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Conflict Between the Classical Equivalence Principle and Quantum Mechanics

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Abstract

As far as external gravitational fields described by Newton's theory are concerned, theory shows that there is an unavoidable conflict between the universality of free fall (Galileo's equivalence principle) and quantum mechanics — a result confirmed by experiment. Is this conflict due perhaps to the use of Newton's gravity, instead of general relativity, in the analysis of the external gravitational field? The response is negative. To show this we compute the low corrections to the cross-section for the scattering of different quantum particles by an external gravitational field, treated as an external field, in the framework of Einstein's linearized gravity. To first order the cross-sections are spin-dependent; if the calculations are pushed to the next order they become dependent upon energy as well. Therefore, the Galileo's equivalence and, consequently, the classical equivalence principle, is violated in both cases. We address these issues here.

Keywords: spin-dependent cross-section, energy-dependent cross-section, violation of the classical equivalence principle, quantum mechanics.

1 Introduction

Undoubtedly, the two great pillars of modern physics are quantum mechanics and general relativity. Nonetheless, the merging of these theories has not yet been accomplished despite the herculean efforts employed by so many distinguished physicists. What is the rationale for such an incompatibility between these two outstanding theories? Perhaps the conceptual difficult of reconciling local general relativity with nonlocal quantum effects or, equivalently, of reconciling the local character of the so-called strong equivalence principle — *An ideal observer in a gravitational field can choose a reference frame in which gravitation goes unnoticed* [1] — with the nonlocal character of the uncertainty principle. It is quite remarkable that this fundamental fact has been overlooked by so many researches in quest of quantum gravity. Actually, if there is indeed a fundamental incompatibility between the strong equivalence principle and quantum mechanics it is likely to be in the realm of strong gravity — black holes, for example. Unfortunately, the only two approaches to quantum gravity that have reached a full mathematical description of the quantum properties of the gravitational field, namely, string theory and loop quantum gravity, have not been tested. This is not so surprising if we take into account that the characteristic scale for quantum gravity is the Planck energy, $E_{\text{P}} \sim 10^{19} GeV$, which is so far out of the range of experiment that the idea of a direct observational test has long seemed a chimerical plan. The claims that we are at the dawn of quantum gravity phenomenology [2], despite being, at least in principle, plausible, are somewhat speculative which seems in a sense to jeopardize the future of this phenomenological approach.

What about the compatibility between the classical equivalence principle (universality of free fall, or equivalence of gravitational and inertial masses) with quantum mechanics? Before analyzing this issue it is of great value for clarity's sake to remark that the classical equivalence principle is actually a nonlocal amalgam of two recognizable different principles, namely, *Galileo's equivalence principle* (all pointlike particles put at a given point with the same velocity fall with the same acceleration in an external gravitational field) and *Newton's equivalence principle* ($m_{\text{inertial}} = m_{\text{gravitational}}$). Although they are considered equivalent in the realm of Newton's gravity, experiment tells us that this is not the case. Indeed, the classical experiments that constitute evidence for the Newton's equivalence only give limited support to the Galileo's equivalence. For this reason we will treat these two principle as distinct ones. After this little digression we return to our inquiry about whether or not quantum mechanics and the classical equivalence principle can exist together successfully. In this work we will restrict our considerations only to external gravitational fields. As far as Newton's gravity is concerned, theory shows that quantum mechanics and the Galileo's equivalence are irreconcilable [3]

— a result confirmed by experiment [4]. Nonetheless, it is worth mentioning that nowadays there exists violation of "$m_{\rm i} = m_{\rm g}$" at 1% level in neutron interferometry experiment [5]. Furthemore, from an operational point of view one cannot claim, even in principle, exact equality of $m_{\rm i}$ and $m_{\rm g}$ (for certain quantum systems) [6, 7]. So, it is reasonable to ask ourselves whether the conflict between these principles has its roots in the fact that we have used Newton's gravity to describe the gravitational interaction. To respond this question it is our main goal here.

