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Avoiding Decoherence for Open System and Renormalizing for Hamiltonian

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Abstract

A different strategy is proposed to avoid decoherence for open system by adjusting external controllable parameters. The results show that the output states in terms of the fidelity are pure states, which correspond to the state vectors that are given by a renormalized Hamiltonian. Thus, the output states may perfectly preserve memory of initial single-qubit states at some evolving time points.

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In a real system, noise and decoherence are big problems. The interaction between a quantum system with its surrounding environment may lead to an irreversible loss of information on the system. The process limits the ability to maintain pure quantum states in quantum information [1]. Decoherence is the most important limiting factor for quantum computation [2] because its effect is that quantum superpositions decay into statistical mixtures, which results in a relatively short coherence time. On the other hand, the decoherent process by which coherent superpositions of Schrödinger cat states are transformed into statistical mixtures is at the heart of the quantum measurement theory and plays an essential role in the classical limit of quantum mechanics [3].

Moreover, it is well-known that the decoherence strongly affects the quantum gate speed and gate error rate. Thus, the environment-induced decoherence has been a main obstacle for the practical application of superconducting qubits and nuclear-magnetic resonance in quantum computation [4]. In the practical applications, therefore, it is important that a typical gate operation time must be much smaller than the decoherent time. Thus, it is a fundamental issue how to avoid the decoherence.

The decay of quantum information, due to the interaction of a system with its environment, can be described by a superoperator. If the environment frequently scatters off the quantum system, and the environment state is not monitored, then off-diagonal elements in the density matrix of the system decay rapidly in a preferred basis[5]. The time scale for the decoherence is set by the scattering rate, which may be much larger than the damping rate for the system. The great challenge facing any such information processing in the quantum regime lies in avoiding, controlling or overcoming the effects of the decoherence.

It has become a major challenge how to fight the decoherence over the past decades. Several schemes have been proposed in the theory of quantum computation and communication, which included quantum error correction strategies [6, 7], feedback implementations [8], the realization of qubits in symmetric subspaces decoupled from the environment [9], dynamical decoupling techniques [10], and engineering of pointer states [11]. They are dependent on the possibility of preserving quantum coherence. Differently from the approaches suppressed the decoherence by the artificial reservoirs, which may lead to more errors and new decoherent sources, we propose to overcome the effects of the decoherence by controlling an external magnetic field.

When a relevant dynamical time scale of the open quantum system is long compared to the time for the interaction with environment, the evolution of system is effectively local in time (the Markovian approximation). Much as general unitary evolution is generated by a master equation[12],

$$\frac{\partial \rho}{\partial t} = -i[\hat{\mathcal{H}}, \rho] + \mathcal{L}\rho, \tag{1}$$

where $\mathcal{L}$ is a superoperator associated with the interaction,

$$\mathcal{L}\rho = \sum_{\mu} \left(\Gamma_{\mu} \rho \Gamma_{\mu}^{+} - \frac{1}{2}\{\Gamma_{\mu}^{+}\Gamma_{\mu}, \rho\} \right). \tag{2}$$

The first term on the right of Eq. (1) is a usual Schrödinger term that generates a unitary evolution. The second term describes all possible transitions that the open system may undergo due to interactions with the reservoir. The operators Γ_μ in Eq. (2) are called Lindblad operators or quantum jump operators.

Let us apply Eq. (1) to a two-level system under the magnetic field with Hamiltonian $H = \frac{1}{2}\omega\hbar\sigma_z$ and a dephasing source with $\Gamma = \sqrt{\lambda}\sigma_z$, where $\omega = g(\mu)B/\hbar$ with $g(\mu)$ that is the gyromagnetic, B acts as an external controllable parameter and can be experimentally changed, σ_z is a pauli operator of z-direction, and λ is a decay rate.

