Adv. Studies Theor. Phys., Vol. 1, 2007, no. 7, 343 - 356

Poincaré Charges for Chiral Membranes

Alberto Escalante

Departamento de física
Centro de investigación y de Estudios Avanzados del I.P.N.
Apartado Postal 14-740, 0700 México, D.F., México
aescalante@fis.cinvestav.mx

Abstract

Using basic ideas of simplectic geometry, we find the covariant canonically conjugate variables, the commutation relations and the Poincaré charges for chiral superconducting membranes (with null currents), as well as, we find the stress tensor for the theory under study.

I. Introduction

The interest in physical systems characterized by extended structures goes back to the XIX th Century and to Lord Kelvin's "aether atoms", for which a spatial extension was postulated in order to accommodate a complex structure which would be have both as an elastic solid (conveying the transverse wave motion of electromagnetism) and viscous liquid (dragged by the earth in its orbital motion).

In the XX Century, there have been three active motivations leading to either classical or quantum extendons. On the other hand, the physics of condensed matter (including biological systems) have revealed that membranes and two-dimensional layers play an important role; in some case, there also appear one-dimensional filaments (or strings). Similar structures appear in astrophysics and cosmology, one example being the physics of Black holes, in which the "membrane" is the boundary layer between the hole and the embedding spacetime, and another example is represented by the hypothetical cosmic strings [1].

In this context, it is believed that cosmic strings are fundamental bridges on the understanding of the Universe formation due to that several cosmological phenomena can be described by means of cosmic strings properties. Besides of these there are other kinds of cosmic objects possessing different properties of those inherited to ordinary cosmic strings, for example: domains walls and hybrid structures. They can arise in several Grand Unified Theories whenever there exists an appropriate symmetry breaking scheme. However, there is other class of cosmological objects that can emerge with ability to carry some sort of charge. For instance, as was suggested by Witten [2], cosmic strings could behave like superconductors.

Since that time, the vast research on super conducting strings has thrown a new variety of cosmic objects. The cosmological results of supersymmetric theories was also considered, yielding to another class of cosmic strings, namely chiral cosmic strings. These objects are the result of symmetry breaking in supersymmerty (SUSY) where a $U(1)$ symmetry is broken with a Fayet-lliopoulos D term, turning out a sole fermion zero mode traveling in only one direction in the string core [3]. In other words, when the current along the superconducting string shows a light-like causal structure then we have a chiral string. The dynamics of the chiral strings model has been recognized to be an intermediate stage between Dirac-Nambu-Goto [DNG] model and that of generic elastic model [4, 5, 6], which has interesting cosmological implications. The microphysics of this kind of topological defects has been investigate, opening up the possibility to have chiral vortons more stable than vortons of other kinds.

In this manner, the purpose of this article is establish the basis for the study of symmetries and the quantization aspects of chiral membranes. Such basis consists in the construction of a covariant and gauge invariant symplectic structure on the corresponding quotient phase space Z (the space of solutions of classical equations of motion divided by the symmetry group volume) instead of choosing a spatial coordinate system on the phase space. For example, recent letters [7, 8, 9, 10, 11] the basic elements to quantize extended objects (in particular bosonic p-branes) has been explored; in [8] we established the basis to study the quantization aspects of p-branes with thickness, because, when we add it to the [DNG] action has an important effect in QCD [12], among other things. In [9] has been demonstrated that the presence of Gauss-Bonnet [GB] topological term in the [DNG] action describing strings, has a dramatic effect on the covariant phase space formulation of the theory, in this manner, we shall obtain a completely different quantum field theory. Recently, using the results given in [9] we identify the covariant canonical variables for [DNG] p-branes and [GB] strings, among other things [10]. Thus, in this paper we extend the results for chiral strings reported in [13, 14, 15], using a Kaluza-Klein (KK) reduction mechanism [16] and following closely the results given in [8, 9, 10].

This paper is organized as follows. In Sect. II, We discuss the deformation formalism for the geometry of chiral membranes, which is crucial for the develop of this work. In Sect. III, using the results given in [9, 10] and basic ideas of symplectic geometry [17], we identify the covariant canonically conjugate variables, the relevant Poison brackets, the Poncaré charges and its corresponding laws of conservation. In the Sect. IV, we study a generalization of auxiliary variables for relativistic membranes for an arbitrary co-dimension , which is a generalization of the results given in [18]. We find in this section the stress tensor for the theory under study and confirm the results given in the last section. In Sect. V, we establish some remarks and prospects. .