The plan of this work is as follows. In Section 2 we show that to first order the cross-sections for the scattering of different quantum particles by a weak gravitational field, treated as an external field, are spin dependent, which is obviously in disagreement with the classical equivalence principle. The cross-section for the scattering of a massive scalar boson by the aforementioned gravitational field is analyzed in Section 3 by pushing the calculations to the next order. Now the cross-section depends on the mass which is also in disagreement with the classical equivalence principle. We conjecture at last about the likelihood of measuring the aforementioned deviation from the classical equivalence in the foreseeable future in Section 4.

For simplicity's sake we assume in all our computations the equality of inertial and gravitational masses.

In our convention the signature is $(+---)$. The curvature tensor is defined $R^{\alpha}{}_{\beta\gamma\delta} = -\partial_{\delta}\Gamma^{\alpha}{}_{\beta\gamma} + ...$, the Ricci tensor by $R_{\mu\nu} = R^{\alpha}{}_{\mu\nu\alpha}$ and the curvature scalar by $R = g^{\mu\nu}R_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor. Natural units are used throughout.

2 A General Theorem

To begin with, we will calculate to first order the differential cross-section for the scattering of a massive scalar boson by a weak gravitational field, treated as an external field. The purpose of this calculation is twofold: on the one hand, it will serve as a paradigm for the evaluation to first order of the cross-sections for the scattering of different quantum particles by an external gravitational field; on the other hand it will be utilized in Section 3 for computing the cross-section for the very and same process but to second order. Nevertheless, in order to avoid boring repetitions we will not exhibit the calculations concerning the evaluation of the differential cross-sections for the scattering of both massless particles of spin $\frac{1}{2}$, 1, and 2, and massive particles of spin $\frac{1}{2}$ and 1; instead we will present only the final results.

2.1 Scattering of a massive scalar boson by an external gravitational field to first order

In the weak field approximation, i. e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu},$$

with $\eta_{\mu\nu} = diag(+1,-1,-1,-1)$, and in the de Donder gauge, the general solution of the linearized Einstein's equations, having as source a point particle of mass M located at $\mathbf{r} = \mathbf{0}$ can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2MG}{r}\left(\eta_{\mu\nu} - 2\eta_{\mu 0}\eta_{\nu 0}\right),$$

where $\kappa^2 = 32\pi G$, while the momentum space gravitational field, namely, $h_{\mu\nu}(\mathbf{k}) \equiv \int d^3\mathbf{r} e^{-i\mathbf{k}.\mathbf{r}} h_{\mu\nu}(\mathbf{r})$ reads

$$h_{\mu\nu}(\mathbf{k}) = \frac{\kappa M \eta_{\mu\nu}}{4\mathbf{k}^2} - \frac{\kappa M}{2}\frac{\eta_{\mu 0}\eta_{\nu 0}}{\mathbf{k}^2}.$$

To first order, the Feynman amplitude, $\mathcal{M}^{(0)}$, for the scattering of a scalar boson of mass m by a weak gravitational field, treated as an external field (See Fig. 1), is given by

$$\begin{aligned} i\mathcal{M}^{(0)} &= i h_{\alpha\beta}(\mathbf{q}) V^{\alpha\beta}(p,p') \\ &= -\frac{4\pi MG}{\mathbf{q}^2}(2m^2 + 4\mathbf{p}^2), \end{aligned}$$

where $V^{\alpha\beta}(p,p')$ — the graviton-scalar-scalar vertex — is given by

$$V^{\alpha\beta}(p,p') = -\frac{i\kappa}{2}\left[p'^{\alpha}p^{\beta} + p'^{\beta}p^{\alpha} - \eta^{\alpha\beta}(p'\cdot p - m^2)\right].$$

For convenience sake's we have multiplied the Feynman amplitude by i in order to end up with a real expression.

Accordingly, the differential cross-section can be written as

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_0 &= \frac{|i\mathcal{M}|^2}{(4\pi)^2} \\ &= \left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\left(1 + \frac{\alpha}{2}\right)^2, \end{aligned} \quad (1)$$

where $\alpha \equiv \frac{m^2}{\mathbf{p}^2} = \frac{1-\mathbf{v}^2}{\mathbf{v}^2}$, with $\mathbf{v}$ being the the velocity of the ingoing scalar boson, and θ is the scattering angle.