The initial state of two-level system is assumed as $|\Psi(0)\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$, which may be rewritten in terms of the density matrix, i.e.,

$$\rho(0) = \begin{pmatrix} \cos^2(\theta/2) & \frac{1}{2}\sin\theta \\ \frac{1}{2}\sin\theta & \sin^2(\theta/2) \end{pmatrix}, \tag{3}$$

where θ is an initial azimuthal angle in the poincaré sphere. For this system, the solution of master equation (1) can be obtained by

$$\rho(t) = \begin{pmatrix} \cos^2(\theta/2) & \frac{1}{2}\sin\theta e^{-i\omega t-2\lambda t} \\ \frac{1}{2}\sin\theta e^{i\omega t-2\lambda t} & \sin^2(\theta/2) \end{pmatrix}, \tag{4}$$

under the initial condition (3). Here the dephasing factor $\exp(-2\lambda t)$ parameterizes the amount of decoherence. The effect of dephasing is to decrease the size of the nondiagonal elements of density matrix in a basis determined by the dephasing interaction with the environment so that a single-qubit is corrupted by the dephasing.

The properties of quantum information through noisy quantum channels are quantified by the fidelity which measures the overlap between the initial and time-developed state vectors[13, 14, 15, 16]. For an initially pure state $|\Psi(0)\rangle$, the fidelity is in fact a probability to find the system in the initial state at a later time. Now we take the density matrix in Eq. (3) as an initial state at the time $t = 0$ and in Eq. (4) as a finial state at the time $t = \tau$ respectively, the fidelity of the physical system with the dephasing is written as

$$\begin{aligned} F(\tau) &= \text{Tr }(\rho(0)\rho(\tau)) \\ &= \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + \frac{1}{2}\sin^2\theta\cos(\omega\tau)e^{-2\lambda\tau}, \end{aligned} \tag{5}$$

which depends on the dephasing factor $\exp(-2\lambda\tau)$. In general, $0 \le F(\tau) \le 1$. For the case of $F(\tau) = 0$, the quantum information is completely distorted in the quantum computation. In the case of $F(\tau) = 1$, the quantum information is perfectly preserved in process of the information. If $0 < F(\tau) < 1$, some memories of initial physical state are lost in the quantum information process so that the output state only preserves a part of memory about the initial state.

Our objective is to have the decoherent factor disappear during the evolution. Thus we seek the solution of following transcendental equation,

$$\cos(\omega\tau)e^{-2\lambda\tau} = \cos(\Omega_1\tau), \tag{6}$$

where $\Omega_1 = (\omega^2 - 4\lambda^2)^{1/2}$. It is worth noting that the conditional frequency ω satisfied Eq. (6) is dependent on the dephasing rate and may be controlled by the experimenters. The numeral results at Fig. 1 show that there exist, indeed, some physically meaningful solutions in Eq. (6). Inserting Eq. (6) into Eq. (5), the fidelity becomes

$$F(\tau) = \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + \frac{1}{2}\sin^2\theta\cos(\Omega_1\tau). \tag{7}$$

From Eq. (7), we see that, though the the fidelity depends on the dephasing rate λ in terms of the conditional frequency Ω_1, the dephasing factor $\exp(-2\lambda\tau)$ disappears.

It is obvious that the corresponding output state for Eq. (7) may be described by

$$\rho(\tau) = \begin{pmatrix} \cos^2(\theta/2) & \frac{1}{2}\sin\theta e^{-i\Omega_1\tau} \\ \frac{1}{2}\sin\theta e^{i\Omega_1\tau} & \sin^2(\theta/2) \end{pmatrix}, \tag{8}$$

which is an output of pure state with the state vector,

$$|\Psi(\tau)\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\Omega_1\tau}\sin\frac{\theta}{2}|1\rangle. \tag{9}$$

Thus, we obtain an effective scheme for the perfect recovery of a pure state. Especially, if the output state is controlled at the times $\tau = 2n\pi/\Omega_1 (n = 1, 2, \cdots)$, one may obtain a perfect fidelity $F(\tau) = 1$ according to Eq. (7). This implies that a qubit state is perfectly preserved in our approach. In other words, by controlling the external magnetic filed according to Eq. (6) and Fig. 1, one may effectively avoid the decoherence in the output state in the case of dephasing.