II. Geometry and deformations for chiral membranes

In this section, utilizing the formalism given in [19] we will discuss the deformations of the embedding for chiral membranes possesing nullcurrents on the worldvolume ($\varpi = \gamma^{ab}\varphi,_a \varphi,_b$) based in the Kaluza-Klein approach used in [16] . For ours purposes , we consider a relativistic membrane of dimension d, whose worldsheet $\{m, \Gamma_{ab}\}$ is an oriented timelike $d+1$ dimensional manifold embedded in a N-dimensional extended arbitrary fixed background space-time $\{M, g_{\bar{\mu}\bar{\nu}}\}$. We describe the worldsheet by the extended embedding as

$$X^{\bar{\mu}} = \begin{pmatrix} X^{\mu}(\xi^a) \\ \varphi(\xi^a) \end{pmatrix}, \tag{1}$$

where φ is a field living on the worldsheet, $\bar{\mu} = 0, 1, ...N$, and ξ^a are coordinates on the worldsheet, $a = 1, 2...d$.

With the former embedding, we can make contact with the Kaluza-Klein description for the background spacetime metric

$$g_{\bar{\mu}\bar{\nu}} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & g_{44} \end{pmatrix}, \tag{2}$$

where $g_{\mu\nu}$ is the original background spacetime and g_{44} is a constant.

We have for the embedding a tangent basis defined by $e_a = X^{\bar{\mu}},_a D_{\bar{\mu}}$, where $D_{\bar{\mu}}$ is the covariant derivative compatible with $g_{\bar{\mu}\bar{\nu}}$. The tangent vectors $e_a^{\bar{\mu}}$ associated with the embedding (1) can be written as

$$e_a^{\bar{\mu}} = \begin{pmatrix} e_a^{\mu} \\ \varphi,_a \end{pmatrix}. \tag{3}$$

Thus, the metric induced on m is iven by $\Gamma_{ab} = g_{\bar{\mu}\bar{\nu}} e_a^{\bar{\mu}} e_b^{\bar{\nu}} = \gamma_{ab} + g_{44}\varphi,_a \varphi,_b$, where γ_{ab} is the metric of the embedding without the field ϕ [19].

We will denote the $n^{J\bar{\mu}}$ as the J-th unit normal to the worldsheet, $I = 1, ..., N-d$ given by

$$n^{\bar{\mu}i} = \begin{pmatrix} n^{\mu i} \\ 0 \end{pmatrix}, \qquad n^{\bar{\mu}4} = \sqrt{g_{44}} \begin{pmatrix} e^{\mu}_{a}\varphi'^{a} \\ -g^{44} \end{pmatrix}, \tag{4}$$

and defined intrinsically by

$$g_{\bar{\mu}\bar{\nu}} n^{\bar{\mu}I} e^{\bar{\nu}}_{a} = 0, \qquad g_{\bar{\mu}\bar{\nu}} n^{\bar{\mu}I} n^{\bar{\nu}J} = \delta^{I}{}_{J}. \tag{5}$$

The collection of vectors $\{e^{\bar{\mu}}_{a}, n^{\bar{\nu}I}\}$ can be used as a basis for the spacetime vectors appropriate for the geometry under consideration.
We can calculate the world sheet gradients of the basis vectors, given by

$$\begin{aligned} D_a e^{\bar{\mu}}{}_{b} &= \Gamma_{ab}{}^{c} e^{\bar{\mu}}{}_{c} - K_{ab}{}^{I} n^{\bar{\mu}}{}_{I}, \\ D_a n^{\bar{\mu}I} &= K_{ab}{}^{I} e^{b\bar{\mu}} + \omega_{a}{}^{IJ} n^{\bar{\mu}}{}_{J}, \end{aligned} \tag{6}$$

where the I-th extrinsic curvature $K_{ab}{}^{I}$ is given by

$$K_{ab}{}^{I} = K_{ba}{}^{I} = -D_a e^{\bar{\mu}}{}_{b} n_{\bar{\mu}}{}^{I}, \tag{7}$$

and the extrinsic twist potential by [19]

$$\omega_{a}{}^{IJ} = -\omega_{a}{}^{JI} = D_a n^{\bar{\mu}I} n_{\bar{\mu}}{}^{J}. \tag{8}$$

Now we will calculate the deformations of the intrinsic and extrinsic geometry. For we aims we consider the neighboring surface described by the deformation

$$x^{\bar{\mu}} = X^{\bar{\mu}}(\xi^{a}) + \delta X^{\bar{\mu}}(\xi^{a}), \tag{9}$$

we can decompose the infinitesimal deformation vector field $\delta X^{\bar{\mu}}$ with respect to the spacetime basis $\{e_a, n^{I}\}$ as

$$\delta X^{\bar{\mu}} = \phi^{a} e^{\bar{\mu}}{}_{a} + \phi^{I} n^{\bar{\mu}}{}_{I}, \tag{10}$$

therefore, we can find that the deformations of the intrinsic geometry of the embedding are given by [8, 9, 10, 19]

$$\mathbf{D} e_a = (K_{ab}{}^{I}\phi_I) e^{b} + (\widetilde{\nabla}_a \phi_I) n^{I} + (\nabla_a \phi^{b}) e_b - K_{ab}{}^{I} \phi^{b} n_I, \tag{11}$$