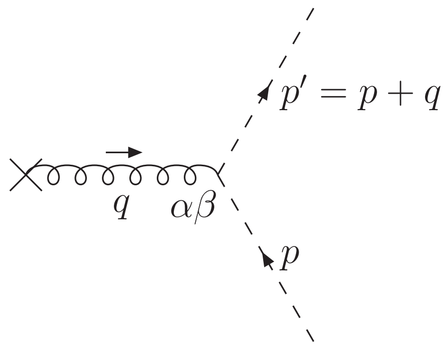

Figure 1: Feynman graph for the scattering of a massive scalar boson by an external gravitational field; $|\mathbf{p}| = |\mathbf{p}'|$.

2.2 Unpolarized differential cross-sections for the scattering of different quantum particles by an external gravitational field to first order

Utilizing a procedure similar to that employed to compute the preceding cross-section, as well as much algebra, we obtained the differential cross-sections for the scattering of the remaining quantum particles mentioned in the beginning of Section 2. These results are collected in Table 1.

The analytical results displayed in Table 1 are certainly robust theoretically; nonetheless, they are at variance with the Galileo's equivalence and, consequently, with the classical equivalence principle since they clearly depend on the spin of the quantum particle deflected by the gravitational field. We are now ready to enunciate a general theorem.

Theorem 2.1 *To first order the unpolarized differential cross-section for the scattering of different quantum particles by an external gravitational field described by Einstein's linearized field equations is spin dependent, which is in disagreement with the classical equivalence principle.*

Why the gravitational field perceives the spin? Because there is the presence of a nonzero momentum transfer ($\mathbf{q}$) in the scattering, responsible for probing the internal structure (spin) of the particle. Therefore, in order to recover Einstein's geometrical results, we must have $\mathbf{q} \rightarrow \mathbf{0}$; in other words, in the nontrivial limit of small momentum transfer, which corresponds to a nontrivial small angle limit since $|\mathbf{q}| = 2|\mathbf{p}| \sin\frac{\theta}{2}$, the massive (massless) particles behave in the same way, regardless the spin. In fact, when the spin is "switched off", we obtain from Table 1 that for $m = 0$,

$$\left(\frac{d\sigma}{d\Omega}\right)_0 \approx \frac{16G^2M^2}{\theta^4}, \tag{2}$$

while for $m \neq 0$,

$$\left(\frac{d\sigma}{d\Omega}\right)_0 \approx \frac{16G^2M^2}{\theta^4}\left(1+\frac{\alpha}{2}\right)^2. \tag{3}$$

These cross-sections can be related to a classical trajectory with impact parameter b via the relation $bdb = -\frac{d\sigma}{d\Omega}\theta d\theta$. Accordingly, we conclude that for $m = 0$,

$$\theta_0 \approx \frac{4GM}{b}, \tag{4}$$

and for $m \neq 0$,

$$\theta_0 \approx \frac{4GM}{b}(1+\frac{\alpha}{2}). \tag{5}$$

The former equation gives the gravitational deflection angle for a massless classical test particle — a result foreseen by Einstein a long time ago; whereas the latter gives just the prediction of general relativity for the deflection of a massive classical test particle by an external gravitational field [8]. The results of Table 1, in short, reduce for small angles to those predicted by Einstein's geometrical theory, confirming in this way the accuracy of our analytical calculations. Note that Equation (5) tells us that for $|\mathbf{v}| \ll 1$, $\theta_0 \longrightarrow \frac{2GM}{b\mathbf{v}^2}$, which is nothing but Newton's prediction for the gravitational deflection angle; it also reproduces Equation (4) in the limit $|\mathbf{v}| \longrightarrow 1$. Note also that for $\frac{m}{E} \ll 1$, where E is the energy of the incoming particle, Equation (5) leads to

$$\theta_0 \approx \frac{4GM}{b}\left(1+\frac{m^2}{2E^2}\right), \tag{6}$$

a result that was recently utilized to find an upper bound for the photon mass [9, 10].

