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Five Dimensional Perfect Fluid Cosmological Models in Barber's Second Self-Creation Theory

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Abstract. In this paper we investigated Bianchi type-I cosmological model in Barber's second self creation theory in five dimensional space-time. For complete determination of the model it is assumed that the component σ_4^4 of shear tensor $\left(\sigma_i^j\right)$ is proportional to scalar of expansion θ. Several cases have been studied in this context. Some physical and kinematical features of the models are also discussed.

Keywords: String, Bianchi type-I, Five dimensional

Introduction

Barbar [1] proposed two self creation cosmologies by modifying the Brans-Dicke theory and general relativity. These modified theories create the universe out of self contained gravitational and matter fields. Barber has included continues creation of matter in these theories. The universe is seen to be created out of self contained gravitational, scalar and matter fields.

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Brans [2] has pointed out that Barber's first theory is not only in disagreement with experiment, as well as inconsistent, since the equivalence principle is violated. Barber's second theory is a modification of general relativity to a variable G-theory. The second theory retains the attractive features of the first theory and overcomes previous objections. In this theory the scalar field does not directly gravitate, but simply divides the matter tensor, acting as a reciprocal gravitational constant. It is postulated that this scalar couples to the trace of energy momentum tensor. This theory predicts the same precession of the perihelian of the planets on general relativity and in that respect agrees with observation to within one percentage. In the limit, the theory approaches the standard general relativity in every respect.

Pimental [3] and Soleng [4] have presented the Robertson Walker solutions of Barber's second self creation theory of gravitation by using power law relation between the expansion factor of the universe and the scalar field. Soleng [5], Singh [6], Reddy [7, 8, 9], Reddy et al. [10, 11] and Maharaj and Beesham [12] are some of the authors who have studied various aspects of the two self creation theories. Venkateswarlu and Reddy [13] presented Bianchi type-V radiating model in Barber’s first theory, while Venkateswarlu and Reddy [14] have obtained an anisotropic cosmological model in this theory. Reddy and Venkateswarlu [15, 16] have obtained spatially homogeneous and anisotropic Bianchi type-VI cosmological models in Barber’s second self creation theory of gravitation both in vacuum and in presence of perfect fluid with pressure equal to energy density.

Rao and Sanyasi Raju [17] and Sanyasi Raju and Rao [18] have discussed Bianchi type VIII and IX in zero mass scalar fields and self creation cosmology. Shanti and Rao [19] obtained Bianchi type-II and III models in self creation cosmology. Micro and Macro cosmological model in Barber’s second self creation theory have been studied by Mohanty et al. [20]. Venkateswarlu et al. [21] have studied Bianchi type I, II, VIII and IX string cosmological solutions in self creation theory of gravitation, Rao et al. [22] have discussed exact Bianchi type II, VIII and IX string cosmological models in general relativity and self creation theory of gravitation. In this paper we investigated Bianchi type-I cosmological model in Barber's second self creation theory in five dimensional space-time.

The Field Equations

We consider the five dimensional LRS Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2\left(dy^2 + dz^2\right) + C^2dw^2 \tag{1}$$

where A, B, C are functions of cosmic time 't' alone.

The field Equations in Barber's second self creation theory are given by

$$G_{ij} = -\frac{8\pi T_{ij}}{\phi} \tag{2}$$

and the scalar field ϕ satisfies the equation

$$\Box\phi = \frac{8\pi}{3}\eta T \tag{3}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, η is a coupling constant, ϕ is a Barber's scalar and T is the trace of energy momentum tensor.

The energy momentum tensor is given by

$$T_i^j = (p+\rho)u_i u^j + p g_i^j \tag{4}$$

where ρ is the energy density, p is isotropic pressure and u^i is the flow vector satisfying

$$g_{ij}\, u^i\, u^j = -1 \tag{5}$$

Using equation (5), the energy momentum tensor (4) for the line element (1) is given by

$$T_0^{0'} = -\rho,\quad T_1^1 = T_2^2 = T_3^3 = T_4^4 = p$$

$$T_i^j = 0 \text{ for } i \neq j \text{ and } T = -(\rho - 4p) \tag{6}$$

The Barber's field equations (2) and (3) for the line element (1) leads to

$$\frac{2A_4B_4}{AB} + \frac{2B_4C_4}{BC} + \frac{A_4C_4}{AC} + \frac{B_4^2}{B^2} = \frac{8\pi\rho}{\phi} \tag{7}$$

