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Einstein Equation and Self-Graviting Objects.

Connotations for High Density Stars

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Abstract

We analyze a type of self-gravitating object, described by a soliton to the coupled system of the Einstein equation and a matter field equation. The existence of such objects could have important connotations in astrophysics, specially by high density stars.

Keywords: oscillations, density, stars

1 Introduction

In astronomy and cosmology, dark matter is hypothetical matter that is undetectable by its emitted radiation, but whose presence can be inferred from gravitational effects on visible matter. According to present observations of structures larger than galaxies, as well as Big Bang cosmology, dark matter and dark energy could account for the vast majority of the mass in the observable universe.

Dark matter is postulated to partially account for evidence of “missing mass" in the universe, including the rotational speeds of galaxies, orbital velocities of galaxies in clusters, gravitational clusters, gravitational lensing of background objects by galaxy clusters, and the temperature distribution of hot gas in galaxies and clusters of galaxies.

Dark matter is believed to play a central role in structure formation and galaxy, and has measurable effects on the anisotropy of the cosmic microwave background. All these lines of evidence suggest that galaxies, clusters of galaxies, and the universe as a whole contain far more matter than that which interacts with electromagnetic radiation: the remainder is frequently called the "dark matter component," even though there is a small amount of baryonic dark matter. The largest part of dark matter which does not interact with electromagnetic radiation is not only "dark" but also by definition utterly transparent; in recognition of this, it has been referred to as transparent matter by some astronomers.

As important as dark matter is believed to be in the universe, direct evidence of its existence and a concrete understanding of its nature have remained elusive. Though the theory of dark matter remains the most widely accepted theory to explain the anomalies in observed galactic rotation, some alternative theories such as modified newtonian dynamics and tensor-vector-scalar gravity have been proposed.

Modern cosmology is in a state of crisis, it started with dark matter and erupted with an indication that most of the universe is made up of dark energy. But what is the dark matter, and what does it mean for the universe? No one at this time can definitely state what dark matter is other than to assure us that it really does exist and that most of our galaxy is made of it. At present there are two prevailing thoughts as to what dark matter could be, the most popular thought being that it represents a kind of exotic state of normal matter [1, 3, 7, 12] (or perhaps more accurately that “normal matter” is an exotic representation of the more abundant form of matter in the universe) that we have yet to identify. The least popular thought to what dark matter is –is simply that it does not exist and that it originates due to over reliance on the assumption that Newtonian gravity applies everywhere at the same magnitude that it does within the Solar System. The overall problem with dark matter is that it is questions one of the most sacred paradigms of physics, that the laws of physics on Earth are no different than those on the Moon, the Sun, and the whole of the universe. Yet the existence of dark matter seems to challenge the idea that the laws of physics are the same everywhere, as either some unknown physics can generate dark matter non universally or that law of gravitation does not apply universally.

2 The model

The classical field theories [4-10] admit nontopological soliton solutions, solutions which have finite and nonzero masses, confined to finite regions of space for all time, free of singularity, and which are nontopological in nature [6]. The interest in these solutions systems to issue from the dark-matter problem in cosmology. It is by now well accepted, that visible, baryonic matter can account for only a small fraction of the total mass of the Universe, and there are strong indications that the dark matter is nonbaryonic in nature. Various kinds of nontopological soliton configurations of nonbaryonic matter have been proposed and studied for their possible astrophysical roles.
In this work we investigate the existence of soliton solutions for classical field theories. As an example, I show that a massive real scalar field satisfying the Klein-Gordon equation can form a self-gravitating solitonic object when coupled to Einstein gravity. I call such objects oscillating soliton stars, emphasizing their possible astrophysical role.

3 The metric and equations

As a simple example of an oscillating soliton star, I consider a massive, real Klein-Gordon scalar field, coupled only to gravity. I expect, in the absence of angular momentum, that the soliton solution to be spherically symmetric. The metric can then be written in the form,

$$ds^2 = - N^2 (t,r)\, dt^2 + g^2 (t,r)\, dr^2 + r^2\, d\Omega^2$$

The coupled Einstein-Klein-Gordon equations lead to

$$(N^2)' = N^2 \left[\frac{g^2 - 1}{r}\right] + 4\pi G r \left(N^2 \phi'^2 - N^2 g^2 \phi^2 + g^2 \dot{\phi}^2\right)$$