Another interesting example is to consider spontaneous decay $\Gamma = \sqrt{\lambda}\sigma_-$ as a source of decoherence for the two-level system. According to Eq. (1), thus, the density matrix may be written as,

$$\rho(t) = \begin{pmatrix} \cos^2(\theta/2)e^{-\lambda t} & \frac{1}{2}\sin\theta e^{-i\omega t - \frac{1}{2}\lambda t} \\ \frac{1}{2}\sin\theta e^{i\omega t - \frac{1}{2}\lambda t} & 1 - \cos^2(\theta/2)e^{-\lambda t} \end{pmatrix}. \tag{10}$$

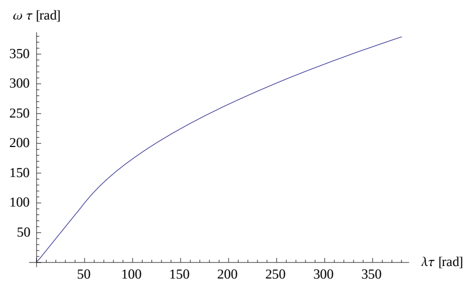

Figure 1: In dephasing case, the controlling magnetic field is as a function of decay rate.

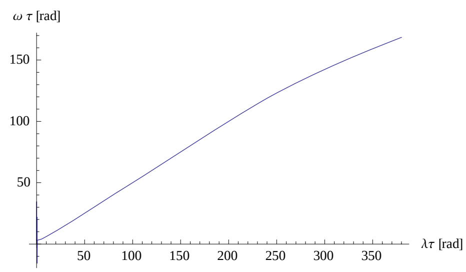

Figure 2: In spontaneous decay $\Gamma = \sqrt{\lambda}\sigma_-$, the controlling magnetic field is as a function of decay rate with $\theta = \pi/4$.

Differently from the dephasing case, the effects of spontaneous decay are to decrease the sizes of both the diagonal and nondiagonal elements of the density matrix in terms of the decoherent factors $\exp(-\lambda t)$ and $\exp(-\lambda t/2)$, respectively.

Similarly, we find that, when

$$\cos^2\frac{\theta}{2}\cos\theta e^{-\lambda\tau} + \frac{1}{2}\sin^2\theta\cos(\omega\tau)e^{-\frac{1}{2}\lambda\tau} = \cos^4\frac{\theta}{2} + \frac{1}{2}\sin^2\theta\cos(\Omega_2\tau), \tag{11}$$

where $\Omega_2 = (\omega^2 - \lambda^2/4)^{1/2}$, the fidelity for the two-level system with the spontaneous decay may be written as the same form as Eq. (7) replaced Ω_1 by Ω_2. It is noted that, differently from the dephasing case, the conditional frequency ω satisfied Eq. (11) in case of the spontaneous decay is determined by both the initial angle θ and decay rate λ as shown at Fig. 2. It is obvious that the corresponding output state is a pure state according to the fidelity so that a perfect memory may be preserved for the output state at the evolving time points $\tau = 2n\pi/\Omega_2 (n = 1, 2, \cdots)$.

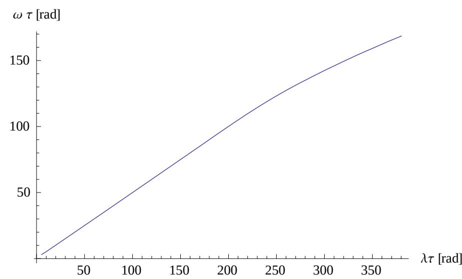

Figure 3: In spontaneous decay $\Gamma = \sqrt{\lambda}\sigma_+$, the controlling magnetic field is as a function of decay rate with $\theta = \pi/4$.

