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Relativistic Distance

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Abstract

In this paper, we prove that two different observers don't equally measure the distance between two points A and B. For this, we introduce some postulates and obtain a new formula to show distance between A and B. In this formula radius of universe, $\mathbf{n}$, is entered such that if $\mathbf{n}$ tends to infinity the ordinary distance is obtained.

Mathematics Subject Classification: 83D05, 78A02

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1 Introduction

In this paper we introduce a new theory about distance, if radius of universe be finite. This theory can result in a completely new view of the nature of distance. Ordinary measurement is so much a part of our daily life that almost everyone has some conceptual difficulty in understanding our idea of distance when he (she) first studies it. Einstein may have put his finger on the difficulty when he said "Common sense is that layer of prejudices laid down in the mind prior to the age of eighteen"[1]. Indeed, it has been said that every great theory begins as a heresy and ends as a prejudice. More than a half-century of experimentation and application has removed special relativity theory from the heresy stage and put it on a sound conceptual and practical basis.

In this article, we show that there is a careful analysis of new theory about distance. Furthermore, we obtain a formula to measure distance between two points x and y. In this study, we measure distance between two points and compare it for two fixed observers. For simplicity, we consider the problem in one dimensional space. Assume that universe is bounded, however if the universe is not bounded, universe radius has a finite value. Some scientists such as Einstein and Rimman have believed unboundedness of universe does not mean it is infinite [2]. Einstein has said "Coming into existence of non-Euclidean geometry resulted in this reality that infiniteness of space is dubitable"[2]. He also has said "Closed but unbounded spaces are imaginable"[2]. We can measure relative distance between two frames may be by comparing measurement between frames. But then we have not deduced the relative distance from observations confined to a single frame. Furthermore, there is no way at all of determining the absolute distance of an inertial reference frame. No inertial frame is preferred over any other, for the laws of mechanics are the same in all. We say that all inertial frame are equivalent as far as mechanics is concerned.

2 Postulates

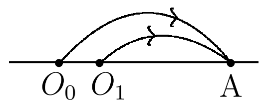
In this section, for our theory we introduce some assumptions. Their simplicity and generality are characteristics of this theory. In this theory what is troublesome, apparently is the philosophic notion that length and time in the abstract are absolute quantities and the belief that relativity contradicts this notion. In this fitting, in emphasizing the common sense of relativity, to conclude with this quotation from Bondi [3] on the presentation of relativity theory: "At first, relativity was considered shocking, anti-establishment and highly mysterious, and all presentations intended for the population at large were meant to emphasize these shocking and mysterious aspects, which is hardly conductive to easy teaching and good understanding."

We explain our postulates as follows:

- **Postulate 1.** We can select any point as a reference frame (Relativity).
- **Postulate 2.** Any point in itself frame lies in the center of universe and measures **n** as the universe radius (Absolute infinity).

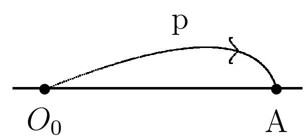
By these postulates we introduce some principles as follows:

- **Principle 1.** Universe is symmetric in directions (Symmetry principle).
- **Principle 2.** Every point agrees in the order of place of points (Agreement principle).
- **Principle 3.** Consider three points O_0, O_1 and A as the following order



then distance of O_0A in O_0 point of view is not less than distance of O_1A in O_1 point of view (Intuition principle).

Notation. Henceforth, if distance of O_0A in O_0 point of view is equal to p, we say that point view of O_0 from A is p and we will show this by $(O_0 \mapsto A) = p$. We show this graphically by



Theorem 1. If N is a point such that $(O \mapsto N) = \mathbf{n}$ for a fixed point O, then for any arbitrary point O_1 such that $(O_1 \mapsto N_1) = \mathbf{n}$, N_1 in point view of O_1 is N in point view of O.
Proof. Suppose that O extends the universe as long as $\mathbf{n}$ to reach N. Also suppose that O_1 extends the universe as long as $\mathbf{n}$ to reach N_1. If N_1 lies after N in point view of O_1 such as Figure 1

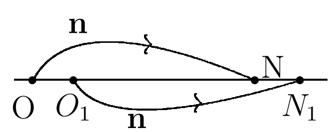


Figure 1

hence by agreement principle N_1 in point view of O lies after N. Therefore, for O the radius of universe is greater than $\mathbf{n}$, this contradicts with second postulate. Therefore, N_1 in point view of O_1 lies before N or coincides it. If N_1 in point view of O_1 lies before N then O_1 measures the universe radius greater than $\mathbf{n}$. This contradicts with second postulate. Consequently, N_1 in point view of O_1 is N in point view of O. ■
Corollary. For all points, N is a fixed point. ■
Consider the following figure, the fundamental question is: what is $(A \mapsto B)$?