$$(g^2)' = - g^2 \left[\frac{g^2 - 1}{r\, N^2}\right] + 4\pi G r g^2 \left[g^2 \phi'^2 + \phi'^2 + g^2 m^2 \phi^2\right]$$

$$\ddot{\phi} = - \frac{(N^2)^{\cdot}\, \dot{\phi}}{2N^2} + \frac{(N^2)'\, \phi'}{2g^2} + \frac{N^2}{g^2} \left[\phi'' - \frac{(g^2)'\, \phi'}{2g^2} - \frac{(g^2)^{\cdot}\, \dot{\phi}}{2N^2}\right] + \frac{2}{r}\frac{N^2}{g^2}\phi'$$

$$- m^2 N^2 \phi$$

where an overdot denotes $\partial/\partial t$ and a prime denotes $\partial/\partial r$.
It is tempting to search for time-independent solutions. However, the pseudovirial theorem of Rosen [8] implies that no such solution is possible in the Newtonian limit, and in the strong-field case. All known static solutions in the previous system of equations has no singularities [2]. The structure of equations suggests periodic expansions of the form

$$N^2(t,r) = 1 + \Sigma^{\infty}_{j=0} N_{2j}(r) \cos(2j\omega_0 t)$$
$$g^2(t,r) = 1 + \Sigma^{\infty}_{j=0} g_{2j}(r) \cos(2j\omega_0 t)$$
$$\phi(t,r) = \Sigma^{\infty}_{j=1} \phi_{2j-1}(r) \cos[(2j-1)\omega_0 t]$$

I put these expansions into equations, set the coefficients of each Fourier component to zero, and obtain a system of coupled nonlinear ordinary first-order differential equations for $N_{2j}(r)$ and $g_{2j}(r)$, and second-order differential equations for $\phi_{2j-1}(r)$. The boundary conditions are given by the following requirements: asymptotic flatness requires $N_{2j}(\infty) = g_{2j}(\infty) = \phi_{2j+1}(\infty) = 0$, at $r = 0$, the absence of a conical singularity implies $g_{2j}(r = 0) = 0$. The requirement that the metric coefficients be finite at $r = 0$ implies that $(d/dr)\phi_{2j-1}(r = 0) = 0$.
This is an eigenvalue problem; is there a nontrivial solution to the set of ordinary differential equations satisfying the above boundary conditions for some particular values of $N_{2j}(0)$, $\phi_{2j-1}(0)$, and ω_0 have an impossible analytic solution.

4 Numerical results

The system of equations it is truncate after a certain maximum $j=j_{max}$, numerically solve the eigenvalue problem, and study the convergence of the series as a function of $j = j_{max}$. We find that for each value of $\phi_1(r = 0)$ there exists a set of values for the other initial data such that a solution satisfying the appropiate boundary conditions at $r = \infty$ exists. A typical radial metric function $g^2(t=0, r) - 1$, for the case of $\phi_1(0) = 0.20$, is plotted as a solid line in Figure 1, the individual components $g_{2j}(r)$ are plotted as dashed lines for the first few values of j. The series expansion converge. It can see that the scalar field energy densities (measured by an observer at fixed radius r) at $\omega_0 t = 0, \pi/2$, and π.
In Figure 2 the mass M of the star is plotted against the radius containing 97 % of its mass. This mass curve is similar it those of white dwarfs and neutron stars, with maximum mass given by $M_c \approx 0.6\ M^2_{Planck/m}$., and the total mass of the oscillating soliton star (in units of M^2_{Planck}/m) is calculated as a function of its radius R (in units of $1/m$).

It is clear that this oscillating soliton solution cannot be obtained as a post-Newtonian expansion, even for those weak-field configurations having small total mass *M* and large radii. In the Newtonian expansion time derivatives of the metric functions are treated as one order higher in smallness than spatial derivatives. This is not true for oscillating soliton stars, for which temporal derivatives are of zeroth order ($\partial/\partial t \approx \omega_0 \approx m$). The oscillation is an intrinsic character of the solution.
Given the explicit construction of the first few terms in the expansion, one would like to investigate the importance of the rest of the terms which were neglected. Second, it is important to know whether the solution is stable with respect to perturbations. Such an initial configuration can be regarded as an exact oscillating soliton solution with a small perturbation.