For the two-level system with another spontaneous decay $\Gamma = \sqrt{\lambda}\sigma_+$ as a source of decoherence, the solution of Eq. (1) is

$$\rho(t) = \begin{pmatrix} 1 - \sin^2(\theta/2)e^{-\lambda t} & \frac{1}{2}\sin\theta e^{-i\omega t - \frac{1}{2}\lambda t} \\ \frac{1}{2}\sin\theta e^{i\omega t - \frac{1}{2}\lambda t} & \sin^2(\theta/2)e^{-\lambda t} \end{pmatrix}. \tag{12}$$

By controlling the magnetic field to satisfy

$$\begin{aligned} &-\sin^2(\theta/2)\cos\theta e^{-\lambda\tau} + \frac{1}{2}\sin^2\theta\cos(\omega\tau)e^{-\frac{1}{2}\lambda\tau} \\ &= \sin^4(\theta/2) + \frac{1}{2}\sin^2\theta\cos(\Omega_3\tau), \end{aligned} \tag{13}$$

where $\Omega_3 = (\omega^2 - \lambda^2/4)^{1/2}$, we can avoid the coherence and obtain the output of pure state. As shown at Fig. 3, the solutions of Eq. (13) exist so that the scheme may be realized to overcome the decoherence. It is noted that, though Ω_3 is the same form as Ω_2, the conditional frequencies ω in Eq. (11) and Eq. (13) satisfy the different equations, respectively.

Our approach may be a general way to avoid the decoherence. Let us include two sources of decoherence with $\Gamma_1 = \sqrt{\lambda}\sigma_z$ and $\Gamma_2 = \sqrt{\lambda}\sigma_-$. The solution of Eq. (1) to the physical system may be obtained by

$$\rho(t) = \begin{pmatrix} \cos^2(\theta/2)e^{-\lambda t} & \frac{1}{2}\sin\theta e^{-i\omega t - \frac{5}{2}\lambda t} \\ \frac{1}{2}\sin\theta e^{i\omega t - \frac{5}{2}\lambda t} & 1 - \cos^2(\theta/2)e^{-\lambda t} \end{pmatrix}. \tag{14}$$

Similarly, if

$$\begin{aligned} &\cos^2(\theta/2)\cos\theta e^{-\lambda\tau} + \frac{1}{2}\sin^2\theta\cos(\omega\tau)e^{-\frac{5}{2}\lambda\tau} \\ &= -\cos^4(\theta/2) + \frac{1}{2}\sin^2\theta\cos(\Omega_4\tau), \end{aligned} \tag{15}$$

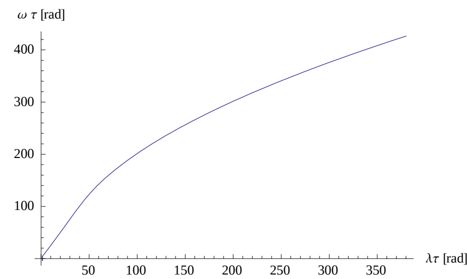

Figure 4: In two sources of decoherence with $\Gamma_1 = \sqrt{\lambda}\sigma_z$ and $\Gamma_2 = \sqrt{\lambda}\sigma_-$, the controlling magnetic field is as a function of decay rate with $\theta = \pi/4$.

where $\Omega_4 = (\omega^2 - 25\lambda^2/4)^{1/2}$, the output state is a pure state expressed by Eq. (9), where Ω_1 should be replaced by Ω_4. The numeral results at Fig. 4 show that there exist, indeed, solutions of transcendental equation (15).

