a\Big)\,L_{0a}\varepsilon^{0jk} \\
& -\frac{1}{2}B_i^c B^{ld}\Big(\partial_j L^{ia} - \partial^i L_j^a\Big)(L_{0c}L_{la} - L_{lc}L_{0a})\,L_{kd}\varepsilon^{0jk} \\
& +\frac{1}{2}B_0^c\partial_k\Big(L^{0a}L^{ib}L_{jc}\Big)(B_{ia}L_{0b} - B_{ib}L_{0a})\,\varepsilon^{0jk} + \frac{1}{2}\partial_j L_{ka}\Big(B_c^i L_{ib} - B_b^i L_{ic}\Big)\,\varepsilon^{abc}\varepsilon^{0jk} \\
& +\frac{1}{2}B^{kd}B_i^a\lambda_0^{(1)c}(L_{0a}L_{kc} - L_{ka}L_{0c})\,L_{jd}\varepsilon^{0ij} + \frac{1}{2}B_i^a\lambda_0^{(1)c}(B_{jc}L_{0a} - B_{ja}L_{0c})\,\varepsilon^{0ij}
\end{aligned}
$$

$$
\begin{aligned}
&+B^{0b}B_i^a\Big(L_a^l(\partial_j L_{lb}-\partial_l L_{jb})-L_b^l(\partial_j L_{la}-\partial_l L_{ja})+L_a^0\partial_j L_{0b}-L_b^0\partial_j L_{0a}\Big)\,\varepsilon^{0ij}\\
&+B^{0b}B_i^a\Big(L_a^0L_b^lL_j^c\partial_l L_{0c}-L_a^kL_b^0L_j^c\partial_k L_{0c}-L_a^kL_b^lL_j^c(\partial_k L_{lc}-\partial_l L_{kc})\Big)\,\varepsilon^{0ij}\\
&+\frac{1}{2}\Big(B_i^b\partial_j L^{ia}-B_i^c\partial_j\Big(L^{0a}L^{ib}L_{0c}\Big)-B_i^c\partial_0\Big(L^{0a}L^{ib}L_{jc}\Big)\Big)\\
&\times\Big(-L_{0a}B_{mb}+L_{0b}B_{ma}-B_{lc}L_{0a}L_b^lL_m^c+B_{kc}L_a^kL0bL_m^c\Big)\,\varepsilon^{0jm}\\
&+\frac{1}{2}L_j^c\Big(B_k^bL^{0d}-B_k^dL^{0b}-B_{lc}L^{0b}L^{ld}L_k^c+B_{lc}L^{lb}L^{0d}L_k^c\Big)\\
&\times\Big(B_{id}L^{ia}-B_i^aL_d^i-B_i^cL^{0a}L_d^iL_{0c}\Big)\,\varepsilon^{abc}\varepsilon^{0jk}\\
&-L^{ia}B_i^b\Big(B_{jb}L_0^c-B_j^cL_{0b}-B_{ld}L_{0b}L^{lc}L_j^d\Big)\Big(L_c^0L_a^mL^{ke}\partial_m L_{0e}\Big)\,\varepsilon^{0jk}\\
&-\frac{1}{2}L^{ib}(\partial_i B_k^a-\partial_k B_i^a)\Big(B_{ja}L_{0b}-B_{jb}L_{0a}-\Big(B_{lc}L_{0a}L_b^l+B_{lc}L_a^lL_{0b}\Big)\,L_j^c\Big)\,\varepsilon^{0jk}\\
&-4B_i^bL^{ia}\Big(\partial_m L_b^mL_j^c\Big)\Big(\frac{1}{2}B_{kc}L_a^0-B_{ka}L_c^0-\frac{1}{2}B_{lb}L_c^0L_a^l\Big)\,\varepsilon^{0jk}\\
&+i\partial_i\bar\psi_0\,\psi_j\;\varepsilon^{0ij}-i\partial_i\bar\psi_j\,\psi_0\;\varepsilon^{0ij}+i\partial_j\bar\psi_\mu\,\tau_\gamma\tau_\nu\,\partial_k\psi_\sigma\,\varepsilon^{\gamma\sigma k}\varepsilon^{\mu\nu j}\\
&+\frac{1}{4}\partial_i L_{0a}\Big(\bar\psi_j\,\tau_b\,\psi_c-\bar\psi_j\,\tau_c\,\psi_b+\bar\psi_b\,\tau_j\,\psi_c\Big)\,\varepsilon^{abc}\varepsilon^{0ij}\\
&+\frac{1}{4}\partial_i L_{ja}\Big(\bar\psi_0\,\tau_b\,\psi_c-\bar\psi_0\,\tau_c\,\psi_b+\bar\psi_b\,\tau_0\,\psi_c\Big)\,\varepsilon^{abc}\varepsilon^{0ij}\\
&+\frac{1}{4}\Big(\partial_i L^{0a}\lambda_0^{(1)c}-B_i{}^a\lambda_0^{(1)c}\Big)\Big(\bar\psi_j\,\tau_a\,\psi_c-\bar\psi_j\,\tau_c\,\psi_a+\bar\psi_a\,\tau_j\,\psi_c\Big)\,\varepsilon^{0ij}\\