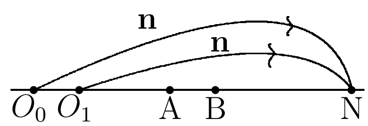


Figure 2

In fact, we look for a formula to show distance of A and B. Suppose

$$(O_0 \mapsto O_1) = p, \ (O_0 \mapsto A) = x$$

if $(O_1 \mapsto A) = x_1$, we set $x_1 = f(x, p)$ and we obtain function f (Figure 3).

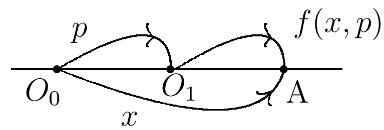


Figure 3

By above notations, in Figure 3 we have the following relations

$$\left.\begin{array}{l}(O_0 \mapsto O_1) = p \\ (O_0 \mapsto A) \ \ = x\end{array}\right\} \Rightarrow (O_1 \mapsto A) = f(p, x).$$

If $(O_1 \mapsto O_0) = p'$, we have

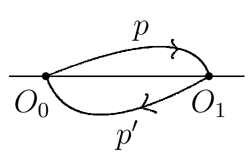


Figure 4

$$\left.\begin{array}{l}(O_0 \mapsto O_1) = p \\ (O_0 \mapsto O_0) = 0\end{array}\right\} \Rightarrow (O_1 \mapsto O_0) = f(p, 0)$$

hence $p' = f(p, 0)$. Consider points O_0, O_1, A, O_1' and A' like Figure 5

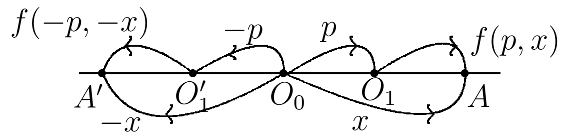


Figure 5

$$\left.\begin{array}{l}(O_0 \mapsto O_1) = p \\ (O_0 \mapsto A) \ = x\end{array}\right\} \Rightarrow (O_1 \mapsto A) = f(p, x)$$

$$\left.\begin{array}{l}(O_0 \mapsto O_1') = -p \\ (O_0 \mapsto A') \ = -x\end{array}\right\} \Rightarrow (O_1' \mapsto A') = f(-p, -x).$$

By symmetric principle universe is symmetric in directions. This results in

$$(O_1 \mapsto A) \ = -(O_1' \mapsto A')$$

or

$$f(p, x) = -f(-p, -x). \tag{1}$$

Hence function f must have property (1). Consider $(O_0 \mapsto O_1) = p$ and $(O_1 \mapsto O_0) = p'$. If $|p| = |p'|$ there isn't anything to prove. Suppose that $|p| < |p'|$ hence there is a point, for example O', such that $(O_1 \mapsto O') = -p$. As $|p| < |p'|$ in O_1 point of view O_0 lies between O_1 and O'. Therefore by agreement principle for any other points O_0 lies between O_1 and O' (Figure 6). Now we obtain $(O' \mapsto O_1)$

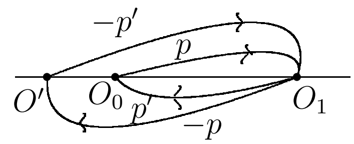


Figure 6

$$\left.\begin{array}{l}(O_1 \mapsto O') = -p \\ (O_1 \mapsto O_1) = 0\end{array}\right\} \Rightarrow (O' \mapsto O_1) = f(-p, 0)$$

as f has property (1) we have

$$\begin{array}{l}(O' \mapsto O_1) = -f(-(-p), 0) \\ (O' \mapsto O_1) \ = -f(p, 0)\end{array}$$

