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The Electromagnetic Force on the Walls of a Rectangular Waveguide

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Abstract

The electromagnetic force distribution on the walls of a rectangular waveguide is determined when *TE* or *TM* wave transmits inside the waveguide. The distribution properties of electric force, magnetic force and electromagnetic force are analyzed for both *TE* and *TM* propagation.. The effect of mode numbers is shown and TE_{10} is taken as a typical case. The major differences of the force distribution between *TE* and *TM* are stood out. The result demonstrates the symbolic symmetry of corresponding quantities and the general meaning of the formulas for surface distribution of the forces on the walls inside waveguide.

Keywords: electromagnetic force; rectangular waveguide; *TE* wave; *TM* wave

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1. Introduction

When *TE* or *TM* waves propagate inside a rectangular waveguide, there are electric and magnetic forces acting on the wall. Ref. [1] and [2] dealt with the electrostatic forces on the walls of some coaxial transmission lines [1], [2]. Since *TEM* wave never transmit inside any waveguide[3], the methods used in static field, such as conformal mapping, can not be applied to solve the problems inside waveguide when electromagnetic waves transmits. The attention of this thesis is focused on the electromagnetic forces on the walls as *TE* or *TM* wave propagates inside a rectangular waveguide. The distribution of the force is determined and illustrated. The properties of the force are analyzed, and the comparison of the distribution of electromagnetic force between *TE* wave and *TM* wave are displayed as well. The symbolic symmetry and general meaning in the proceeding are pointed out.

2. Field components of TE_{mn} wave

Fig.1 shows the cross section of a rectangular waveguide in *XOY*-plane. The waveguide is straight along *Z*-direction. Its width and height of the rectangular wall are denoted by a and b, respectively.

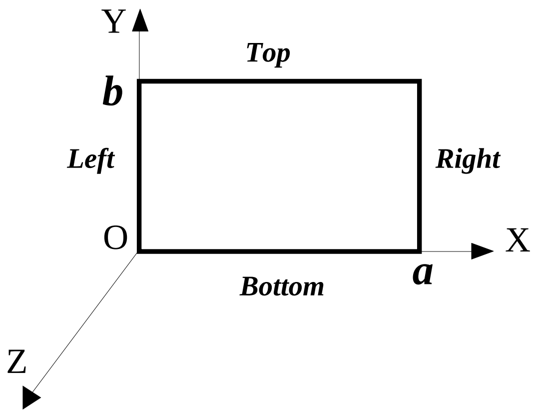

Fig.1 The cross section of a rectangular waveguide

When *TE*mn wave propagates inside the rectangular waveguide, the field components of the wave are as follows[4]:

$$\begin{cases} H_z = H_0 \cos\dfrac{m\pi}{a}x \cdot \cos\dfrac{n\pi}{b}y \cdot \cos(k_z z - \omega t) \\ H_x = \dfrac{m\lambda_c}{2a}\sqrt{\dfrac{\lambda_c^2}{\lambda^2} - 1} \cdot H_0 \sin\dfrac{m\pi}{a}x \cdot \cos\dfrac{n\pi}{b}y \cdot \sin(k_z z - \omega t) \\ H_y = \dfrac{n\lambda_c}{2b}\sqrt{\dfrac{\lambda_c^2}{\lambda^2} - 1} \cdot H_0 \cos\dfrac{m\pi}{a}x \cdot \sin\dfrac{n\pi}{b}y \cdot \sin(k_z z - \omega t) \\ E_x = \dfrac{n\eta\lambda_c{}^2}{2b\lambda} H_0 \cos\dfrac{m\pi}{a}x \cdot \sin\dfrac{n\pi}{b}y \cdot \sin(k_z z - \omega t) \\ E_y = -\dfrac{m\eta\lambda_c{}^2}{2a\lambda} H_0 \sin\dfrac{m\pi}{a}x \cdot \cos\dfrac{n\pi}{b}y \cdot \sin(k_z z - \omega t) \end{cases} \tag{1}$$

where λ is the wavelength of the electromagnetic wave traveling in free space, λ_c is the cut-off wavelength of TE_{mn} wave inside the waveguide[5]

$$\lambda_c = \frac{1}{\sqrt{\left(\dfrac{m}{2a}\right)^2 + \left(\dfrac{n}{2b}\right)^2}} \tag{2}$$

and

$$k_z = \frac{2\pi}{\lambda}\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}\ ; \quad \omega = \frac{2\pi}{\lambda\sqrt{\varepsilon_0\mu_0}}\ ; \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{3}$$