Table 1: Unpolarized differential cross-sections for the scattering of different quantum particles by an external gravitational field to first order. Here m is the particle mass, s the spin, θ the scattering angle, and $\alpha \equiv \frac{m^2}{\mathbf{p}^2} = \frac{1-\mathbf{v}^2}{\mathbf{v}^2}$, with $\mathbf{v}$ and $\mathbf{p}$ being the velocity and three-momentum, in this order, of the incident particle.

m	s	$\left(\frac{d\sigma}{d\Omega}\right)_0$
0	0	$\left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2$
$\neq 0$	0	$\left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\left(1+\frac{\alpha}{2}\right)^2$
0	$\frac{1}{2}$	$\left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\cos^2\frac{\theta}{2}$
$\neq 0$	$\frac{1}{2}$	$\left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\left[\cos^2\frac{\theta}{2}+\frac{\alpha}{4}(1+\alpha+3\cos^2\frac{\theta}{2})\right]$
0	1	$\left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\cos^4\frac{\theta}{2}$
$\neq 0$	1	$\left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\left[\frac{1}{3}+\frac{2}{3}\cos^4\frac{\theta}{2}-\frac{\alpha}{3}(1-\frac{3\alpha}{4}-4\cos^2\frac{\theta}{2})\right]$
0	2	$\left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\left(\sin^8\frac{\theta}{2}+\cos^8\frac{\theta}{2}\right)$

2.3 Discussion

If we choose two expressions, no matter which, from those listed in Table 1, we get that the difference between them is always extremely small for typical deflection angles. To show this for the massless particles, for instance, we perform a quantitative analysis of this difference via the study of the behavior of

$$\frac{\Delta\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{s=0}} \equiv \frac{\left(\frac{d\sigma}{d\Omega}\right)_s - \left(\frac{d\sigma}{d\Omega}\right)_{s=0}}{\left(\frac{d\sigma}{d\Omega}\right)_{s=0}}$$

as a function of the scattering angle θ (See Fig. 2). It is trivial to show that for small angles the preceding expression reduces to

$$\frac{\Delta\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{s=0}} \approx -\frac{s\theta^2}{2}.$$

This is not so surprising because for small angles the differential cross-sections for gravitational deflection of massless particles obtained via Einstein's theory are expected practically to coincide with those calculated to first order using quantum field theory. In fact, for a typical deflection angle, namely, $\theta \sim 10^{-6}$, we get $\frac{\Delta\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{s=0}} \sim 10^{-12}$, implying that $\frac{\Delta\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{s=0}} \ll 1$, as expected.