$$\mathbf{D}\Gamma_{ab} = 2K_{ab}{}^{J}\phi_J + \nabla_a\phi_b + \nabla_b\phi_a, \tag{12}$$

$$\mathbf{D}\Gamma^{ab} = -2K^{abJ}\phi_J - \nabla^{a}\phi^{b} - \nabla^{b}\phi^{a}, \tag{13}$$

$$\mathbf{D}\sqrt{-\Gamma} = \sqrt{-\Gamma}[\nabla_a\phi^{a} + K^{I}\phi_I], \tag{14}$$

finally, the deformation of the extrinsic curvature is given by

$$\mathbf{D}K_{ab}{}^{I} = \widetilde{\nabla}_a\widetilde{\nabla}_b\phi^{I} + K_{ac}{}^{I}K^{c}{}_{bJ}\phi^{J} + R((e_a, n_J), e_b, n^{I})\phi^{J}. \tag{15}$$

where $R((e_a, n_J), e_b, n^I) = R_{\bar{\alpha}\bar{\beta}\bar{\mu}\bar{\nu}} e_a^{\bar{\beta}} n_J^{\bar{\alpha}} e_b^{\bar{\mu}} n^{I\bar{\nu}}$. $R_{\bar{\alpha}\bar{\beta}\bar{\mu}\bar{\nu}}$ is the riemann tensor of the spacetime with metric $g_{\bar{\mu}\bar{\nu}}$. For this work these are all deformations that we will need.

III. The Poincaré charges for chiral membranes by means of symplectic geometry

In this section, we will find the Poincaré charges and the relevant Poisson brackets for the theory under study. In the literature [13, 14], we can find an equivalence between the chiral membrane dynamics and the DNG dynamics in an extended background spacetime plus a chirality condition, we can see this equivalence if we considered an action which is invariant under reparametrizations of the worldvolume like DNG with the induced metric Γ_{ab}

$$S = -\mu_0 \int d^{d+1}\xi \sqrt{-\Gamma}, \tag{16}$$

here Γ is the determinant of the extended induced metric Γ_{ab}, μ_0 is a constant and the determinant is given by $\Gamma = \gamma(1 + g_{44}\varpi)$, where γ is the determinant of the induced metric γ_{ab} without the field φ. In this manner, we can observe from action (16) that is one for superconducting strings involving the Nielsen model whit $L(\varpi) = \sqrt{1 + g_{44}\varpi}$ [16]. Thus, with the embedding (1) we have unified the strings and superconducting string theory. In the next lines, we will find the covariant Poincaré charges and we will establish the covariant quantization aspects of the theory under study.

Such as we commented in previous lines, in recent works using a covariant description of the canonical formalism for quantization has been established the basis for the study of the symmetries and the quantization aspects of many systems such as DNG p-branes, DNG p-branes with thickness and the topological strings governed for the Gauss-Bonnet action [8, 9, 10]. Such basis consists in constructing a covariant and gauge invariant symplectic structure on the corresponding quotient phase space Z instead of choosing a special coordinate system on the phase space, with coordinates p_i and q^i as we usually find in the literature.

In view of the action for our system is like to DNG action, for our purposes, we can use the results found in [8, 9] and the deformation formalism given in [19] to construct our simplectic structure .

We follow the steps given in [8, 9] first we will find the equations of motion, which determinate the covariant phase space defined as: *the space of solutions to the classical equations of motion* [9, 10, 17]. Using the action (17) and the deformations (11-15) we find

$$\delta S = -\mu_0 \int d^{d+1}\xi \sqrt{-\Gamma}\Gamma^{ab} K_{ab}{}^I \phi_I + \mu_0 \int d^{d+1} \nabla_a (\sqrt{-\Gamma}\phi^a), \tag{17}$$

where we can identify the equations of motion

$$\Gamma^{ab} K^{I}_{ab} = 0, \tag{18}$$

and the divergence term $\Psi^a = \sqrt{-\Gamma}\phi^a$ we can identify as a simplectic potential, because of the variation of this potential will generate the simplectic structure for our theory [8].
On the other hand, if we take the variation of (18), we obtain the linearized equations, this is

$$\widetilde{\Delta}\phi^I + K_{ac}{}^{I} K^{ac}{}_{J}\phi^J + R((e_a, n_J), e_a, n^I)\phi^J = 0, \tag{19}$$

which has the same form like DNG for bosonic p-branes [9, 10, 19].
From the equation (19) and using the self-adjoint operators method we can find a conserved current and with this current we can construct our simplectic structure, however, following [8], we can construct the same simplectic structure taking the variation of the simpletic potential, this is