To guarantee that TE_{mn} wave propagates inside the rectangular waveguide, λ should satisfy

$$\lambda < \lambda_c \tag{4}$$

3. Distribution of charge and current on the wall

Note the wall is the boundary of the field region in the waveguide. Eq.(1) enable us to determine the distribution of charge and current on the wall. Let n be the unit vector of normal on a wall. inside the waveguide. The density of the surface charge on the wall should be

$$\sigma = \boldsymbol{n} \cdot \boldsymbol{D} = \varepsilon \boldsymbol{n} \cdot \boldsymbol{E} \tag{5}$$

Employing Eq.(5), the surface density of charge on the top, bottom, right and left wall is, respectively

$$\sigma_U = -\varepsilon E_y = (-1)^n \frac{m\varepsilon\eta\lambda_c^{\ 2}}{2a\lambda} H_0 \sin\frac{m\pi}{a} x \cdot \sin(k_z z - \omega t) \tag{6}$$

$$\sigma_D = \varepsilon E_y = -\frac{m\varepsilon\eta\lambda_c^{\ 2}}{2a\lambda} H_0 \sin\frac{m\pi}{a} x \cdot \sin(k_z z - \omega t) \tag{7}$$

$$\sigma_R = -\varepsilon E_x = (-1)^{m+1} \frac{n\varepsilon\eta\lambda_c^{\ 2}}{2b\lambda} H_0 \sin\frac{n\pi}{b} y \cdot \sin(k_z z - \omega t) \tag{8}$$

$$\sigma_L = \varepsilon E_x = \frac{n\varepsilon\eta\lambda_c^{\ 2}}{2b\lambda} H_0 \sin\frac{n\pi}{b} y \cdot \sin(k_z z - \omega t) \tag{9}$$

Let J represent the surface current on the walls inside the rectangular waveguide. Applying

$$\boldsymbol{J} = \boldsymbol{n} \times \boldsymbol{H} \tag{10}$$

the surface current on the four walls is achieved. The components are as follows:

$$J_{U\,x} = -H_z = (-1)^{n+1} H_0 \cos\frac{m\pi}{a} x \cdot \cos(k_z z - \omega t)\,;$$

$$J_{U\,z} = H_x = (-1)^n \frac{m\lambda_c}{2a}\sqrt{\frac{\lambda_c^2}{\lambda^2} - 1} \cdot H_0 \sin\frac{m\pi}{a} x \cdot \sin(k_z z - \omega t) \tag{11}$$

$$J_{D\,x} = H_z = H_0 \cos\frac{m\pi}{a} x \cdot \cos(k_z z - \omega t)\,;$$

$$J_{D\,z} = -H_x = -\frac{m\lambda_c}{2a}\sqrt{\frac{\lambda_c^2}{\lambda^2} - 1} \cdot H_0 \sin\frac{m\pi}{a} x \cdot \sin(k_z z - \omega t) \tag{12}$$

$$J_{R\,y} = H_z = (-1)^m H_0 \cos\frac{n\pi}{b} y \cdot \cos(k_z z - \omega t)\,;$$

$$J_{R\,z} = -H_y = (-1)^{m+1} \frac{n\lambda_c}{2b}\sqrt{\frac{\lambda_c^2}{\lambda^2} - 1} \cdot H_0 \sin\frac{n\pi}{b} y \cdot \sin(k_z z - \omega t) \tag{13}$$

$$J_{L\,y} = -H_z = -H_0 \cos\frac{n\pi}{b} y \cdot \cos(k_z z - \omega t)\,;$$

$$J_{Lz} = H_y = \frac{n\lambda_c}{2b}\sqrt{\frac{\lambda_c^2}{\lambda^2}-1}\cdot H_0 \sin\frac{n\pi}{b}y\cdot\sin(k_z z-\omega t) \tag{14}$$