$$\frac{2B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{B_4^2}{B^2} = -\frac{8\pi p}{\phi} \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{C_4A_4}{CA} = -\frac{8\pi p}{\phi} \tag{9}$$

$$\frac{A_{44}}{A} + \frac{2B_{44}}{B} + \frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} = -\frac{8\pi p}{\phi} \tag{10}$$

and

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{2B_4}{B} + \frac{C_4}{C}\right) = \frac{8\pi}{3}\eta(\rho - 4p) \tag{11}$$

where the suffix '4' after A, B, C and ϕ denote ordinary differentiation with respect to cosmic time t.

Solution of Field Equations

Here we have six unknowns A, B, C, ρ, p and ϕ in five independent field equations. To get a determinate solution, we assume that component σ_4^4 of shear tensor σ_i^j is proportional to scalar of expansion θ i.e.

$\sigma_4^4 \propto \theta$

which leads to

$$\sigma_4^4 = n\theta \quad (12)$$

where n is constant of proportion

For the metric (1), we have

$$\sigma_4^4 = \frac{1}{3}\left(\frac{2C_4}{C} - \frac{2B_4}{B} - \frac{A_4}{A}\right) \quad (13)$$

And $\theta = \frac{A_4}{A} + \frac{2B_4}{B} + \frac{C_4}{C}$ (14)

without loss of generality, the constant of proportion n can be taken as 1/6. Using equations (13) and (14) in (12), we have

$$C = AB^2 \quad (15)$$

From equation (8) and (10), we have

$$\frac{A_{44}}{A} - \frac{C_{44}}{C} + \frac{2B_4}{B}\left(\frac{A_4}{A} - \frac{C_4}{C}\right) = 0 \quad (16)$$

Using equation (15) in (16), we have

$$\frac{B_4}{B}\left[\frac{B_{44}}{B_4} + \frac{3B_4}{B} + \frac{2A_4}{A}\right] = 0 \quad (17)$$

which leads to following three cases

Case (I) $B_4 = 0$

Case (II) $\frac{B_{44}}{B_4} + \frac{3B_4}{B} + \frac{2A_4}{A} = 0$

Case (III) $B_4 = 0$ and $\frac{B_{44}}{B_4} + \frac{3B_4}{B} + \frac{2A_4}{A} = 0$

Case (I) when $B_4 = 0$ (18)

which leads to

$B = K_1$ (19)

where K_1 is constant of integration.

Using equation (19) in (15), we have

$$C = K_1^2 A \tag{20}$$

Using equations (19) and (20) in equations (7) - (11), we have

$$\frac{A_4^2}{A^2} = \frac{8\pi\rho}{\phi} \tag{21}$$

$$\frac{A_{44}}{A} = -\frac{8\pi p}{\phi} \tag{22}$$

$$\frac{2A_{44}}{A} + \frac{A_4^2}{A^2} = -\frac{8\pi p}{\phi} \tag{23}$$

$$\phi_{44} + \frac{2A_4}{A}\phi_4 = \frac{8\pi\eta}{3}(\rho - 4p) \tag{24}$$

Using equations (21) and (22) in (24), we get

$$\phi_{44} + \frac{2A_4}{A}\phi_4 = \frac{\eta}{3}\left[\frac{A_4^2}{A^2} + \frac{4A_{44}}{A}\right] \tag{25}$$

Equations (22) and (23) lead to

$$\frac{A_4}{A}\left[\frac{A_{44}}{A_4} + \frac{A_4}{A}\right] = 0 \tag{26}$$

which leads to following three subcases

Subcase (Ia) $A_4 = 0$

Subcase (Ib) $\frac{A_{44}}{A_4} + \frac{A_4}{A} = 0$

Subcase (Ic) $A_4 = 0$ and $\frac{A_{44}}{A_4} + \frac{A_4}{A} = 0$

Subcase (Ia) when $A_4 = 0$

which leads to

$A = K$ (27)

where K is constant of integration.

Using equation (27) in (20) and (25), we have

$$C = K_1^2 K \tag{28}$$

$$\phi = (at + b) \tag{29}$$

where a and b are constants of integration.