&+\frac{1}{4}L_c^k\lambda_i^{(2)c}\Big(\bar\psi_j\,\tau_k\,\psi_0-\bar\psi_j\,\tau_0\,\psi_k+\bar\psi_k\,\tau_j\,\psi_0\Big)\,\varepsilon^{0ij}\\
&+\frac{1}{4}L_{kc}\lambda^{(2)ic}\Big(\bar\psi_j\,\tau_i\,\psi_0-\bar\psi_j\,\tau_0\,\psi_i+\bar\psi_i\,\tau_j\,\psi_0\Big)\,\varepsilon^{0kj}\\
&+\frac{1}{4}\delta_0\Big(\bar\psi_j\,\tau_0\,\psi_k-\bar\psi_j\,\tau_k\,\psi_0+\bar\psi_0\,\tau_j\,\psi_k\Big)\,\varepsilon^{0jk}\\
&+\frac{i}{2}\delta_i(\tau_0\,\psi_k+\tau_k\,\psi_0)\,\bar\psi_j\varepsilon^{0jk}+\frac{i}{2}\delta_i(\tau_0\,\psi_k+\tau_k\,\psi_0)\,\partial_l\bar\psi_a\,\tau_b\,\tau_j\varepsilon^{abl}\varepsilon^{0jk}\\
&+i\Big(B_{ia}L_b^i-B_{ib}L_a^i\Big)\,\bar\psi_j\,\tau_c\psi_k\varepsilon^{abc}\varepsilon^{0jk}-2i\,\bar\psi_j\,\tau^c\,\psi_kL^{ia}B_i^b\varepsilon_{abc}\,\varepsilon^{0jk}\\
&-\frac{1}{4}B^{0c}\partial_i L_0{}^a\Big(\bar\psi_j\,\tau_a\,\psi_c-\bar\psi_j\,\tau_c\,\psi_a+\bar\psi_a\,\tau_j\,\psi_c\Big)\,\varepsilon^{0ij}\\
&+\frac{1}{4}B_{ic}\Big(\bar\psi_j\,\tau_a\,\psi_b-\bar\psi_j\,\tau_b\,\psi_a+\bar\psi_a\,\tau_j\,\psi_b\Big)\,\varepsilon^{abc}\varepsilon^{0ij}\\
&+\frac{1}{4}B^{ic}\Big(\;\partial_j L_i^a-\partial_i L_j^a\Big)\Big(\bar\psi_k\,\tau_c\,\psi_a-\bar\psi_k\,\tau_a\,\psi_c+\bar\psi_a\,\tau_k\,\psi_c\Big)\,\varepsilon^{0jk}\\
&+\frac{1}{8}\Big(B_i^cL^{0a}L^{kb}(B_{kc}-\partial_k L_{0c})+B_i^cL^{ka}L^{0b}(\partial_k L_{0c}-B_{kc})\Big)\\
&\times\Big(\bar\psi_j\,\tau_a\,\psi_b-\bar\psi_j\,\tau_b\,\psi_a+\bar\psi_a\,\tau_j\,\psi_b\Big)\,\varepsilon^{0ij}\\
&+B^{0b}B_i^a\Big(L_{0b}B_{ja}-L_{0a}B_{jb}+\Big(L_a^kL_{0b}B_{kc}-L_{0a}L_b^kB_{kc}\Big)\,L_{jc}\Big)\,\varepsilon^{0jk}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{2}B_i^c\Big(B^{0a}L^{ib}L_{jc}+B^{ib}L^{0a}L_{jc}+B_{jc}L^{0a}L^{ib}\Big)\\
&\times\Big(B_{ka}L_{0b}-B_{kb}L_{0a}+B_{ld}L_a^lL_{0b}L_k^d-B_{ld}L_b^lL_{0a}L_k^d\Big)\,\varepsilon^{0jk}\\
&+\frac{i}{2}B_i^bL_a^iL_{kc}\Big(\bar\psi_j\,\tau_b\,\psi_d-\bar\psi_j\,\tau_d\,\psi_b+\bar\psi_b\,\tau_j\,\psi_d\Big)\,\varepsilon^{0jk}\varepsilon^{acd}\\
&+\frac{i}{2}\Big(B_{ia}L_b^i-B_{ib}L_a^i\Big)\,\bar\psi_j\,\tau_c\,\psi_k\varepsilon^{abc}\varepsilon^{0jk}\\
&+\frac{1}{4}L^{kb}(\partial_iB_k^a-\partial_kB_i^a)\Big(\bar\psi_j\,\tau_a\,\psi_b-\bar\psi_j\,\tau_b\,\psi_a+\bar\psi_a\,\tau_j\,\psi_b\Big)\,\varepsilon^{0ij}\\
&+\frac{1}{8}L^{kb}L^{la}L_{ic}(\partial_kB_l^c-\partial_lB_k^c)\Big(\bar\psi_j\,\tau_a\,\psi_b-\bar\psi_j\,\tau_b\,\psi_a+\bar\psi_a\,\tau_j\,\psi_b\Big)\,\varepsilon^{0ij}\\