4. Electric and magnetic forces on the walls

Electric force acts perpendicularly to the wall. The surface density of electric force exerted on the walls is calculated as[6]

$$\boldsymbol{f}_e = \frac{1}{2}\sigma\boldsymbol{E} \tag{15}$$

Substituting Eqs.(5)-(9) respectively, we have the surface density of electric force on the four walls

$$\begin{aligned} f_{Ue} = f_{Uey} &= -\frac{1}{2}\varepsilon E_y^2 = -\frac{1}{2}\varepsilon\left[(-1)^{n+1}\frac{m\eta\lambda_c{}^2}{2a\lambda}H_0\sin\frac{m\pi}{a}x\cdot\sin(k_z z-\omega t)\right]^2 \\ &= -\frac{1}{2}\mu H_0^2\left(\frac{m^2\lambda_c^4}{4a^2\lambda^2}\right)\sin^2\frac{m\pi}{a}x\cdot\sin^2(k_z z-\omega t) \end{aligned} \tag{16}$$

$$\begin{aligned} f_{De} = f_{Dey} &= \frac{1}{2}\varepsilon E_y^2 = \frac{1}{2}\varepsilon\left[-\frac{m\eta\lambda_c{}^2}{2a\lambda}H_0\sin\frac{m\pi}{a}x\cdot\sin(k_z z-\omega t)\right]^2 \\ &= \frac{1}{2}\mu H_0^2\left(\frac{m^2\lambda_c^4}{4a^2\lambda^2}\right)\sin^2\frac{m\pi}{a}x\cdot\sin^2(k_z z-\omega t) \end{aligned} \tag{17}$$

$$\begin{aligned} f_{\mathrm{Re}} = f_{\mathrm{Re}x} &= -\frac{1}{2}\varepsilon E_x^2 = -\frac{1}{2}\varepsilon\left[(-1)^{m}\frac{n\eta\lambda_c{}^2}{2b\lambda}H_0\sin\frac{n\pi}{b}y\cdot\sin(k_z z-\omega t)\right]^2 \\ &= -\frac{1}{2}\mu H_0^2\left(\frac{n^2\lambda_c^4}{4b^2\lambda^2}\right)\sin^2\frac{n\pi}{b}x\cdot\sin^2(k_z z-\omega t) \end{aligned} \tag{18}$$

$$\begin{aligned} f_{Le} = f_{Lex} &= \frac{1}{2}\varepsilon E_x^2 = \frac{1}{2}\varepsilon\left[\frac{n\eta\lambda_c{}^2}{2b\lambda}H_0\sin\frac{n\pi}{b}y\cdot\sin(k_z z-\omega t)\right]^2 \\ &= \frac{1}{2}\mu H_0^2\left(\frac{n^2\lambda_c^4}{4b^2\lambda^2}\right)\sin^2\frac{n\pi}{b}x\cdot\sin^2(k_z z-\omega t) \end{aligned} \tag{19}$$

The surface density of magnetic force on the walls is given by[7]

$$\boldsymbol{f}_m = \frac{1}{2}\mu\boldsymbol{J}\times\boldsymbol{H} \tag{20}$$

Substituting Eqs.(11)-(14) and (1) into Eq.(20) respectively, we derive the surface density of magnetic force on the four walls

$$f_{Um} = f_{Umy} = \frac{1}{2}\mu[J_z H_x - J_x H_z] = \frac{1}{2}\mu[H_x^2 + H_z^2]$$
$$= \frac{1}{2}\mu\left[\left((-1)^n \frac{m\lambda_c}{2a}\sqrt{\frac{\lambda_c^2}{\lambda^2}-1}\cdot H_0 \sin\frac{m\pi}{a}x\cdot\sin(k_z z-\omega t)\right)^2 + \left((-1)^n H_0 \cos\frac{m\pi}{a}x\cdot\cos(k_z z-\omega t)\right)^2\right]$$
$$= \frac{1}{2}\mu H_0^2\left(\frac{m^2\lambda_c^2}{4a^2\lambda^2}(\lambda_c^2-\lambda^2)\sin^2\frac{m\pi}{a}x\cdot\sin^2(k_z z-\omega t) + \cos^2\frac{m\pi}{a}x\cdot\cos^2(k_z z-\omega t)\right) \tag{21}$$

$$f_{Dm} = f_{Dmy} = \frac{1}{2}\mu[J_z H_x - J_x H_z] = -\frac{1}{2}\mu[H_x^2 + H_z^2]$$
$$= -\frac{1}{2}\mu H_0^2\left(\frac{m^2\lambda_c^2}{4a^2\lambda^2}(\lambda_c^2-\lambda^2)\sin^2\frac{m\pi}{a}x\cdot\sin^2(k_z z-\omega t) + \cos^2\frac{m\pi}{a}x\cdot\cos^2(k_z z-\omega t)\right) \tag{22}$$