$$\begin{aligned} \omega &= \int \delta(-\mu_0\sqrt{-\Gamma}\phi^a) d_{\Sigma_a} = \int \delta(-\mu_0\sqrt{-\Gamma} e^a{}_{\bar{\mu}}\tau_a) \wedge \delta X^{\bar{\mu}} d\Sigma \\ &= \int \delta(-\mu_0\sqrt{-\Gamma} e^a{}_{\mu}\tau_a) \wedge \delta X^{\mu} d\Sigma + \int \delta(-\mu_0 g_{44}\sqrt{-\Gamma}\varphi_{,a}\,\tau^a) \wedge \delta\varphi d\Sigma, \end{aligned} \tag{20}$$

where we have used the equations (1), (3) and (10), here δ, is the deformation operator that acts as exterior derivative on the phase space [7, 8] and τ^a is a normalized ($\tau^a\tau_a = -1$) timelike vector field tangent to the world volume. It is remarkable to note that ω given in (20) has the symmetries to be invariant under diffeomorphisms and world volume repametrizations, we can easy prove this in the same way that is presented in [8], also, ω turns out to be independent on the chose of Σ *i.e.,* $\omega_\Sigma = \omega_{\Sigma'}$ where Σ is a Cauchy d-surface. This property will be important, since it allows us to establish a connection between functions and Hamiltonian vector fields on Z.
From (20) we can identify the canonical conjugate coordinates, $p_\mu \equiv -\mu_0\sqrt{-\Gamma}e^a{}_{\mu}\tau_a$ is canonical conjugate to $q^\mu \equiv X^\mu$ and $p' \equiv -\mu_0 g_{44}\sqrt{-\Gamma}\varphi_{,a}\,\tau^a$ to $q' \equiv \varphi$. In this manner, if $\varpi = 0$ we identify the p_μ and X^μ as the canonical variables for DNG bosonic membranes such as is reported in [9, 10].
On the other hand, using equation (20) we can group the canonical variables in only one $\hat{P}^{\bar{\mu}}$ and $q^{\bar{\mu}}$, this is

$$\hat{P}_{\bar{\mu}} = -\mu_0\sqrt{-\Gamma} e_{\bar{\mu}a}\tau^a, \quad q^{\bar{\mu}} = X^{\bar{\mu}}, \tag{21}$$

and ω takes the form

$$\omega = \int_\Sigma \delta\hat{P}_{\bar{\mu}} \wedge \delta X^{\bar{\mu}} d\Sigma. \tag{22}$$

We can see that the embedding functions depend of local coordinates for the world-volume (ξ^a) , and we have that $\hat{P}_{\bar{\mu}} = \hat{P}^{\bar{\mu}}(\vec{\xi}, \tau)$, and $X^{\bar{\mu}} = X^{\bar{\mu}}(\vec{\xi}, \tau)$, where we split the local coordinates for the world-volume in an arbitrary evolution parameter τ and coordinates $\vec{\xi}$ for Σ at fixed values of τ. Thus, any function f on the phase space depends of $f = f(\hat{P}_{\bar{\mu}}, X^{\bar{\mu}})$.
Now, using basic ideas of symplectic geometry, we known that if the symplectic structure ω is invariant under a group of transformations G, (in our case ω is invariant under space-time diffeormorphisms) which corresponds to the gauge transformations of the theory [8, 9], therefore, the Lie derivative along a vector V tangent to a gauge orbit G of ω vanishes, this is,

$$\pounds_V \omega = V \rfloor \delta\omega + \delta(V \rfloor \omega) = 0, \tag{23}$$

where $\rfloor$ denotes the operation of contraction with V. Since ω is an exact and in particular closed two-form [9, 10], $\delta\omega = 0$, and we have that Eq. (23), at least locally, takes the form

$$V \rfloor \omega = -\delta H, \tag{24}$$

where H is a function on Z which we call the generator of the G transformations. In this manner the relation (24) allows us to establish a connection between functions and Hamiltonian vector fields on Z.
On the other hand, if h and g are functions on the phase space, we can define using the simplectic structure ω a new function $[f, g]$, the Poisson bracket of h and g, as

$$[h, g] = V_h \rfloor g = -V_g \rfloor h, \tag{25}$$

where V_h and V_g correspond to the Hamiltonian vector fields generated by h, and g respectively through Eq. (23).
Now, we will calculate the fundamental Poisson brackets. For this, we take the particular case of the flat spacetime ($g_{\mu\nu} = \eta_{\mu\nu}$ see Eq. (2)) and using (24) we find

$$\begin{aligned} X^{\bar{\alpha}} \quad &\longrightarrow \quad V_{X^{\bar{\alpha}}} = -\frac{\partial}{\partial p_{\bar{\alpha}}}, \\ \hat{P}_{\bar{\alpha}} \quad &\longrightarrow \quad V_{P_{\bar{\alpha}}} = \frac{\partial}{\partial X^{\bar{\alpha}}}, \end{aligned} \tag{26}$$

in this manner, using (6) we have,

$$[X^{\bar{\mu}}(\overrightarrow{\xi}, \tau), X^{\bar{\nu}}(\overrightarrow{\xi}', \tau)] = [\hat{P}^{\bar{\mu}}(\overrightarrow{\xi}, \tau), \hat{P}^{\bar{\nu}}(\overrightarrow{\xi}', \tau)] = 0, \tag{27}$$