&-\frac{1}{4}\partial_kL^{0a}B_i^c\Big(\bar\psi_0\,\tau_a\,\psi_c-\bar\psi_0\,\tau_c\,\psi_a+\bar\psi_a\,\tau_0\,\psi_c\Big)\,\varepsilon^{0ik}\\
&-\frac{1}{4}\partial_k\Big(L^{0a}L^{ib}L_{jc}\Big)\,B_i^c\Big(\bar\psi_0\,\tau_a\,\psi_b-\bar\psi_0\,\tau_b\,\psi_a+\bar\psi_a\,\tau_0\,\psi_b\Big)\,\varepsilon^{0jk}\\
&+\frac{1}{4}\Big(B_i^bL^{ia}-B_i^cL^{0a}L^{ib}L_{0c}\Big)\,\partial_j\Big(\bar\psi_k\,\tau_a\,\psi_b-\bar\psi_k\,\tau_b\,\psi_a+\bar\psi_a\,\tau_k\,\psi_b\Big)\,\varepsilon^{0jk}\\
&+\frac{1}{4}L_{jc}\Big(B_{id}L_a^i-B_{ia}L_d^i-B_i^eL_a^0L_d^iL_{0e}\Big)\Big(\bar\psi_k\,\tau_b\,\psi^d-\bar\psi_k\,\tau^d\,\psi_b+\bar\psi_b\,\tau_k\,\psi^d\Big)\,\varepsilon^{abc}\varepsilon^{0jk}\\
&-B^{ic}(\,L_{0a}L_{ib}-L_{ia}L_{0b})\,L_{kc}L_{0e}\omega_{jd}^{\ \ b}\varepsilon^{0jk}\varepsilon^{ade}+\Big(B_i^bL_a^i-B_{ia}L^{ib}\Big)\,\omega_{kbd}L_{jc}\varepsilon^{acd}\varepsilon^{0jk}\\
&-\frac{1}{2}B^{ic}\Big(\,B^{lb}L_{le}-B_{le}L^{lb}\Big)\,L_{0a}L_{ib}L_{jc}\omega_k^{\ ea}\varepsilon^{0jk}+\frac{1}{2}B_i^cB_{kc}\Big(L_a^kL_{0b}-L_{0a}L_b^k\Big)\,\omega_j^{\ ab}\varepsilon^{0ij}\\
&+\frac{1}{2}B^{ic}\Big(\,B^{la}L_{le}-B_{le}L^{la}\Big)\,L_{0a}L_{ib}L_{jc}\omega_k^{\ be}\varepsilon^{0jk}+B_i^{\ c}\,\partial_j\Big(L^{0a}L^{ib}L_{0c}\Big)\,\omega_{kab}\varepsilon^{0jk}\\
&-2i\bar\psi_0\,\tau_c\,\psi_i\omega_{jab}\varepsilon^{abc}\varepsilon^{0ij}-i\bar\psi_j\,\tau_c\,\psi_i\omega_{0ab}\varepsilon^{abc}\varepsilon^{0ij}\\
&-i\bar\psi_i\,\tau_e\tau_\gamma\tau_j\,\partial_k\psi_\sigma\,\varepsilon^{\gamma k\sigma}\omega_{0cd}\varepsilon^{cde}\varepsilon^{0ij}\\
&-i\bar\psi_i\tau_e\tau_\gamma\tau_j\Big(\omega_{kcd}\tau_f\psi_\sigma\varepsilon^{cdf}\Big)\,\omega_{0ab}\varepsilon^{\gamma k\sigma}\varepsilon^{abe}\varepsilon^{0ij}\\
&+i\Big(\omega_{jab}\tau_c\bar\psi_\mu\varepsilon^{abc}\Big)\,\tau_\gamma\tau_\rho\Big(\omega_{kde}\tau_f\psi_\nu\varepsilon^{def}\Big)\,\varepsilon^{\gamma\nu k}\varepsilon^{\mu\rho j}\\
&-\Big(\omega_i^{\ ab}\partial_j\omega_{0ab}+\omega_0^{\ ab}\partial_i\omega_{jab}-2\omega_0^{\ ab}\omega_{ib}^{\ \ c}\,\omega_{jca}\Big)\,\varepsilon^{0ij}\\
&-4L^{ia}B_i^{\ b}\omega_{jb}^c\,\omega_{kca}\,\varepsilon^{0jk}+\omega_{idb}\Big(\omega_{ja}^{\ \ d}\,L_{0c}-2\omega_{0a}^{\ \ d}\,L_{jc}\Big)\,\varepsilon^{abc}\varepsilon^{0ij}\\
&+\text{ 2 bosons and 2 fermions terms} + \text{4 fermions terms}\\
&+\text{ 3 bosons and 2 fermions terms} + \text{1 bosons and 4 fermions terms}\\
&+\text{ 4 bosons and 2 fermions terms} + \text{6 fermions terms}
\end{aligned}
\tag{32}
$$