$$f_{Rm} = f_{Rmx} = \frac{1}{2}\mu[J_y H_z - J_z H_y] = \frac{1}{2}\mu[H_y^2 + H_z^2]$$
$$= \frac{1}{2}\mu\left[\left((-1)^m \frac{n\lambda_c}{2a}\sqrt{\frac{\lambda_c^2}{\lambda^2}-1}\cdot H_0 \sin\frac{n\pi}{b}x\cdot\sin(k_z z-\omega t)\right)^2 + \left((-1)^m H_0 \cos\frac{n\pi}{b}x\cdot\cos(k_z z-\omega t)\right)^2\right]$$
$$= \frac{1}{2}\mu H_0^2\left(\frac{n^2\lambda_c^2}{4b^2\lambda^2}(\lambda_c^2-\lambda^2)\sin^2\frac{n\pi}{b}x\cdot\sin^2(k_z z-\omega t) + \cos^2\frac{n\pi}{b}x\cdot\cos^2(k_z z-\omega t)\right) \tag{23}$$

$$f_{Lm} = f_{Lmx} = \frac{1}{2}\mu[J_y H_z - J_z H_y] = -\frac{1}{2}\mu[H_y^2 + H_z^2]$$
$$= -\frac{1}{2}\mu H_0^2\left(\frac{n^2\lambda_c^2}{4b^2\lambda^2}(\lambda_c^2-\lambda^2)\sin^2\frac{n\pi}{b}x\cdot\sin^2(k_z z-\omega t) + \cos^2\frac{n\pi}{b}x\cdot\cos^2(k_z z-\omega t)\right) \tag{24}$$

5. Distribution of electromagnetic force on the walls

Electromagnetic force on the wall should be the sum of electric force and magnetic force on that wall. Based on the results in the above sections the distribution of electromagnetic force on the four walls of the waveguide can be obtained. For each wall the force has only normal component rather than tangential component. Designate the surface density of electromagnetic force acting on the walls as *f*

$$\boldsymbol{f} = \boldsymbol{f}_E + \boldsymbol{f}_M \tag{25}$$

For the four walls, using Eq.(16)-(19) and (21)-(24) in Eq.(25) yields

$$f_U = f_{Uy} = f_{Uey} + f_{Umy}$$
$$= \frac{1}{2}\mu H_0^2 \left(\cos^2 \frac{m\pi}{a} x \cdot \cos^2(k_z z - \omega t) - \frac{m^2 \lambda_c^2}{4a^2} \sin^2 \frac{m\pi}{a} x \cdot \sin^2(k_z z - \omega t) \right) \tag{26}$$
$$= \frac{1}{2}\mu H_0^2 \left(\cos^2 \frac{m\pi}{a} x \cdot \cos^2(k_z z - \omega t) - \frac{1}{1 + \left(\frac{an}{bm} \cdot \right)^2} \sin^2 \frac{m\pi}{a} x \cdot \sin^2(k_z z - \omega t) \right)$$

$$f_D = f_{Dy} = f_{Dey} + f_{Dmy}$$
$$= -\frac{1}{2}\mu H_0^2 \left(\cos^2 \frac{m\pi}{a} x \cdot \cos^2(k_z z - \omega t) - \frac{1}{1 + \left(\frac{an}{bm} \cdot \right)^2} \sin^2 \frac{m\pi}{a} x \cdot \sin^2(k_z z - \omega t) \right) \tag{27}$$

$$f_R = f_{Rx} = f_{\mathrm{Re}x} + f_{Rmx}$$
$$= \frac{1}{2}\mu H_0^2 \left(\cos^2 \frac{n\pi}{b} y \cdot \cos^2(k_z z - \omega t) - \frac{n^2 \lambda_c^2}{4b^2} \sin^2 \frac{n\pi}{b} y \cdot \sin^2(k_z z - \omega t) \right) \tag{28}$$
$$= \frac{1}{2}\mu H_0^2 \left(\cos^2 \frac{n\pi}{b} y \cdot \cos^2(k_z z - \omega t) - \frac{1}{1 + \left(\frac{bm}{an} \cdot \right)^2} \sin^2 \frac{n\pi}{b} y \cdot \sin^2(k_z z - \omega t) \right)$$

$$f_L = f_{Lx} = f_{Lex} + f_{Lmx}$$
$$= -\frac{1}{2}\mu H_0^2 \left(\cos^2 \frac{n\pi}{b} y \cdot \cos^2(k_z z - \omega t) - \frac{1}{1 + \left(\frac{bm}{an} \cdot \right)^2} \sin^2 \frac{n\pi}{b} y \cdot \sin^2(k_z z - \omega t) \right) \tag{29}$$

Introduce the dimensionless surface density of force as

$$\hat{f} = \frac{f}{\frac{1}{2}\mu H_0^2} \tag{30}$$

Under the condition of $b = 0.5a$, Fig. 2 shows the function $\hat{f}_U$ of TE_{12} versus x/a and φ, where $\varphi = kz - \omega t$.