$$[X^{\bar{\mu}}(\overrightarrow{\xi}, \tau), \hat{P}_{\bar{\nu}}(\overrightarrow{\xi}', \tau)] = \delta^{\bar{\mu}}{}_{\bar{\nu}} \delta(\overrightarrow{\xi} - \overrightarrow{\xi}'), \tag{28}$$

where $\delta^{\mu}{}_{\nu}$ is the Kronecker symbol and $\delta(\overrightarrow{\xi} - \overrightarrow{\xi}')$ the Dirac delta function. We can see from last equation that in particular we have the following cases,

for example; $[X^{\mu}(\overrightarrow{\xi},\tau), X^{\nu}(\overrightarrow{\xi}',\tau)] = [\hat{P}^{\mu}(\overrightarrow{\xi},\tau), \hat{P}^{\nu}(\overrightarrow{\xi}',\tau)] = 0$, which correspond to the Poisson brackets found in [10] for [DNG] system, and the cases; $[\varphi,\varphi] = [\varphi,_{a}\tau^{a}, \varphi,_{b}\tau^{b}] = 0$, which corresponds to the Poisson brackets of the field φ living on the worldvolume.
On the other hand, if we choose $V = \epsilon^{\bar{\alpha}}\frac{\partial}{\partial X^{\bar{\alpha}}}$, where $\epsilon^{\bar{\alpha}}$ is a constant in the Kaluza-Klein spacetime, and using (22), (24) we find

$$V\rfloor\omega = -\delta(-\epsilon_{\bar{\mu}}\tau_{a}(\mu_{0}\sqrt{-\Gamma}e^{a\bar{\mu}})), \tag{29}$$

where we can identify the linear momentum density

$$P^{a\bar{\mu}} = -\mu_{0}\sqrt{-\Gamma}e^{a\bar{\mu}}, \tag{30}$$

using (6) we can prove that the lineal momentum for chiral membranes are covariantly conserved

$$\nabla_{a}P^{a\bar{\mu}} = 0, \tag{31}$$

such as is found in [10, 20] for the [DNG] system.
We can express the total lineal momentum $P^{\bar{\mu}}$ as

$$P^{\bar{\mu}} = \int_{\Sigma} P^{a\bar{\mu}}d\Sigma_{a} = \int \hat{P}^{\bar{\mu}}d\Sigma, \tag{32}$$

where, $\hat{P}^{\bar{\mu}}$ is the canonical momentum. The total lineal momentum and the canonical momentum coincide.
Now, for a vector field given by $V = \frac{a_{\bar{\alpha}\bar{\beta}}}{2}[X^{\bar{\alpha}}\frac{\partial}{\partial X^{\bar{\beta}}} - X^{\bar{\beta}}\frac{\partial}{\partial X^{\bar{\alpha}}}]$, with $a_{\bar{\alpha}\bar{\beta}} = -a_{\bar{\beta}\bar{\alpha}}$, the contraction $V\rfloor\omega$ gives

$$V\rfloor\omega = \delta(\frac{a_{\bar{\alpha}\bar{\beta}}}{2}P^{a\bar{\mu}}X^{\bar{\nu}} - P^{a\bar{\alpha}}X^{\bar{\nu}}), \tag{33}$$

thus, we can identify the angular momentum of the chiral membrane

$$M^{a\bar{\mu}\bar{\nu}} = \frac{1}{2}[P^{a\bar{\mu}}X^{\bar{\nu}} - P^{a\bar{\alpha}}X^{\bar{\nu}}], \tag{34}$$

using the gradients of the vector basis(Eq. (6)), we find that

$$\nabla_{a}M^{a\bar{\mu}\bar{\nu}} = 0, \tag{35}$$

this is, the angular momentum is covariantly conserved too [10, 20].
We can define the total angular momentum $M^{\alpha\bar{\beta}}$ as

$$M^{\bar{\alpha}\bar{\beta}} = \int_{\Sigma} M^{a\bar{\alpha}\bar{\beta}}d\Sigma_{a} = \int_{\Sigma}(\hat{P}^{\bar{\beta}}X^{\bar{\alpha}} - \hat{P}^{\bar{\alpha}}X^{\bar{\beta}})d\Sigma, \tag{36}$$