Next, let us consider another two sources of decoherence with $\Gamma_1 = \sqrt{\lambda}\sigma_z$ and $\Gamma_3 = \sqrt{\lambda}\sigma_+$, the density matrix is

$$\rho(t) = \begin{pmatrix} 1 - \sin^2(\theta/2)e^{-\lambda t} & \frac{1}{2}\sin\theta e^{-i\omega t - \frac{5}{2}\lambda t} \\ \frac{1}{2}\sin\theta e^{i\omega t - \frac{5}{2}\lambda t} & \sin^2(\theta/2)e^{-\lambda t} \end{pmatrix}. \tag{16}$$

In this case, the magnetic field is controlled to satisfy

$$\begin{aligned} & -\sin^2(\theta/2)\cos\theta e^{-\lambda\tau} + \frac{1}{2}\sin^2\theta\cos(\omega\tau)e^{-\frac{5}{2}\lambda\tau} \\ & = -\sin^4(\theta/2) + \frac{1}{2}\sin^2\theta\cos(\Omega_5\tau), \end{aligned} \tag{17}$$

where $\Omega_5 = (\omega^2 - 25\lambda^2/4)^{1/2}$, the corresponding output state is a pure state. The solutions of Eq. (17) are shown at Fig. 5, where Eq. (17) is different from Eq. (15) so that the conditional frequencies ω in Ω_4 and Ω_5 are different each other.

For two sources of decoherence with spontaneous decay $\Gamma_2 = \sqrt{\lambda}\sigma_-$ and $\Gamma_3 = \sqrt{\lambda}\sigma_+$, the density matrix is

$$\rho(t) = \begin{pmatrix} \frac{1}{2}(1 + \cos\theta e^{-2\lambda t}) & \frac{1}{2}\sin\theta e^{-i\omega t - \lambda t} \\ \frac{1}{2}\sin\theta e^{i\omega t - \lambda t} & \frac{1}{2}(1 - \cos\theta e^{-2\lambda t}) \end{pmatrix}. \tag{18}$$

In order to avoid the coherence, we control the magnetic field to satisfy

$$\begin{aligned} & \frac{1}{2} + \frac{1}{2}\cos^2\theta e^{-2\lambda t} + \frac{1}{2}\sin^2\theta\cos(\omega\tau)e^{-\lambda t} \\ & = \cos^4(\theta/2) + \sin^4(\theta/2) + \frac{1}{2}\sin^2\theta\cos(\Omega_6\tau), \end{aligned} \tag{19}$$

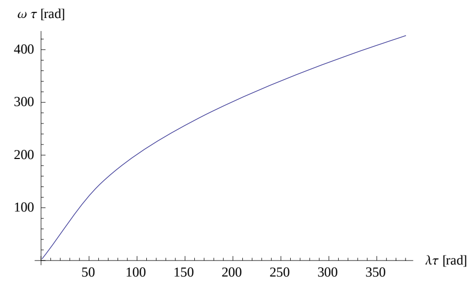

Figure 5: In two sources of decoherence with $\Gamma_1 = \sqrt{\lambda}\sigma_z$ and $\Gamma_2 = \sqrt{\lambda}\sigma_+$, the controlling magnetic field is as a function of decay rate with $\theta = \pi/4$.

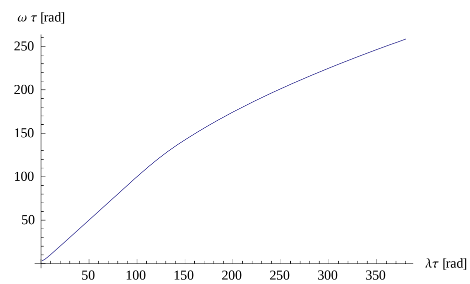

Figure 6: In two sources of spontaneous decay with $\Gamma_1 = \sqrt{\lambda}\sigma_-$ and $\Gamma_2 = \sqrt{\lambda}\sigma_+$, the controlling magnetic field is as a function of decay rate with $\theta = \pi/4$.

where $\Omega_6 = (\omega^2 - \lambda^2)^{1/2}$. Thus, the fidelity will be independent of the deconherent factors so that the output state is a pure state. Fig. 6 shows that one can find the solutions of transcendental equation (19).