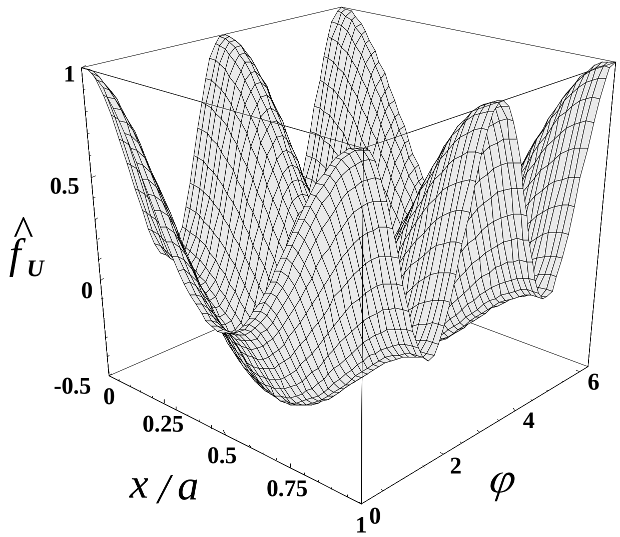

Fig.2 $\hat{f}_U$ of TE_{12} versus x/a and φ $(b = 0.5a)$

6. Discussion

When *TE*mn wave transfers inside a rectangular waveguide, the distribution of electric force at any point on each wall is normal tension, which is found from Eqs.(16)-(19). By contrast, Eqs.(21)-(24) indicate that the distribution of magnetic force is normal pressure.

However, it is deduced from Eqs.(26)-(29) that the distribution of electromagnetic force on each wall, although along the normal, is partly pressure and partly tension, decided by the coordinates of that point on the wall. In addition, the maximum in distribution depends on the amplitude of the guided wave instead of frequency or wavelength of the wave. The force distribution transfers in *Z*-direction along the waveguide with the wave. The surface density of the force is related to the geometric dimensions of the waveguide, *a* and *b*, as well as the mode numbers of the wave, *m* and *n*.

When the mode number increases, the magnitude of surface density of the force on the corresponding two walls decreases and the one on the other two walls increases. That implies the electromagnetic force shifts as the mode number varies.

If one of *m* and *n* is not zero, the force distribution on the walls is not uniform in *X* and *Y*-direction in cross section. However, if one of mode numbers is zero, the force distribution is uniform on the corresponding two walls in cross section, and the surface density of force on the two walls reaches the maximum but on the other two walls reaches the minimum. Saying $n = 0$, the force distributes uniformly on the right and left walls in *Y*-direction, and the surface density of force on right and left walls is its maximum but the force on top and bottoms walls is its minimum. The magnitude of maximum can be over $\frac{1}{2}\mu H_0^2$.

The results further demonstrate the symbolic symmetry of *a, m* and *x* with *b, n* and *y*.

For example, replacing *a, m* and *x* with *b, n* and *y* respectively in Eqs.(16) and (17) leads to the results in Eqs.(18) and (19). Then from the surface force distribution on top and bottom walls we obtain the one on right and left walls. Similarly, replacing in Eqs.(21) and (22) yields the results in Eqs.(23) and (24), doing so in Eqs.(26) and (27) leads to the results in Eqs.(28) and (29). This consistency attests to the correction of the results.