In addition, we can use the equation (33) to find the Hamiltonian vector field associated to the angular momentum, using (24) with $H^{\bar{\alpha}\bar{\beta}} = (\hat{P}^{\bar{\beta}} X^{\bar{\alpha}} - \hat{P}^{\bar{\alpha}} X^{\bar{\beta}})$ we find

$$V^{\bar{\beta}\bar{\alpha}} = g^{\bar{\lambda}\bar{\beta}} \left(X^{\bar{\alpha}} \frac{\partial}{\partial X^{\bar{\lambda}}} + \hat{P}^{\bar{\alpha}} \frac{\partial}{\partial \hat{P}^{\bar{\lambda}}} \right) - g^{\bar{\lambda}\bar{\alpha}} \left(X^{\bar{\beta}} \frac{\partial}{\partial X^{\bar{\lambda}}} + \hat{P}^{\bar{\beta}} \frac{\partial}{\partial \hat{P}^{\bar{\lambda}}} \right), \tag{37}$$

thus, using the definition of the Poisson's brackets, we can find $[M^{\bar{\mu}\bar{\nu}}, M^{\bar{\alpha}\bar{\beta}}]$ and $[M^{\bar{\mu}\bar{\nu}}, P^{\bar{\alpha}}]$, this is

$$\begin{aligned} [M^{\bar{\mu}\bar{\nu}}, M^{\bar{\alpha}\bar{\beta}}] &= g^{\bar{\nu}\bar{\alpha}} M^{\bar{\mu}\bar{\beta}} + g^{\bar{\mu}\bar{\alpha}} M^{\bar{\beta}\bar{\nu}} + g^{\bar{\nu}\bar{\beta}} M^{\bar{\alpha}\bar{\mu}} + g^{\bar{\mu}\bar{\beta}} M^{\bar{\nu}\bar{\alpha}}, \\ [M^{\bar{\mu}\bar{\nu}}, P^{\bar{\alpha}}] &= g^{\bar{\mu}\bar{\alpha}} P^{\bar{\nu}} - g^{\bar{\alpha}\bar{\nu}} P^{\bar{\mu}}, \end{aligned} \tag{38}$$

We can see that the Poincaré charges, P and M, indeed close correctly on the Poincaré algebra, such as the [DNG] theory [10], the difference with the [DNG] theory is that the Poison brackets given in (38) contains the case for chiral membranes, and in particular the results found in [10].

IV. Generalization of auxiliary variables for relativistic membranes

In this section, we will extend the results given in [18] for an arbitrary co-dimension, and we will find the stress tensor for the theory under study proving that it coincide with the found in equation (30).
In [18], has been considered a Hamiltonian which depends on the metric and extrinsic curvature induced on the surface. The metric and the curvature, along with the basis vector which connect them to the embedding functions defining the surface, are introduced as auxiliary variables by adding appropriate constraints, all of them quadratic. This elegant treatment, allow us study the response of the Hamiltonian to a deformations in each of the variables and the relationship between the multipliers implementing the constraints a the conserved stress tensor.
First, we will consider any reparametrization invariant functional of the variables Γ_{ab} and ${K_{ab}}^I$ defined in the equations (3) and (7) respectively

$$H[X] = \int \sqrt{-\Gamma} H d^{d+1}\xi. \tag{39}$$

Such as in [18], we are interested in determining the response of H to a deformation of the worldvolume $X \longrightarrow \delta X$, and will be to distribute the burden on X among the basis normal n^I, tangent e^a and the extrinsic curvature ${K_{ab}}^I$ treating latter as independent auxiliary variables.
Now, to introduce Lagrange multipliers function to implement the constraints.

We thus construct a new functional
$H_c[\Gamma_{ab}, K^I_{ab}, n^I, e_a, X^\mu, f^a, \Lambda^{ab}{}_I, \lambda^{ab}, \lambda^a{}_{\perp I}, \lambda_{IJ}, \phi^a{}_{IJ}]$ this is,

$$\begin{aligned} H_c = \ & H[\Gamma_{ab}, K^I_{ab}] + \int [f^a(e_a - \partial_a X) \\ + \ & \lambda^a{}_{\perp I}(e_a \cdot n^I) + \lambda_{IJ}(n^I \cdot n^I - \delta^{IJ}) + \Lambda^{ab}{}_I(K^I_{ab} - e_a \cdot \widetilde{\nabla}_b n^I) \\ + \ & \lambda^{ab}(g_{ab} - e_a \cdot e_b) + \phi^a{}_{IJ}(\omega_a{}^{IJ} - \nabla_a n^J \cdot n^I)]\sqrt{-\Gamma} d^{d+1}\xi, \end{aligned} \tag{40}$$

here, the original functional is considered as a function of the independent variables Γ_{ab} and $K_{ab}{}^I$, the auxiliary variables f^a, $\lambda^a{}_{\perp I}$, λ_{IJ}, $\Lambda^{ab}{}_I$, λ^{ab}, $\phi^a{}_{IJ}$ are Lagrange multipliers. In the expression (40) we introduced a new worldsheet covariant derivative $\widetilde{\nabla}_a$ defined on fields transforming as tensors under frame rotations as [19]

$$\widetilde{\nabla}_a \phi^I{}_J = \nabla_a \phi^I{}_J - \omega_a{}^I{}_k \phi^k{}_J - \omega_{aJ}{}^k \phi^I{}_k, \tag{41}$$