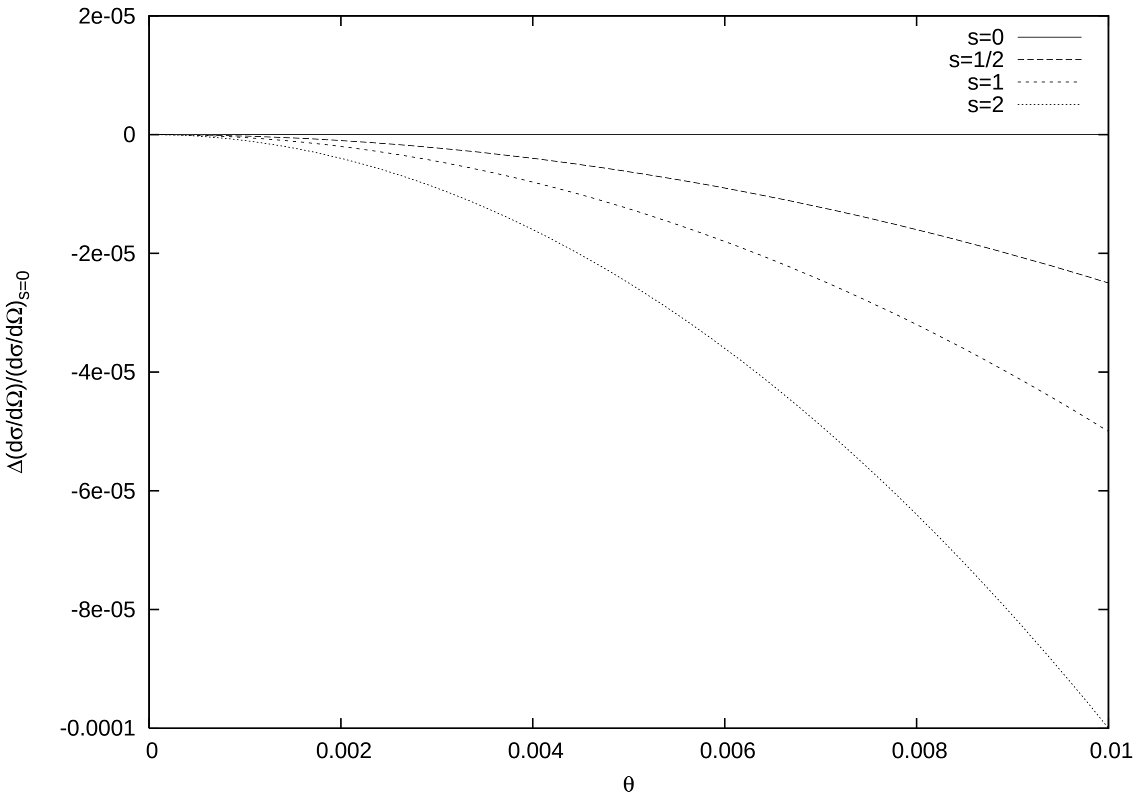

Figure 2: $\frac{\Delta\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{s=0}}$ as a function of the scattering angle θ.

3 An Unusual Result

What would happen if the calculations that gave origin to Table 1 were pushed to the next order? To avoid the terrible computations involved in the evaluation of all these cross-sections, we shall concentrate our attention only on the determination of the cross-section for the massive scalar particle.

To second order we have three graphs that contribute to the scattering process at hand (See Fig. 3). It is noteworthy that *bremsstrahlung* and *radiative corrections* need not to be taken into account because, of course, $M \gg E$, where M is the mass of the the gravitational source — a macroscopic body — and E is the energy of the scattered particle, which implies that these effects are on the ratio of $\frac{E}{M}$ in relation to the diagrams of Fig. 3. For instance, for the solar gravitational deflection of radio waves with a frequency, say, of order $1GHz$, $\frac{E}{M} \sim 10^{-71}$.

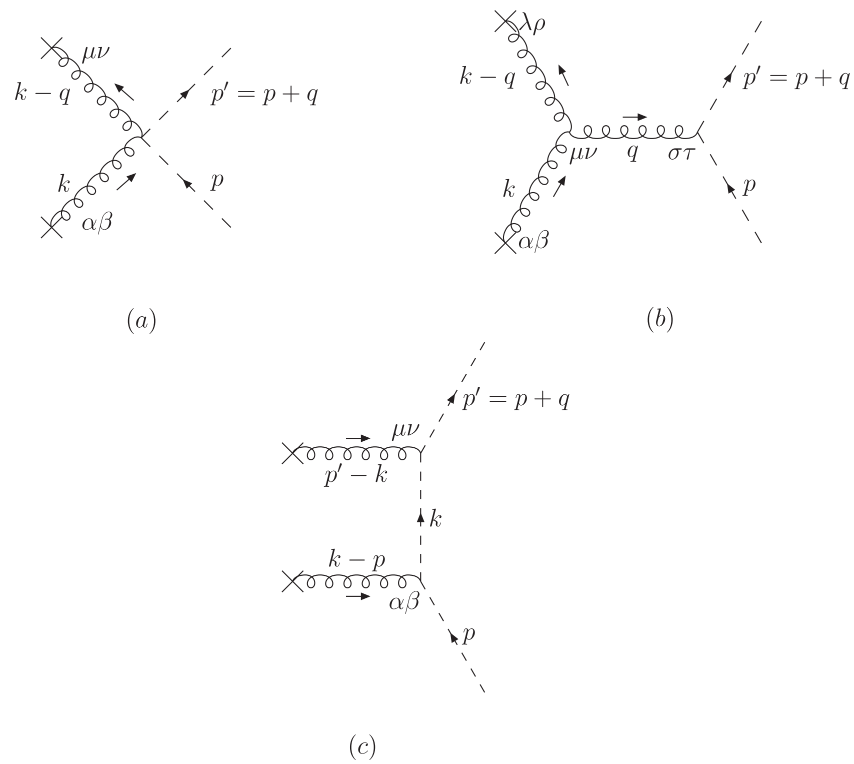

Figure 3: Feynman graphs for the scattering of a massive scalar boson by an external gravitational field to second order; $|\mathbf{p}| = |\mathbf{p}'|$.