7. TE_{10} wave

Special attention now is given to the special case of TE_{10} wave under the condition of $a > b$. For the transmission of electromagnetic wave inside rectangular waveguide, TE_{10} wave has its own notable advantages[8]. Hence, in most of cases, TE_{10} wave is adopted to propagate inside rectangular waveguide. To guarantee that TE_{10} wave travels inside the rectangular waveguide in single mode, λ should satisfy

$$a < \lambda < 2a \tag{31}$$

Letting $m = 1$ and $n = 0$ in Eq.(1), it is readily found that the electric field has E_y component only, which implies that TE_{10} wave is polarized in single direction. Thus, the surface charge only distributes on the top and bottom walls but no surface charge on the other two walls, which is verified in Eqs.(6)-(9). Besides, Eqs.(13) and (14) show that the surface current on right and left walls only has *y*-component. Consequently, electric force acts only on the top and bottom walls but the magnetic force exerts on the four walls. Since electric force is tension and magnetic force is pressure, the electromagnetic force on right and left walls certainly reaches its maximum. Eq.(1) tells us that all components of the field are independent of *y*-coordinate as $n = 0$. Therefore, the force contribution on right and left wall is inevitably uniform in *Y*-direction, which has been mentioned in previous section.

8. For *TM* wave

Furthermore, we consider the case of *TM* wave[9]

$$\begin{cases} E_z = E_0 \sin\dfrac{m\pi}{a}x \cdot \sin\dfrac{n\pi}{b}y \cdot \cos(k_z z - \omega t) \\ E_x = -\dfrac{m\lambda_c}{2a}\sqrt{\dfrac{\lambda_c^2}{\lambda^2} - 1} \cdot E_0 \cos\dfrac{m\pi}{a}x \cdot \sin\dfrac{n\pi}{b}y \cdot \sin(k_z z - \omega t) \\ E_y = -\dfrac{n\lambda_c}{2b}\sqrt{\dfrac{\lambda_c^2}{\lambda^2} - 1} \cdot E_0 \sin\dfrac{m\pi}{a}x \cdot \cos\dfrac{n\pi}{b}y \cdot \sin(k_z z - \omega t) \\ H_x = \dfrac{n\lambda_c{}^2}{2b\eta\lambda} E_0 \sin\dfrac{m\pi}{a}x \cdot \cos\dfrac{n\pi}{b}y \cdot \sin(k_z z - \omega t) \\ H_y = -\dfrac{m\lambda_c{}^2}{2a\eta\lambda} E_0 \cos\dfrac{m\pi}{a}x \cdot \sin\dfrac{n\pi}{b}y \cdot \sin(k_z z - \omega t) \end{cases} \tag{32}$$

The electromagnetic force distribution on the four walls would be

$$
\begin{aligned}
f_U &= f_{Uy} = f_{Uey} + f_{Umy} = -\frac{1}{2}\varepsilon E_y^2 + \frac{1}{2}\mu H_x^2 \\
&= -\frac{1}{2}\varepsilon\left((-1)^{n+1}\frac{n\lambda_c}{2b}\sqrt{\frac{\lambda_c^2}{\lambda^2}-1}\cdot E_0 \sin\frac{m\pi}{a}x\cdot\sin(k_z z-\omega t)\right)^2 + \frac{1}{2}\mu\left((-1)^n\frac{n\lambda_c{}^2}{2b\eta\lambda}E_0\sin\frac{m\pi}{a}x\cdot\sin(k_z z-\omega t)\right)^2 \\
&= \frac{1}{2}\varepsilon E_0^2\left(\frac{n^2\lambda_c^2}{4b^2}\sin^2\frac{m\pi}{a}x\cdot\sin^2(k_z z-\omega t)\right) = \frac{1}{2}\varepsilon E_0^2\left(\frac{1}{1+\left(\frac{bm}{an}\right)^2}\sin^2\frac{m\pi}{a}x\cdot\sin^2(k_z z-\omega t)\right)
\end{aligned}
\tag{33}
$$

$$
f_D = f_{Dy} = f_{DEy} + f_{DMy} = \frac{1}{2}\varepsilon E_y^2 - \frac{1}{2}\mu H_x^2 = -\frac{1}{2}\varepsilon E_0^2\left(\frac{1}{1+\left(\frac{bm}{an}\right)^2}\sin^2\frac{m\pi}{a}x\cdot\sin^2(k_z z-\omega t)\right)
\tag{34}
$$