5 Conclusions

Preliminary studies shows that this can be formed under very general initial conditions, and we have been obtained similar results that others authors [11]. Therefore, even if the object is just quasi-periodic, the existence of this type of self-graviting object could have significant astrophysical implications.
The existence of such objects give rise to the possibility that the dark matter is made up of oscillating soliton stars, and the condensation of, e.g., axions or pseudo Higgs bosons into very compact, high-density oscillating soliton stars may significantly enhance their annihilation rates, which could in turn rule them out as dark-matter candidates. Whether one of these interesting possibilities turns out to be the case hinges on the formation process of the oscillating soliton stars.

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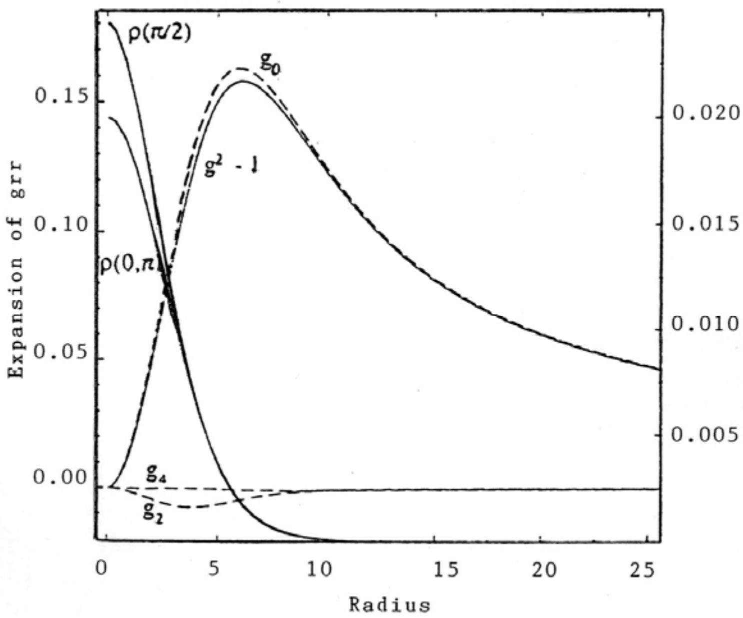

Figure 1: A solution to the truncated eigenvalue equations ($j_{mas} = 2$) of the metric quantity g_{rr} is shown (left vertical axis) for a solution with a mass $M = 0.56\ M^2_{Planck}/m$. The solid line shows $g_{rr} - 1$, while the dashed lines show the first three terms of its cosine series expansion. This rapid convergence of the series is typical of all the configurations calculated.

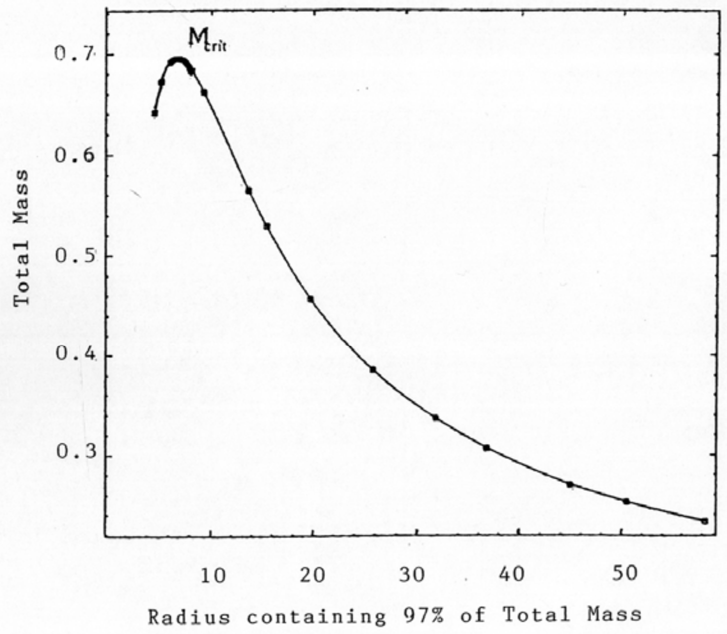

Figure 2: The total mass M of the oscillating soliton star (in units of $M^2_{\text{Planck/m}}$) plotted as a function of its radius R (in units of $1/m$). The squares represent actual configuration resulting from solutions to the eigenvalue equations. Configurations to the right of the maximum mass $M_{\text{critic}} \approx 0.7$ are stable, while those to the left are unstable.

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