3.1 The $\mathcal{M}^{(a)}$ amplitude

From Fig. 3(a), we get

$$i\mathcal{M}^{(a)} = \frac{i}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3} h_{\alpha\beta}(\mathbf{k}) h_{\mu\nu}(\mathbf{k}-\mathbf{q}) V^{\mu\nu,\,\alpha\beta}(p,p'),$$

where $V^{\mu\nu,\,\alpha\beta}(p,p')$ — the graviton-graviton-scalar-scalar vertex — has the form

$$i\kappa^2 \left\{ \left[I^{\mu\nu,\lambda\sigma} I_{\sigma}^{\ \rho,\alpha\beta} - \frac{1}{4}(\eta^{\mu\nu} I^{\alpha\beta,\lambda\rho} + \eta^{\alpha\beta} I^{\mu\nu,\lambda\rho}) \right] (p'_\lambda p_\rho + p'_\rho p_\lambda) - \frac{1}{2}(p \cdot p' - m^2) \mathcal{P}^{\alpha\beta,\mu\nu} \right\},$$

with $\mathcal{P}_{\alpha\beta,\mu\nu} = I_{\alpha\beta,\mu\nu} - \frac{1}{2}\eta_{\alpha\beta}\eta_{\mu\nu}$, and $I_{\alpha\beta,\mu\nu} = \frac{1}{2}(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu})$.

Performing the computations, we obtain

$$i\mathcal{M}^{(a)} = -\frac{4\pi^2 G^2 M^2}{|\mathbf{q}|}\left(4E^2 - \frac{\mathbf{q}^2}{2}\right).$$

3.2 The $\mathcal{M}^{(b)}$ amplitude

A cursory glance at Fig. 3(b) allows us to conclude that

$$i\mathcal{M}^{(b)} = \frac{i}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3} h^{\lambda\rho}(\mathbf{k}-\mathbf{q})h^{\alpha\beta}(\mathbf{k})V_{\alpha\beta,\,\mu\nu,\,\lambda\rho}(k,q,k-q)\frac{i\mathcal{P}^{\mu\nu,\sigma\tau}}{q^2}V_{\sigma\tau}(p,p'),$$

where $V_{\alpha\beta,\,\mu\nu,\,\lambda\rho}(k,q,k-q)$ — the graviton-graviton-graviton vertex — is given by

$$\begin{aligned}
&+ \;\; i\kappa\left\{-\frac{1}{2}(k^2+q^2+(k-q)^2)\left[I^{\sigma}_{\mu,\alpha\beta}I_{\lambda\rho,\sigma\nu} + I^{\sigma}_{\nu,\alpha\beta}I_{\lambda\rho,\sigma\mu} + \frac{1}{4}\eta_{\alpha\beta}\eta_{\mu\nu}\eta_{\lambda\rho}\right.\right.\\
&- \;\; \left.\frac{1}{2}(\eta_{\alpha\beta}I_{\mu\nu,\lambda\rho} + \eta_{\mu\nu}I_{\alpha\beta,\lambda\rho} + \eta_{\lambda\rho}I_{\alpha\beta,\mu\nu})\right]\\
&+ \;\; q^{\sigma}(k-q)^{\tau}\left[I_{\mu\nu,\lambda\rho}I_{\alpha\beta,\sigma\tau} - \frac{1}{2}(I_{\alpha\beta,\rho\sigma}I_{\mu\nu,\lambda\tau} + I_{\alpha\beta,\lambda\sigma}I_{\mu\nu,\rho\tau} + I_{\lambda\rho,\nu\sigma}I_{\alpha\beta,\mu\tau} + I_{\lambda\rho,\mu\sigma}I_{\alpha\beta,\nu\tau})\right]\\
&- \;\; k^{\sigma}(k-q)^{\tau}\left[I_{\alpha\beta,\lambda\rho}I_{\mu\nu,\sigma\tau} - \frac{1}{2}(I_{\mu\nu,\rho\sigma}I_{\alpha\beta,\lambda\tau} + I_{\mu\nu,\lambda\sigma}I_{\alpha\beta,\rho\tau} + I_{\lambda\rho,\alpha\sigma}I_{\mu\nu,\beta\tau} + I_{\lambda\rho,\beta\sigma}I_{\mu\nu,\alpha\tau})\right]\\
&- \;\; \left.k^{\sigma}q^{\tau}\left[I_{\alpha\beta,\mu\nu}I_{\lambda\rho,\sigma\tau} - \frac{1}{2}(I_{\lambda\rho,\beta\tau}I_{\mu\nu,\alpha\sigma} + I_{\lambda\rho,\alpha\tau}I_{\mu\nu,\beta\sigma} + I_{\lambda\rho,\nu\sigma}I_{\alpha\beta,\mu\tau} + I_{\lambda\rho,\mu\sigma}I_{\alpha\beta,\nu\tau})\right]\right\}.
\end{aligned}$$