$$
\begin{aligned}
f_R &= f_{Rx} = f_{\mathrm{Re}\,x} + f_{Rmx} = -\frac{1}{2}\varepsilon E_x^2 + \frac{1}{2}\mu H_y^2 \\
&= -\frac{1}{2}\varepsilon\left((-1)^{m+1}\frac{m\lambda_c}{2a}\sqrt{\frac{\lambda_c^2}{\lambda^2}-1}\cdot E_0\cdot\sin\frac{n\pi}{b}y\cdot\sin(k_z z-\omega t)\right)^2 + \frac{1}{2}\mu\left((-1)^{m+1}\frac{m\lambda_c{}^2}{2a\eta\lambda}E_0\cdot\sin\frac{n\pi}{b}y\cdot\sin(k_z z-\omega t)\right)^2 \\
&= \frac{1}{2}\varepsilon E_0^2\left(\frac{m^2\lambda_c^2}{4a^2}\sin^2\frac{n\pi}{b}y\cdot\sin^2(k_z z-\omega t)\right) = \frac{1}{2}\varepsilon E_0^2\left(\frac{1}{1+\left(\frac{an}{bm}\right)^2}\sin^2\frac{n\pi}{b}y\cdot\sin^2(k_z z-\omega t)\right)
\end{aligned}
\tag{35}
$$

$$
f_L = f_{Lx} = f_{Lex} + f_{Lmx} = \frac{1}{2}\varepsilon E_x^2 - \frac{1}{2}\mu H_y^2 = -\frac{1}{2}\varepsilon E_0^2\left(\frac{1}{1+\left(\frac{an}{bm}\right)^2}\sin^2\frac{n\pi}{b}y\cdot\sin^2(k_z z-\omega t)\right)
\tag{36}
$$

Under the same condition as in Fig..2, the function $\hat{f}_U$ of TM_{12} versus x/a and φ is depicted in Fig.3.

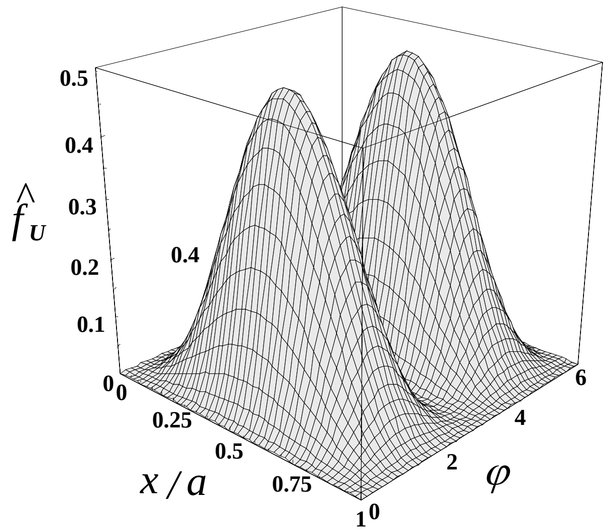

Fig.3 $\hat{f}_U$ of TM_{12} versus x/a and φ ($b = 0.5a$)

Comparing the cases of *TM* wave and *TE* wave, the major differences are worth of note. Since *m* or *n* can not be zero *for TM* wave [10], in all cases the electromagnetic force distribution on a wall can not be uniform in any direction. The maximum of the surface density of force can not reach $\frac{1}{2}\varepsilon E_0^2 = \frac{1}{2}\mu H_0^2$. It must be less than that value. As mentioned in Sec.6, The force distribution on right and left wall is uniform in *Y*-direction for TE_{10}. The magnitude of the surface density of force can be greater than $\frac{1}{2}\mu H_0^2$

As *TM*mn wave travels inside a rectangular waveguide, the distribution of electromagnetic force is normal pressure always, in any time and at any point on any wall. This conclusion is easy drawn from Eqs.(33)-(36). In contrast with it, the direction of electromagnetic force on a wall alternates as *TE* wave runs. Figs.2 and 3 clearly show that striking contrast. Due to the direction alternation of the force, it has no significance to calculate the resultant force on a wall for *TE* wave propagation. But it is worth to do that for *TM* wave transmission. Hence, along Z-direction, the longitudinal linear density of electromagnetic force on the four walls is computed as

$$F_U = F_{Uy} = \int_0^a f_{Uy}\,dx = \frac{1}{4}a\varepsilon E_0^2\left(\frac{1}{1+\left(\frac{bm}{an}\right)^2}\right)\cdot\sin^2(k_z z - \omega t) \tag{37}$$

$$F_D = F_{Dy} = \int_0^a f_{Dy}\,dx = -\frac{1}{4}a\varepsilon E_0^2\left(\frac{1}{1+\left(\frac{bm}{an}\right)^2}\right)\cdot\sin^2(k_z z - \omega t) \tag{38}$$

$$F_R = F_{Rx} = \int_0^b f_{Rx} dy = \frac{1}{4} b\varepsilon E_0^2 \left(\frac{1}{1+\left(\frac{an}{bm}\right)^2} \right) \cdot \sin^2(k_z z - \omega t) \tag{39}$$

$$F_L = F_{Lx} = \int_0^b f_{Lx} dy = -\frac{1}{4} b\varepsilon E_0^2 \left(\frac{1}{1+\left(\frac{an}{bm}\right)^2} \right) \cdot \sin^2(k_z z - \omega t) \tag{40}$$