the expression (41) and the Lagrange multiplier $\phi^a{}_{IJ}$, are fundamentals for the extension of the results given in [18] for arbitrary co-dimensions.
Using the equation (40) we can calculate the Euler-Lagrange equations corresponding to X

$$\frac{\delta H_c}{\delta X} = \int \nabla_a(\sqrt{-\Gamma} f^a) d^D \xi, \tag{42}$$

where in equilibrium we have

$$\nabla_a(\sqrt{-\Gamma} f^a) = 0, \tag{43}$$

this is, $\sqrt{-\Gamma} f^a$ is covariantly conserved. The physical interpretation of f^a is as a stress tensor [18].
On the other hand, the Euler-Lagrange equations for e_a are given by

$$\frac{\delta H_c}{\delta e_a} = \int [f^a + \lambda^a{}_{\perp I} n^I - \Lambda^{ab}{}_I \nabla_b n^I + \Lambda^{ab}{}_I \omega_b{}^{IJ} n_J - 2\lambda^{ab} e_b], \tag{44}$$

in the equilibrium this implies

$$f^a = -\lambda^a{}_{\perp I} n^I + \Lambda^{ab}{}_I \nabla_b n^I - \Lambda^{ab}{}_I \omega_b{}^{IJ} n_J + 2\lambda^{ab} e_b, \tag{45}$$

taking account equation (41) we have

$$f^a = -\lambda^a{}_{\perp I} n^I + \Lambda^{ab}{}_I \widetilde{\nabla}_b n^I + 2\lambda^{ab} e_b, \tag{46}$$

using the Gauss-Weingarten equations which describe completely the extrinsic geometry of the worldsheet Eq. (6), the last equation takes the form

$$f^a = (\Lambda^{ab}{}_I K_b{}^{cI} + 2\lambda^{ac}) e_c - \lambda^a{}_{\perp I} n^I, \tag{47}$$

these expressions contains as particular case the found in [18].
In the same way, the Euler-Lagrange equations for n^I we have

$$\begin{aligned}\frac{\delta H_c}{\delta n^I} &= \lambda^a{}_{\perp I} e_a + 2\lambda_{IJ} n^J + \nabla_a(\Lambda^{ab}{}_I e_b) + \Lambda^{ab}{}_I \omega_b{}^{IJ} e_a + \nabla_a(\phi^a{}_{IJ} n^J) \\ &= (\widetilde{\nabla}_a \Lambda^{ab}{}_I + \lambda^b{}_{\perp i} n^I) e_b + (2\lambda^J{}_I - \Lambda^{ab}{}_I K_{ab}{}^J) n_J + \nabla_a \phi^a{}_{IJ} n^J + \phi^a{}_{IJ} \nabla_a n^J, \quad (48)\end{aligned}$$

and the Euler-Lagrange equation for $\omega_a{}^{IJ}$ we find

$$\frac{\delta H_c}{\delta \omega_a{}^{IJ}} = \Lambda^{ab}{}_I n_J \cdot e_b + \phi^a{}_{IJ}, \tag{49}$$

from last equation we can see that in equilibrium

$$\phi^a{}_{IJ} = -\Lambda^{ab}{}_I n_J \cdot e_b. \tag{50}$$

in this manner, enforcing the constraints (see Eqs. (5)) and using the expression (48) we have

$$(\widetilde{\nabla}_a \Lambda^{ab}{}_I + \lambda^b{}_{\perp I}) e_b + (2\lambda_I{}^J - \Lambda^{ab}{}_I K_{ab}{}^J) n_J = 0, \tag{51}$$

this implies

$$\begin{aligned}\widetilde{\nabla}_a \Lambda^{ab}{}_I + \lambda^b{}_{\perp I} &= 0 \\ 2\lambda_I{}^J - \Lambda^{ab}{}_I K_{ab}{}^J &= 0.\end{aligned} \tag{52}$$

Finally, the Euler-Lagrange equations for $K_{ab}{}^I$ and Γ_{ab} is given by

$$\begin{aligned}\Lambda^{ab}{}_I &= -\frac{\partial H_c}{\partial K_{ab}{}^I} \\ \lambda^{ab} &= \frac{T^{ab}}{2},\end{aligned} \tag{53}$$

where

$$T^{ab} = -2(\sqrt{-\Gamma})^{-1} \frac{\partial \sqrt{-\Gamma} H}{\partial \Gamma_{ab}}. \tag{54}$$

Thus, to study the case of chiral membranes we use the expression (16) and taking $H = -\mu_0$ in the expression (54) we find

$$\lambda^{ab} = -\mu_0 \Gamma^{ab} = 2T^{ab}, \tag{55}$$

using (55) in (47) we have

$$f^a = -\mu_0 \Gamma^{ab} e_b = -\mu_0(\gamma^{ab} - g_{44} \nabla^a \varphi \nabla^b \varphi) e_b, \tag{56}$$

where we can see that the stress tensor given in (56) coincides with the found in (30) using simplectic geometry.