The effective Lagrangian density implied complex algebraic calculations and its explicit expression is very complex. Therefore, terms that give us the bosonic and fermionic propagators, the three and four legs vertices, are explicitly written. In the case of remaining terms with five and six legs vertices, terms of their origin are indicated in order to be located by the reader.

From equation (29) we can see that the non-local operator M^{ab} is written in a non covariant way. Using the Faddeev-Popov trick to go over a covariant gauge, the covariant form defined by the formula $M_L\alpha(x) = \alpha - \partial_\mu(L^{\mu a}, \alpha)$, can be obtained. To that purpose, we must transfer the integration measure defined on the surface $f_3 = \partial_i L^{ia} = 0$, to the surface $f_3' = \partial_\mu L^{\mu a} = 0$, which defines the Lorentz gauge. The same argument can be used on the surface f_2.

The path integral over the field B_μ is performed in the equation (30), the surface f_2 becomes the time derivative of the surface f_3.

Moreover, we must write the functional $detM^{ab}(A)$ by using the integral representation:

$$detM = \int exp\left(i \int d^3x \bar{\eta}^a(x) M_{ab}(A) \eta^b(x)\right) \mathcal{D}\bar{\eta}\mathcal{D}\eta \ , \tag{33}$$

where $\bar{\eta}(x)$ and $\eta(x)$ are the auxiliary anticommuting scalar functions called the Faddeev-Popov ghost.

Finally, the partition function (31) remains given by:

$$Z = \int \mathcal{D}L_{\nu a}\, \mathcal{D}\bar{\psi}_{(\alpha)}\, \mathcal{D}\bar{\eta}\, \mathcal{D}\eta\; exp\; i\,[\mathcal{L}^*] \ , \tag{34}$$

where

$$\mathcal{L}^* = \mathcal{L}_{eff} - \Lambda_1 f_1 - \Lambda_3 f_3 \ , \tag{35}$$

for the Lagrange multipliers Λ_1 and Λ_3.

From equation (34) we can see that the quantum problem remains defined in terms of a path integral, in which the independent fields are the original gauge field $L_{\nu a}$, the matter Dirac spinor field ψ and the unphysical ghost fields $\bar{\eta}$ and η. Consequently, now it is possible to apply diagrammatic technique by defining proper Feynman rules for propagators and vertices corresponding to these fields.