The wavelength in waveguide is[11]

$$\lambda_g = \frac{2\pi}{k_z} = \frac{1}{\sqrt{\frac{1}{\lambda^2} - \left(\frac{m}{a}\right)^2 - \left(\frac{n}{b}\right)^2}} \tag{41}$$

Then, the resultant electromagnetic force of a segment with longitudinal length λ_g on the four wall for is

$$\begin{aligned}\tilde{F}_U = \tilde{F}_{Uy} &= \int_z^{z+\lambda_g} F_{Uy} dz = \int_z^{z+\lambda_g} \frac{1}{4} a\varepsilon E_0^2 \left(\frac{1}{1+\left(\frac{bm}{an}\right)^2} \right) \cdot \sin^2(k_z z - \omega t) dz \\ &= \frac{1}{8} a\varepsilon E_0^2 \lambda_g \left(\frac{1}{1+\left(\frac{bm}{an}\right)^2} \right) = \frac{1}{8} a\varepsilon E_0^2 \left(\frac{1}{\left(1+\left(\frac{bm}{an}\right)^2\right)\sqrt{\frac{1}{\lambda^2} - \left(\frac{m}{a}\right)^2 - \left(\frac{n}{b}\right)^2}} \right)\end{aligned} \tag{42}$$

$$\tilde{F}_D = \tilde{F}_{Dy} = \int_z^{z+\lambda_g} F_{Dy} dz = -\frac{1}{8} a\varepsilon E_0^2 \left(\frac{1}{\left(1+\left(\frac{bm}{an}\right)^2\right)\sqrt{\frac{1}{\lambda^2} - \left(\frac{m}{a}\right)^2 - \left(\frac{n}{b}\right)^2}} \right) \tag{43}$$

$$\tilde{F}_R = \tilde{F}_{Rx} = \int_z^{z+\lambda_g} F_{Ry} dz = \frac{1}{8} b\varepsilon E_0^2 \left(\frac{1}{\left(1+\left(\frac{an}{bm}\right)^2\right)\sqrt{\frac{1}{\lambda^2} - \left(\frac{m}{a}\right)^2 - \left(\frac{n}{b}\right)^2}} \right) \tag{44}$$

$$\tilde{F}_L = \tilde{F}_{Lx} = \int_z^{z+\lambda_g} F_{Ly}\, dz = -\frac{1}{8} b\varepsilon E_0^2 \left(\frac{1}{\left(1+\left(\frac{an}{bm}\right)^2\right)\sqrt{\frac{1}{\lambda^2} - \left(\frac{m}{a}\right)^2 - \left(\frac{n}{b}\right)^2}} \right) \tag{45}$$

8. Conclusion

When electromagnetic wave, *TE* or *TM*, propagates inside a rectangular waveguide, the electromagnetic force acting on the wall is in the normal line, traveling in Z-direction with double frequency of the wave. The electric force is tension but the magnetic force is pressure. The electromagnetic force distribution is related to the geometric dimensions *a* and *b* as well as mode numbers *m* and *n*. As a mode number increases, the surface force density on the corresponding two walls decreases and on the other walls increases. For *TE* wave, the electromagnetic force distribution on the walls is partly tension and partly pressure, depending on the position coordinates on the wall. However, for *TM*, the electromagnetic force distribution on the four walls is always pressure since the magnitude of the magnetic force is always greater than the magnitude of electric force. Under the condition of $a > b$, TE_{10} is an important case where the force distribution is uniform in *Y*-direction and reaches its maximum on the two walls of length *b*. The symbolic symmetry of the corresponding quantities is demonstrated in the results. It has been proved that $f_e = \frac{1}{2}\varepsilon E^2$ and $f_m = \frac{1}{2}\mu H^2$ can be applied to the wall of the waveguide.

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