Now, following the results given in [18, 20] we can decompose the stress f^a in its normal and tangential parts, $f^a = F^{ab}e_b + F_I{}^a n^I$, so the equations of motion are given by

$$\nabla_a f^a = \widetilde{\nabla}_a f^a = 0, \tag{57}$$

this is

$$\begin{aligned} \nabla_a F^{ab} + K^{bI}{}_a F_I{}^a &= 0 \\ \widetilde{\nabla}_a F^{aI} - F^{ab} K_{ab}{}^I &= 0. \end{aligned} \tag{58}$$

For the chiral membranes case we can see from (56) that $F^{aI} = 0$, therefore, the equations of motion (58) takes the form

$$\Gamma^{ab} K^I_{ab} = 0, \tag{59}$$

this equations is the same found in Eq.(18). Thus, for I=i we have

$$(\gamma^{ab} - g_{44}\nabla^a\varphi\nabla^b\varphi)K_{ab}{}^i = 0, \tag{60}$$

and for I=4

$$\nabla_a \nabla^a \varphi = 0, \tag{61}$$

this results coincide such as is reported in [14].

V. Conclusions and prospects

As we can see, in this paper we have constructed a symplectic structure for chiral membranes in an arbitrary co-dimension. With this geometric structure we could find the Poncaré charges, conservation laws , and construct the relevant Poisson brackets. In this manner, with these results we have the elements to study the quantization aspects in a covariant way and in particular the case of chiral strings theory, which is absent in the literature.

On the other hand, we extended the results given in [18] for an arbitrary co-dimension, thus, with this extension, we can reproduce important results found in the literature, for example, conservation laws for several kind of extended objects that depends of the extrinsic geometry of the worldvolume, and we can find the stress tensor for objects of high-dimension such as the systems studied in [20, 21].

Acknowledgements

This work was supported by CONACYT under grant 44974-F. The author wants to thanks R. Capovilla and Efrain Rojas for the support and the friendship that they have offered me.

References

[1] Elena Eizenberg and Yuval Ne'eman, Membranes and Other Extendons (p-branes), Published 1995 World Scientific.

[2] E. Witten, Nucl, Phys B 249 (1985) 537.

[3] S. C. Davis, A. C. Davis and M. Trodden, Phys. Lett. B 405 (1997) 257.

[4] B. Carter and P. Peter, Phys, Lett. B 466 (1999) 41.

[5] D. A. Steer, Phys. Rev. D 63 (2001) 083517: D. A. Steer, Strings Afther D term Inflation: Evolution and Properties of Chiral Cosmic Strings, astro-ph/0010295.

[6] B. Carter, in formation and Interaction of Topological Defects (Nato ASI B349), Newton Institute, Cambridge, 1994, ed. R. Brandenberger and A. C. Davis, pp 303 (Plenum, New York, 1995).

[7] R. Cartas-Fuentevilla, Class. Quantum Grav., 19, 3571 (2002).

[8] A. Escalante. Mod, Phys, Lett A, (2004), Vol 46, No 43, pp: 1902-1512.

[9] A. Escalante, Int, J. Theor. Phys. (2004), Vol 43, No.6, pp: 1491-1502.

[10] A. Escalante, *Covariant Canonical formalism for Dirac-Nambu-Goto p-branes and the Gauss-Bonnet topological term in string theory*, to be published in International Journal of Modern Physics A, (2006).

[11] R. Cartas Fuentevilla and A. Escalante,*Topological terms and the global symplectic geometry of the phase space in string theory*, Advances in Mathematical Research, Nova Publishing (2004).

[12] A. Polyakov, Nucl. Phys B 268 (1986), 406; H. Kleinert, Phys. Lett B 174 (1986), 335.

[13] Ruben Cordero, Efrain Rojas. Int.J.Mod.Phys. A (2002) 17:73-88.

[14] Ruben Cordero, Efrain Rojas. Rev. Mex.Fis. (2003) 49S1: 44-48, .

[15] J. J. Blanco-Pillado, Ken D. Olum, A. Vilenkin. Phys. Rev. D, (2001) 103513.

[16] N. K. Nielsen, Nucl, Phys. B 167 (1980) 249,

[17] C. Crncović and E. Witten, in *Three Hundred Years of Gravitation*, edited by S. W. Hawking and W. Israel (Cambridge University Press. Cambridge, 1987).

[18] J. Guven, J. Phys. A. (2004), 37:L313-L320.

[19] R. Capovilla and J. Guven, Phys. Rev. D 51, 6736 (1995).

[20] G. Arreaga, R. Capovilla, J. Guven, Ann. Phys. 279 (2000)126.

[21] R. Capovilla, J. Guven, E. Rojas. Class. Quant. Grav. 21:5563-5586, (2004).

Received: April 11, 2007