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Studies in Numerical and Decomposition Solution Profiles of Blasius Equation

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Abstract

The generalized equation of Blasius arises in the boundary layer problems of hydrodynamics. We consider the decomposition solutions [**10**] of the Blasius equation $u''' + auu'' = 0$ and compare them to our numerical solutions.

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1 Introduction

The generalized Blasius equation arises in various boundary value problems in fluid mechanics of laminar flows. The equation takes the form [**8,10**]

$$u''' + auu'' = \beta \left[(u')^2 - 1 \right] \tag{1}$$

The equation (1) in general should be solved numerically [**1** ,**3**, pp. 400 - 445, **6**, **7**]. The equation describes a two dimensional boundary layer of laminar flow of fluid that forms on a semi-infinite plate, which is parallel to a uniform flow u [**2**]. So much toward the analysis of the Blasius has been achieved by various researchers [**4**, **5**, p. 91].

We are particularly interested in the solution obtained by Ranasinghe and Majid [**9**] by the method of decomposition, which can be found in many related works [**1**,**4**,**5**, p. 91].

2 Solution by Decomposition

We consider the case $\beta = 0$ and $a = 1/2$ subjected to the following boundary conditions

$$u(0) = u'(0) = 0 \tag{2}$$

Then the decomposition solution is

$$u = \sum_{n=0}^{\infty} u_n \tag{3}$$

where

$$u_0 = \frac{1}{6}x^2 \quad \text{and} \quad u_n = \left(-\frac{1}{2}\right)^n k_n c^{n+1} \frac{x^{3n+2}}{3n(3n+1)(3n+2)} \quad ; n \geq 1$$

and

$$k_{n+1} = k_n \frac{(3n+1)(3n+2)+2}{3n(3n+1)(3n+2)} + \frac{1}{9}\sum_{i=1}^{n-1} \frac{k_i k_{n-i}}{i(3i+1)(3i+2)(n-i)} \quad ; n \geq 2$$

with $k_0 = 1$, $k_1 = 2$ and $k_2 = 11/30$. The velocity field is given by $u' = \sum_{n=0}^{\infty} u_n'$.

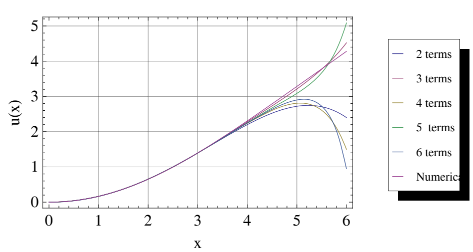

Figure 1: $u(x)$ against x for odd and even n. As $x \to \infty$ u' will not converge in either case.

3 Comparison between Decomposition and Numerical Solution Profiles

The analytical solution to the Blasius equation is consistent with the analytical solutions given in [**1**,**3**]. Numerically we find that $u(x)$ has two classes

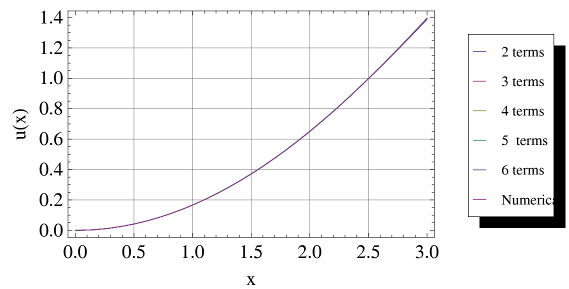

Figure 2: $u(x)$ against x for $x \leq 3$. The decomposition series solution and the numerical solution are indistinguishable from one another. The two solutions agree well in this regime.

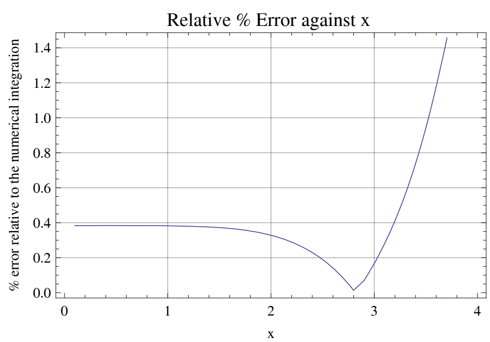

Figure 3: The relative error of the decomposition solution with respect to the numerical solution as a function of x is plotted. The error diminishes steadily until $x = 2.8$ and then rises sharply. Yet, the agreement is excellent with the error $< 0.4\%$ for the practical region of $x \leq 3$.

of solutions, namely for odd and even n. Figure(1) shows these two distinct classes.

If n is even, i.e., the series solution of (3) has even number of terms, then $u(x)$ has a local maximum around $x = 5$.

For odd n, $u(x)$ diverges quite rapidly beyond $x > 3$. As seen in Figure (1), the two classes of solutions diverge from each other too much beyond $x = 3$ making the decomposition solution is substantially inaccurate beyond $x = 3$. For odd n, series solution diverges for large x. For even n, the series solution has a local maximum.

Figure(2) shows that the agreement between the decomposition solution and our numerical calculation is excellent upto $x = 3$. Almost all practical problems in fluid motions described by the Blasius equation falls in the region $x \leq 3$ making the decomposition is still an excellent approximation.

We calculated the relative error of the decomposition solution with respect to our analytical solution. Our results are sown in Figure (3). We note that the relative error decreases steadily up to $x = 2.8$ and then rapidly increases making the analytical approach too inaccurate beyond $x > 3$.

4 Conclusions

We analyzed the solution of decomposition of the Blasius equation obtained by Ranasinghe and Majid [**9**] and compared it to our numerical results. We observe that for all practical purposes $x \leq 3$ and in this region the decomposition solution has a relative error with respect to the numerical solution of $< 0.4\%$. Thus, the decomposition is an excellent approximation to the numerical solution of the Blasius fulfilling all the boundary conditions for the practical region $x \leq 3$.

References

[1] S. Abbasbandy, A Numerical Solution of Blasius Equation by Adomian's Decomposition Method and Comparison with Homotopy Perturbation Method, *Chaos, Solitons and Fractals*, Volume 31 (2007), 257-260.

[2] Z. Belhachmi, B. Brighi and Taous, On The Concave Solutions of the Blasius Equation, *Acta Math. Univ. Comenianae*, Volume LXIX, Issue 2, (2000), 199-214.

[3] H. T. Davis, *Introduction to Nonlinear Differential and Integral Equations*, Dover, 1962.

[4] J. H. He, A simple Perturbation Approach to Blasius Equation, *Applied Mathematics and Computation*, Volume 140 (2003), 217-222.

[5] L. Horace, *Hydrodynamics*, Dover, New York, 1945.

[6] N. Ishimura, On Blowing-Up Solutions of the Blasius Equation, *Discrete and Continuous Dynamical Systems*, Volume 9 (2003).

[7] S. J. Liao, A An Explicit, Totally Analytical Approximate Solution for Blasius Viscous Flow Problem, *International Journal of Non-Linear Mechanics*, Volume 34 (1999).

[8] C. Pozrikidis, *Introduction to Theoretical and Computational Fluid Dynamics*, Oxford, New York, 1998.

[9] A.I. Ranasinghe and F. B. Majid, Solution of Blasius Equation by Decomposition, *Applied Mathematical Sciences*, Volume 3, No 13 (2009), 605-611.

[10] H. Schlichting, *Applied Mathematical Sciences*, Springer, 2004.

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