The metric (1) in this subcase reduces to

$$ds^2 = -dt^2 + K^2dx^2 + K_1^2(dy^2 + dz^2) + K^2K_1^4dw^2 \tag{30}$$

After suitable transformation of coordinates, metric (30) leads to

$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2 + dW^2 \tag{31}$$

Some Physical and Kinematical Features

The energy density (ρ), isotropic pressure (p) and the scalar field (ϕ) for the metric (31) are given by

$\rho = 0$

$p = 0$

$\phi = aT + b$

Subcase (Ib) when $\frac{A_{44}}{A_4} + \frac{A_4}{A} = 0$

which leads to

$$A = (K_2t + K_3)^{1/2} \tag{32}$$

where K_2 and K_3 are constants of integration.

Using equation (32) in (20), we have

$$C = K_1^2(K_2t + K_3)^{1/2} \tag{33}$$

Therefore, the metric (1) in this subcase reduces to

$$ds^2 = -dt^2 + (K_2t + K_3)dx^2 + K_1^2(dy^2 + dz^2) + K_1^4(K_2t + K_3)dw^2 \tag{34}$$

After suitable transformation of coordinates, metric (34) reduces to

$$ds^2 = -dT^2 + TdX^2 + dY^2 + dZ^2 + TdW^2 \tag{35}$$

Some Physical and Kinematical Features

The energy density (ρ), isotropic pressure (p) and the scalar field (ϕ) for the metric (35) are given by

$$\rho = \frac{1}{32\pi T^2}\left[m_1 \cos\{\log(T)\} + m_2 \sin\{\log(T)\}\right]$$

$$P = \frac{1}{32\pi T^2}\left[m_1 \cos\{\log(T)\} + m_2 \sin\{\log(T)\}\right]$$

$$\phi = m_1 \cos\{\log(T)\} + m_2 \sin\{\log(T)\}$$

The scalar of expansion (θ), spatial volume (V) and shear (σ) for the model (35) are given by

$\theta = 1/T$

$V = T$

$$\sigma = \frac{\sqrt{5}}{6T}$$

Subcase (Ic) when $A_4 = 0$ and $\frac{A_{44}}{A_4} + \frac{A_4}{A} = 0$

This case is not acceptable as the second equation becomes indeterminate.

Case II when $\frac{B_{44}}{B_4} + \frac{3B_4}{B} + \frac{2A_4}{A} = 0$

which leads to

$$B = \left[\int \frac{\ell_1}{A^2(t)} dt + \ell_2\right]^{1/4} \tag{36}$$

i.e. for a given A(t), we can easily find out B(t). For deterministic solution, we consider this case in the following manner

$$\frac{B_{44}}{B_4} + \frac{3B_4}{B} = -\frac{2A_4}{A} = \ell_1 (\text{cons}\tan t) \tag{37}$$

which leads to

$$B = \left[4\left(\frac{\ell_2}{\ell_1} e^{\ell_1 t} + \ell_3\right)\right]^{1/4} \tag{38}$$

and $$A = \ell_4 e^{-\frac{\ell_1 t}{2}} \tag{39}$$

where ℓ_2, ℓ_3 and ℓ_4 are constants of integration.

With the help of equations (38) and (39), equation (15) leads to

$$C = \ell_4 e^{-\frac{\ell_1 t}{2}}\left[4\left(\frac{\ell_2}{\ell_1} e^{\ell_1 t} + \ell_3\right)\right]^{1/2} \tag{40}$$

Using equations (7), (10) and (15) in equation (11), we have

$$\phi_{44} + 2\left(\frac{A_4}{A} + \frac{2B_4}{B}\right)\phi_4 = \frac{\eta}{3}\phi\left[\frac{4A_{44}}{A} + \frac{8B_{44}}{B} + \frac{14A_4B_4}{AB} + \frac{9B_4^2}{B^2} + \frac{A_4^2}{A^2}\right] \tag{41}$$

Using equations (38) and (39) in equation (41), we have

$$\phi_{44} + \frac{(-\ell_3)\ell_1^2}{\left(\ell_2 e^{\ell_1 t} + \ell_3\ell_1\right)} + \frac{\eta}{3}\left[\frac{15\ell_1^2\ell_2^2 e^{2\ell_1 t}}{\left[4\left(\ell_2 e^{\ell_1 t} + \ell_3\ell_1\right)\right]^2} - \frac{5}{4}\ell_1^2 - \frac{\ell_1^2\ell_2 e^{\ell_1 t}}{4\left(\ell_2 e^{\ell_1 t} + \ell_3\ell_1\right)}\right]\phi = 0 \tag{42}$$