4 Diagrammatic and Feynman rules. One loop structure.

Looking at the equation (34) and taking into account the expression for $\mathcal{L}^*$, we can easily recognise the propagators defined by the quadratic part of the Lagrangian density and the remaining part of it can be represented by vertices.

The propagator of the fermionic field ψ is the usual one. Therefore, it is more interesting to analyze the gauge field propagator in which the higher derivative feature of the model remains exposed.

The action $\mathcal{L}^*$ can be written in pieces as follows

$$\mathcal{L}^* = \mathcal{L}^*(L_{\nu a}) + \mathcal{L}^*\left(\bar{\psi}\right) + \mathcal{L}^*_{\rm int}\left(L_{\nu a}, \psi\right) + \mathcal{L}^*_{\rm ghost} \ , \tag{36}$$

where $\mathcal{L}^*\left(L_{\nu a}\right)$ defines the gauge field propagator; $\mathcal{L}^*\left(\bar{\psi}\right)$ defines the fermionic field propagator; $\mathcal{L}^*_{\rm int}\left(L_{\nu a}, \psi\right)$ defines the usual interaction vertex $\bar{\psi}\psi L_{\nu a}$ and finally $\mathcal{L}_{\rm ghost}$ contains the ghost field propagator; and the well known vertex $\bar{\eta} L_{\nu a}\partial\eta$ which is linear in momentum.

For instance, the first term on the right hand side of equation (36) corresponding to the gauge field propagator can be written:

$$\mathcal{L}^*(L_{\nu a}) = \int d^3x \left[L^a_\mu \left(D^{-1}\right)^{\mu\nu} L^a_\nu\right] \ . \tag{37}$$

Where the matrix $\left(D^{-1}\right)^{\mu\nu}$ is the inverse of the propagator of the gauge field $L_{\nu a}$. It is Hermitian and non-degenerate and it can be invertible. So, the propagator $D_{\mu\nu}(k)$, in the momentum space, can be straightforward evaluated. The general case for different from zero topological mass of the gauge field, is very tedious to compute and does not bring anything new. The computation of the matrix elements $D_{\mu\nu}(k)$ of the propagator in the general case is straightforward but very tedious. These were obtained, and are long algebraic expressions which we do not write here explicitly.

We can write the Feynman rules, propagator and vertices.

i)Propagators

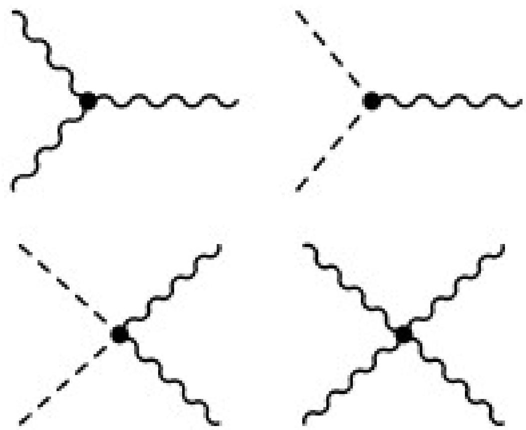
We associate with the propagator $D_{\mu\nu}$ of the bosonic field, a wavy line connecting two generic points.

Figure 1: Propagator $D_{\mu\nu}$ of the boson field $V_{(a)\nu}$

We associate with the propagator G_0 of the fermionic field, a dashed line.

Figure 2: Propagator G_0 of the fermion field $\bar{\psi}$

ii)Vertices



In order to simplify the understanding of the paper we do not write and analyze explicitly the vertices of five six legs. To show improvement of the model convergence, it is sufficient work with only the tree and four legs vertices.

Now, we proceed to examine the perturbative treatment of this gauge theory which describes the interaction of the bosonic object with the Dirac spinor field. Using the above rules, a power-counting analysis shows that the superficial degree of divergence are essentially those of the bosonic model, so we are led to the following one loop diagrams:

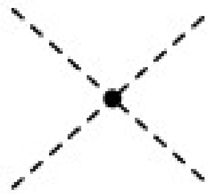
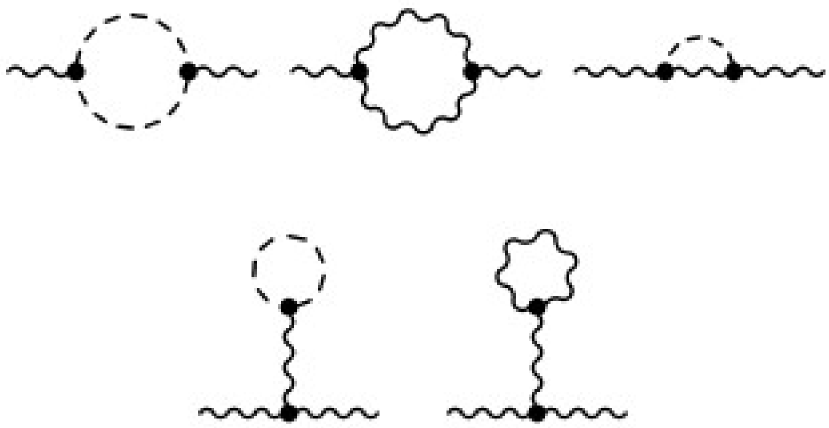


Figure 3: Tree and four legs vertices of the model

iii) One loop Diagrams.



Taking into account only the diagrams to a one loop corresponding to three an four legs vertices, it is possible to appreciate that convergence of theory is improved. This situation becomes more evident and beneficial if we include diagrams with a greater number of legs vertices.

First the correction to the boson line diagram:

Figure 4: One loops diagrams of the model

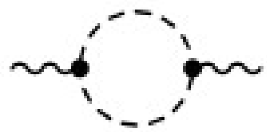


Figure 5: One loop boson-fermion diagram

The integral expression for the correction to the boson line is similar to the non higher model, therefore the ultraviolet behaviour of the integral is the same as in the bosonic model, i.e for large momentum p, the Feynman integral behaves as $\sim \int dp$ and so this diagram is linearly divergent. This must be expected, because this model is higher derivative only with respect to the boson field, and the propagator of that field does not appear in the one loop boson-fermion diagram. Consequently, the evaluation of the integral is carried out by introducing a Feynman parameter and the new loop momentum $p' = p + kx$.

Another diagram to be analized:

Figure 6: Correction to the boson line

The second diagram have a similar ultraviolet behavior to the one loop boson-fermion diagram.

Now, we consider another diagram, correction line with the fermion loop:

The new propagator have an ultraviolet behaviour in such manner that the Feynman integral gives a convergent result. The integral behaves as $\sim \int \frac{dk}{k^3}$.

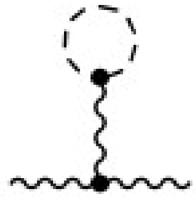


Figure 7: Correction with the fermion loop

The last one loop diagram we consider is the vertex correction:

Figure 8: Diagrams vertex correction. One loop frmionic

We can see that the diagram is also convergent, because for large momentum the integral behaves as $\sim \int \frac{dk}{k^4}$.

At this state we must look at the convenience to add higher derivative terms in the Lagrangian density. The first remark is that for large momentum, the propagator behaves like $\sim \frac{1}{k^4}$.

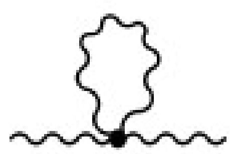


Figure 9: Diagrams vertex correction. One loop bosonic

This diagram have a similar ultraviolet behavior to the vertex correction, one loop fermionic.

We conclude that in the one loop diagrams in which the propagator of the boson takes place, the ultraviolet behaviour is improved and the divergence of these diagrams are eliminated. The remaining diagrammatic with a fermionic loop does not change and the degree of divergence is that of the bosonic model.

Therefore, the use of higher derivatives terms in the Lagrangian, allows to improve the behaviour of the correspondent propagators at large momentum, rendering the theory less divergent.

In the perturbative framework, we are going to consider one loop diagrams, which in the model without higher derivative terms have superficial degree of divergence.

Such diagrams are, for instance, the correction to the fermion line named $\Sigma(p)$ and the vertex correction named $V_\rho(p,q)$, whose analytical expressions have respectively the form:

$$\Sigma(p) \sim \int \frac{d^3k}{(2\pi)^3} \frac{\gamma_\mu \left(\gamma.p - \gamma.k - m\right) \gamma_\nu}{\left(p-k\right)^2 + m^2} \times D_{\mu\nu}\left(k\right) \ , \tag{38}$$

$$V_\rho(p,q) \sim \int \frac{d^3k}{(2\pi)^3} \gamma_\mu \frac{\left(\gamma.p + \gamma.k - m\right)}{\left(p+k\right)^2 + m^2} \gamma_\rho \times \frac{\left(\gamma.q + \gamma.k - m\right)}{\left(q+k\right) + m^2} \gamma_\nu \times D_{\mu\nu}\left(k\right) \ . \tag{39}$$

If we proceed to analyze these diagrams we can see that for large momentum of the gauge field, the integrals in equations (31) and (32) behave as $\sim \int \frac{dk}{k^3}$ and $\sim \frac{dk}{k^4}$ respectively. Consequently, the new gauge field propagator has an ultraviolet behavior so that the Feynman Integrals (31) and (32) give convergent results.