Consequently,

$$i\mathcal{M}^{(b)} = -\frac{4\pi^2 G^2 M^2}{|\mathbf{q}|}\left(-m^2 - \frac{\mathbf{p}^2}{4} + \frac{7\mathbf{q}^2}{16}\right).$$

3.3 The $\mathcal{M}^{(c)}$ amplitude

Fig. 3(c) tells us that

$$i\mathcal{M}^{(c)} = i\int \frac{d^3\mathbf{k}}{(2\pi)^3} h^{\mu\nu}(\mathbf{p}'-\mathbf{k})V_{\mu\nu}(p',k)\frac{i}{k^2-m^2+i\epsilon}V_{\alpha\beta}(k,p)h^{\alpha\beta}(\mathbf{k}-\mathbf{p}).$$

Regularizing the infrared divergence of this graph by regarding the Newtonian potential $\frac{GM}{r}$ as the limit $\mu \to 0$ of the Yukawa potential $\frac{GMe^{-\mu r}}{r}$, yields

$$i\mathcal{M}^{(c)} = \frac{2G^2M^2}{\pi}(2m^2+4\mathbf{p}^2)^2 \int \frac{d^3\mathbf{k}}{[(\mathbf{p}'-\mathbf{k})^2+\mu^2][(\mathbf{p}-\mathbf{k})^2+\mu^2](\mathbf{p}^2-\mathbf{k}^2+i\epsilon)}.$$

Note that we need only the value of the integral $I \equiv \int \frac{d^3\mathbf{k}}{[(\mathbf{p}'-\mathbf{k})^2+\mu^2][(\mathbf{p}-\mathbf{k})^2+\mu^2](\mathbf{p}^2-\mathbf{k}^2+i\epsilon)}$ for $\mu \longrightarrow 0$. In this limit we promptly obtain

$$I \approx -i\frac{2\pi^2}{|\mathbf{p}|\mathbf{q}^2}\ln\frac{|\mathbf{q}|}{\mu},$$

and, as a consequence,

$$i\mathcal{M}^{(c)} \approx -i\frac{4\pi G^2M^2}{|\mathbf{p}|\mathbf{q}^2}(2m^2+4\mathbf{p}^2)^2\ln\frac{|\mathbf{q}|}{\mu}.$$

3.4 The cross-section to second order

To second order the cross-section has the form

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{\left| i\left(\mathcal{M}^{(0)}+\mathcal{M}^{(a)}+\mathcal{M}^{(b)}+\mathcal{M}^{(c)}\right)\right|^2}{(4\pi)^2} \\ &= \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1+\frac{\pi GM|\mathbf{p}|\sin\frac{\theta}{2}}{1+\frac{\alpha}{2}}\left(3\alpha+\frac{15-\sin^2\frac{\theta}{2}}{4}\right)\right], \end{aligned} \quad (7)$$

where $\left(\frac{d\sigma}{d\Omega}\right)_0$ is given by Equation (1).