The metric (1) in this case reduces to

$$ds^2 = -dt^2 + \ell_4^2 e^{-\ell_1 t}dx^2 + \left[4\left(\frac{\ell_2}{\ell_1}e^{\ell_1 t} + \ell_3\right)\right]^{½}\left(dy^2 + dz^2\right) + \ell_4^2 e^{-\ell_1 t}\left[4\left(\frac{\ell_2}{\ell_1}e^{\ell_1 t} + \ell_3\right)\right]dw^2 \tag{43}$$

After suitable transformation of coordinates, metric (43) reduces to

$$ds^2 = -dT^2 + e^{-\ell_1 T}dX^2 + \left(e^{\ell_1 T} - \ell_1 a_1\right)^{½}\left(dY^2 + dZ^2\right) + \left(e^{-\ell_1 T} - \frac{1}{\ell_1 a_1}\right)dW^2 \tag{44}$$

where $a_1 = -\frac{\ell_3}{\ell_2}$

Some Physical and Kinematical Features

The energy density (ρ) and isotropic pressure (p) of the model (44) are given by

$$\rho = \frac{\phi}{8\pi}\left[\frac{5\ell_1^2 e^{2\ell_1 T}}{16\left(e^{\ell_1 T} - a_1\ell_1\right)^2} + \frac{\ell_1^2}{4} - \frac{3\ell_1^2 e^{\ell_1 T}}{4\left(e^{\ell_1 T} - a_1\ell_1\right)}\right]$$

$$p = \frac{\phi}{8\pi}\left[\frac{\ell_1^2\left(e^{2\ell_1 T} + 4a_1^2\ell_1^2 - 12a_1\ell_1 e^{\ell_1 T} - 2e^{\ell_1 T}\right)}{16\left(e^{\ell_1 T} - a_1\ell_1\right)^2}\right]$$

where ϕ is given by

$$\phi_{44} + \frac{\ell_1^2 a_1\phi_4}{\left(e^{\ell_1 T} - a_1\ell_1\right)^2} - \frac{\eta}{3}\phi\left[\frac{5}{4}\ell_1^2 + \frac{\ell_1^2 e^{\ell_1 T}\left[17e^{\ell_1 T} - 32a_1\ell_1 - 28\left(e^{\ell_1 T} - \ell_1 a_1\right)\right]}{16\left(e^{\ell_1 T} - a_1\ell_1\right)^2}\right]$$

The scalar of expansion (θ), spatial volume (V) and shear (σ) for the model (44) are given by

$$\theta = \frac{\ell_1^2 a_1}{\left(e^{\ell_1 T} - a_1\ell_1\right)}$$

$$V = e^{-\ell_1 T/2}\left(e^{\ell_1 T} - \ell_1 a_1\right)^{½}\left(e^{-\ell_1 T} - \frac{1}{\ell_1 a_1}\right)^{½}$$

$$\sigma = \frac{\ell_1^2}{144\left(e^{\ell_1 T} - \ell_1 a_1\right)^2}\left[27e^{2\ell_1 T} + 20a_1^2\ell_1^2 - 36a_1\ell_1 e^{\ell_1 T}\right]$$

Special Case

For equation (42), it is not possible to find general solution for ϕ, so we take

$$\ell_1 = 0 \tag{45}$$

Using equation (45) in (37), we have

$$A = \ell_5 \tag{46}$$

$$B = [\,4(\ell_6 t + \ell_7)]^{1/4} \tag{47}$$

where ℓ_5, ℓ_6, and ℓ_7 are constants of integration.

Using equation (46) and (47) in (15), we have

$$C = \ell_5\,[\,4(\ell_6 t + \ell_7)]^{1/2} \tag{48}$$

Using equation (46) and (47) in (41), we have

$$\phi_{44} + \frac{\ell_6}{(\ell_6 t + \ell_7)}\phi_4 + \frac{5}{16}\frac{\ell_6^2}{(\ell_6 t + \ell_7)^2} = 0$$

which leads to

$$\phi = b_1\cos\{\log(\ell_6 t + \ell_7)\} + b_2\sin\{\log(\ell_6 t + \ell_7)\}$$

where b_1 and b_2 are constants of integration.