Finally, another question we briefly comment is concerning to the part of the action written, which defines the different vertices of the model with gauge field legs. Due to the introduction of higher derivative terms, the appearance of new vertices in the model is the price we must pay. Looking at the expression for $\mathcal{L}_{eff}$, in addition to the usual three legs vertex in the gauge field LLL, we can see that there are new vertices containing more legs in the gauge field and some of them with momentum insertion.

5 Renormalization

The above results and comments do not try to solve the problem of regularization and renormalization of the model. Simply, it was found that the new propagator has a better ultraviolet behavior because for large momentum it behaves like k^{-4} and so we can gain two powers with respect to the usual propagator. Therefore, we can conclude that the inclusion of higher derivative terms in the Lagrangian improves the behavior of propagators at large momentum, rendering the model less divergent.

On the other hand, it is known that the dimensional regularization method is problematic in a field theory containing the volume form $\varepsilon^{\mu\nu\rho}$. In these cases, other gauge invariant regularization methods, for instance the Pauli-Villars procedure, must be used (Alvarez-Gaume et al. 1990). Moreover, on believe that the Chern-Simons field theories belong to the category of finite ones. That is to say, a theory containing a finite number of divergent diagrams. Consequently, at this stage, to solve completely the problem only remains to regularize a renormalizable model. We do not complete here the procedure because it is carried out following the conventional methods.

Another important problem to take into account is the unitarity problem. It is well known (Hawking, 1987), in quantum field theories described by Lagrangians containing higher derivative terms, that the unitarity can be violated. This occurs when ghost states with negative norm are present. In a previous paper (Foussats et al., 1995) where an higher derivative model was analyzed, the unitarity problem was treated carefully at least at free level. Therefore, following the steps given in Foussats et al., 1995 (see also t Hooft and Velman, 1973), we must consider first the gauge field propagator $D_{\mu\nu}\,(k)$. Next, the 3×3 matrix residue $K^R_{\mu\nu}\,(k)$, obtained from the matrix $D_{\mu\nu}$ by leaving out the poles. The matrix residue $K^R_{\mu\nu}\,(k)$ is Hermitian and can be diagonalized and has three different non-zero eigenvalues. Consequently (Faddeev and Slavnov, 1980), a set (α) of real currents $J^{(\alpha)}_\mu\,(k)$ one for every non-zero eigenvalue can be defined. When all the eigenvalues of the matrix residue at the pole are positive the normalization is given by:

$$J^{(\alpha)}_\mu\,(k)\,K^R_{\mu\nu}\,(k)\,J^{(\alpha)}_\nu\,(k) = +1 \;. \tag{40}$$

When the matrix residue has a negative eigenvalue at the pole, it corresponds to states with negative norm (i.e. the unitarity is lost) and they are physically unacceptable. The unitarity is recovered by assuming the existence of a positive metric Hilbert subspace stable under the time evolution. So, the normalization in equation (40) must be done with a minus one. This trick to retrieve the unitarity of a theory, is usually known as indefinite metric prescription.

6 Conclusions

In the present paper the Feynman rules and the diagrammatic for a higher-derivative Chern-Simons theory, were found. This was done in the framework of the path integral quantization method. The definition of the effective Lagrangian density, allows us to define a suitable "bosonic field" and to find the propagator of such bosonic object. The fermionic propagator for the matter field is the usual one. The model has vertex with three, four, five and six legs, and so all the diagrams are obtained by connecting vertices and sources by means of the propagators thus defined. Using the perturbative theory, the one loop structure of the model was analysed. The results obtained for the one loop diagrams in which the boson field propagator takes place, allows to guarantee that the ultraviolet behaviour is improved and the divergence of these diagrams are eliminated. Therefore, we conclude that the presence of higher derivative terms in the Lagrangian density gives rise to a new bosonic propagator, which yield that the theory is less divergent.

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