3.5 Discussion

From Equation (7) we come to the astonishing conclusion that the second-order correction is mass dependent, implying that the Galileo's equivalence and, as a consequence, the classical equivalence principle ceases to hold in quantum mechanics. We remark that to second order the unpolarized differential cross-section for the scattering of a photon by a weak gravitational field, treated as an external field, i.e.,

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0\left[1+\frac{\pi GME\sin\frac{\theta}{2}}{(1+\sin\frac{\theta}{2})^2}\left(\frac{15+14\sin\frac{\theta}{2}+3\sin^2\frac{\theta}{2}}{4}\right)\right], \quad (8)$$

where $\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{GM}{\sin^2\frac{\theta}{2}}\right)^2\cos^4\frac{\theta}{2}$, is energy-dependent [11].

Therefore, we may say that the classical equivalence principle and quantum mechanics are irreconcilable — be gravity described by Newton's theory or general relativity [12]. Why is this so? Because the physical conditions that define equivalence classically are absent in quantum mechanics. In addition, when a system is quantized, energy - dependent effects are introduced which, of course, are in disagreement with the spirit of the classical equivalence principle.

4 On the Possibility of Measuring the Deviations of the Classical Equivalence Principle

Is there any hope of measuring the deviations from the classical equivalence given by Equations (7) and (8) in the foreseeable future? To answer this question we have to know beforehand the expressions for the gravitational deflection angles. A straightforward calculation shows that for small angles Equations (7) and (8) reduce, respectively, to

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{scalar boson}} \approx \left(\frac{4GM}{\theta^2}\right)^2 \left(1+\frac{\alpha}{2}\right)^2 \left[1+\frac{\pi GM|\mathbf{p}|\theta}{2(1+\frac{\alpha}{2})}\left(3\alpha+\frac{15}{4}\right)\right], \tag{9}$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{photon}} \approx \left(\frac{4GM}{\theta^2}\right)^2 \left(1+\frac{15\pi GME\theta}{8}\right); \tag{10}$$

implying that for $\lambda \gg GM$, with λ being the particle wavelength,

$$\theta_{\text{scalar boson}} \approx (1+\frac{\alpha}{2})\left[\frac{4GM}{b}+\frac{16G^3M^3\pi^2}{\lambda b^2}\left(3\alpha+\frac{15}{4}\right)\right], \tag{11}$$

and

$$\theta_{\text{photon}} \approx \frac{4GM}{b}+\frac{60\pi^2G^3M^3}{\lambda b^2}. \tag{12}$$

Therefore,

$$\begin{aligned}\left(\frac{\Delta\theta}{\theta}\right)_{\text{scalar boson}} &\equiv \frac{\theta_{\text{scalar boson}}-\theta_0}{\theta_0} \\ &= \frac{4\pi^2G^2M^2}{\lambda b}\left(3\alpha+\frac{15}{4}\right),\end{aligned}$$

where θ_0 is given by Equation (5), and

$$\left(\frac{\Delta\theta}{\theta}\right)_{\text{photon}} \equiv \frac{\theta_{\text{photon}} - \theta_0}{\theta_0} = \frac{15\pi^2 G^2 M^2}{\lambda b},$$

wherein θ_0 is defined by Equation (4).

Currently, the only measurements of the gravitational deflection we have at our disposal are those related to the solar deflection of photons. Now, for the Sun $GM \sim 1.5km$, which would require photons of very low energy for the measurement of the corresponding bending. Unfortunately, no measurement of gravitational deflection in this energy range is available nowadays. Therefore, the detection of the deviation from the classical equivalence principle predicted by equation (12) seems unlikely in the immediate future.

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[12] For a more thorough discussion see, for instance, A. Accioly, R. Aldrovandi and R. Paszko, Is the equivalence principle doomed forever to Dante's inferno on account of quantum mechanics? (This essay received an "honorable mention" in the Annual Competition of the Gravity Research Foundation for the year 2006); *International Journal of Modern Physics D* **15** (2006), 2249 - 2255.

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