In this case the metric (1) reduces to

$$ds^2 = -dt^2 + \ell_5^2 dx^2 + [4(\ell_6 t + \ell_7)]^{1/2}(dy^2 + dz^2) + \ell_5^2[4(\ell_6 t + \ell_7)]dw^2 \tag{49}$$

After suitable transformation of coordinates, metric (49) reduces to

$$dS^2 = -dT^2 + dX^2 + T^{1/2}(dY^2 + dZ^2) + TdW^2 \tag{50}$$

Some Physical and Kinematical Features

The energy density (ρ), isotropic pressure (p) and the scalar field (ϕ) of the model (50) are given by

$$\rho = \frac{5}{128\pi T^2}\left[b_1\cos\{\log(T)\} + b_2\sin\{\log(T)\}\right]$$

$$p = \frac{5}{128\pi T^2}\left[b_1\cos\{\log(T)\} + b_2\sin\{\log(T)\}\right]$$

$$\phi = \left[b_1\cos\{\log(T)\} + b_2\sin\{\log(T)\}\right]$$

The scalar of expansion (θ), spatial volume (V) and shear (σ) for the model (50) are given by

$$\theta = \frac{1}{T}$$

V = T

$$\sigma = \frac{\sqrt{11}}{12}\frac{1}{T}$$

Case III when $B_4 = 0$ and $\frac{B_{44}}{B_4} + \frac{3B_4}{B} + \frac{2A_4}{A} = 0$

This case is not acceptable as the second equation become indeterminate.

Conclusion

The model (31) is flat in five dimension space time. For T = –b/a, model in Barber second self creation theory reduces to Einstein general theory of relativity. The model represents a vacuum model in Barber's self creation theory.

The expansion in the model (35) decreases as time increases. The model expands with a big-band at T = 0 and stops at $T = \infty$. Volume V increases as time increases. Pressure and energy density tends to vanish as $T \to \infty$ and tends to infinity as $T \to 0$ respectively. It represents a dust perfect fluid and dust is the source of gravitational field.

In the model (44), the fifth dimension contracts and it reduces to 4-dimensional space time in the later stage for $\ell_1 > 0$ where as two dimensions normally expands and remaining one dimension remains constant. If $\ell_1 < 0$, then only two space dimensions increases whereas two dimensions contracts. $\ell_1 = 0$ is the singularity in fifth dimension. Scalar of expansion $\theta = \frac{\ell_1^2 a_1}{1 - a_1\ell_1}$ at T = 0 and $\theta \to 0$ as $T \to \infty$. i.e. model starts to expand as $T \to \infty$. Volume $V \to$ constant as $T \to 0$ and $V \to \infty$ as $T \to \infty$. Energy density $\rho \to \frac{\phi}{32\pi}$ as $T \to 0$ and $\rho = \left(\frac{-3\ell_1^2}{128\pi}\right)$ as $T \to \infty$ i.e. in the absence of field ϕ, energy density tends to zero and field ϕ is only responsible for energy density at the initial stage of evolution of the universe and at the later stages energy conditions are not satisfied. Since $\rho < 0$ for $\phi > 0$ i.e. in the absence of field ϕ, the model remains vacuum. $\rho = p = 0$ for $\ell_1 = 0$. For $T > \frac{1}{\ell_1}\log\left(\frac{\ell_1 a_1}{3}\right)$, the model decelerates when $\ell_{1,}$ $a_1 > 0$ and for $T > \frac{1}{\ell_1}\log\left(\frac{\ell_1 a_1}{3}\right)$ the model inflates if $\ell_1 < 0$ or $a_1 < 0$. Since $\lim_{T\to\infty}\frac{\sigma}{\theta} \neq 0$, therefore model remains anisotropic for large values of T.

The model (50) starts to expand with a big-bang at T = 0 and stops at T = ∞. Volume increases as T increases. Density and pressure tends to ∞ as $T \to 0$ and became vacuum model at $T \to \infty$. Shear tends to infinity as $T \to 0$ and $\sigma \to 0$ as $T \to \infty$. It is a decelerating model and it remain always anisotropic throughout the evolution.

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