At last, let us consider all sources of decoherence with $\Gamma_1 = \sqrt{\lambda}\sigma_z$, $\Gamma_2 = \sqrt{\lambda}\sigma_-$ and $\Gamma_3 = \sqrt{\lambda}\sigma_+$, the density matrix may be expressed by

$$\rho(t) = \begin{pmatrix} \frac{1}{2}(1 + \cos\theta e^{-2\lambda t}) & \frac{1}{2}\sin\theta e^{-i\omega t - 3\lambda t} \\ \frac{1}{2}\sin\theta e^{i\omega t - 3\lambda t} & \frac{1}{2}(1 - \cos\theta e^{-2\lambda t}) \end{pmatrix}. \tag{20}$$

Similarly, let the magnetic field satisfy the following equation,

$$\begin{aligned} &\frac{1}{2} + \frac{1}{2}\cos^2\theta e^{-2\lambda t} + \frac{1}{2}\sin^2\theta\cos(\omega\tau)e^{-3\lambda t} \\ &= \cos^4(\theta/2) + \sin^4(\theta/2) + \frac{1}{2}\sin^2\theta\cos(\Omega_7\tau), \end{aligned} \tag{21}$$

where $\Omega_7 = (\omega^2 - 9\lambda^2)^{1/2}$, so that the output state is a pure state. Fig. 7 shows that one may find the solutions of transcendental equation (21).

In conclusion, a way is proposed to avoid the decoherence for open system by controlling the external parameter, i.e., the magnetic field. Our approach is

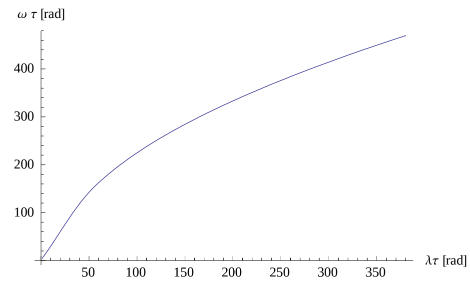

Figure 7: In three sources of decoherence with $\Gamma_1 = \sqrt{\lambda}\sigma_z$, $\Gamma_2 = \sqrt{\lambda}\sigma_-$ and $\Gamma_3 = \sqrt{\lambda}\sigma_+$, the controlling magnetic field is as a function of decay rate with $\theta = \pi/4$.

applied to analyze the two-level system interacting with an external magnetic field including all possible decoherent sources. The results show that, by controlling the external parameters to satisfy some relations with the decoherent rates, the output state may be a pure state. Especially, if some evolving time points $\tau = 2n\pi/\Omega_i (i = 1, 2, \cdots, 7)$ for the different decoherent sources are chosen for the output state, respectively; where the conditional frequency Ω_i have a shift related to the two-level system resonance frequency, one may get a perfect fidelity so that an initial qubit state is completely preserved in the output state. Therefore, it is very helpful for quantum information processing and coherent controlling [17, 18].

From Eqs. (8) and (9), we see that the pure states of output may be described by the Hamiltonian $H_R = \frac{1}{2}\Omega_i\hbar\sigma_z$, where the conditional frequencies $\Omega_i (i = 1, 2, \cdots, 7.)$ are for the different decoherent sources, respectively. Therefore, our approach to avoid the decoherence may be obtained by renormalizing for the free Hamiltonian of two-level system including only the magnetic field, where the magnetic field frequency ω is replaced by the conditional frequencies Ω_i, respectively.

In comparison with the approaches suppressed the decoherence by the artificial reservoirs, our strategy does not need any such process, which leads to a possible reduction in experimental errors as well as in decoherent sources. In contrast to the feedback of implementations, we may completely preserve a qubit state with the perfect fidelity in the output state.

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