or $p' = f(p, 0)$ therefore

$$(O' \mapsto O_1) = -p'.$$

As in Figure 6 order of $O_1,\ O_0$ and O' are similar for any other points, and so as $|p| < |p'|$ we have

$$\left.\begin{array}{l}(O' \mapsto O_1) = -p' \\ (O_0 \mapsto O_1) = p \\ |p| \ < \ |p'|\end{array}\right\} \Rightarrow |(O' \mapsto O_1)| < |(O_0 \mapsto O_1)|$$

this contradicts the intuition principle. Therefore the case of $|p| < |p'|$ is not correct. Similarly assumption of $|p'| < |p|$ results in a contradiction. Hence $p' = -p$, or

$$f(p, 0) = -p. \tag{2}$$

Consequently, function f must have property (2). Therefore we have

$$(A \mapsto B) = -(B \mapsto A).$$

Now we obtain an important condition by Postulate 1. Consider $O_0,\ O_1,\ O_2$ and A as the following figure

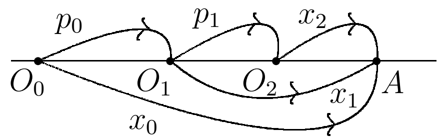


Figure 7

we select O_0 as the reference frame, so

$$\left.\begin{array}{l}(O_0 \mapsto O_1) = p_0\\(O_0 \mapsto A) = x_0\end{array}\right\} \Rightarrow (O_1 \mapsto A) = f(p_0, x_0) = x_1. \tag{3}$$

We can select O_1 as the reference frame, hence by Postulate 1 we have

$$\left.\begin{array}{l}(O_1 \mapsto O_2) = p_1\\(O_1 \mapsto A) = x_1\end{array}\right\} \Rightarrow (O_2 \mapsto A) = f(p_1, x_1) = x_2 \tag{4}$$

from (3) and (4) we have

$$f(p_1, f(p_0, x_0)) = x_2 = (O_2 \mapsto A) \tag{5}$$

also

$$(O_0 \mapsto O_1) = p_0 \Rightarrow \left.\begin{array}{l}(O_1 \mapsto O_0) = -p_0\\(O_1 \mapsto O_2) = p_1\end{array}\right\} \Rightarrow (O_0 \mapsto O_2) = f(-p_0, p_1) \tag{6}$$

and so $(O_0 \mapsto A) = x_0$, therefore

$$\left.\begin{array}{l}(O_0 \mapsto O_2) = f(-p_0, p_1)\\(O_0 \mapsto A) = x_0\end{array}\right\} \Rightarrow (O_2 \mapsto A) = f(f(-p_0, p_1), x_0) = x_2. \tag{7}$$

Finally, from (5) and (7) we have

$$f(f(-p_0, p_1), x_0) = f(p_1, f(p_0, x_0)). \tag{8}$$

Thus function f must have property (8). Consider the following figure

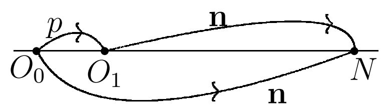


Figure 8

by second postulate there are two points on an axis such that all points lie between and middle them. We call right hand side point as N. For any point such as O we have $(O \mapsto N) = \mathbf{n}$. Suppose that $(O_0 \mapsto O_1) = p$ and O_0 is the reference frame. Therefore

$$\left.\begin{array}{l}(O_0 \mapsto O_1) = p\\ (O_0 \mapsto N) = \mathbf{n}\end{array}\right\} \Rightarrow (O_1 \mapsto N) = f(p, \mathbf{n}) = \mathbf{n}. \tag{9}$$

Hence function f must have property (9). We summarize conditions of f as follows

$$f(x, y) = -f(-x, -y). \tag{10}$$

$$f(x, 0) = -x \tag{11}$$

$$f(f(x, y), z) = f(y, f(-x, z)) \tag{12}$$

$$f(x, \mathbf{n}) = \mathbf{n}. \tag{13}$$

3 Finding of Function f

In this section we present function $f(x, y)$ with properties (10) up to (13). We propose the following function which satisfies conditions (10) up to (12)

$$f(x, y) = \frac{y - x}{1 + \beta xy}\cdot \tag{14}$$

To obtain β we use condition (13), that is

$$f(x, \mathbf{n}) = \mathbf{n}$$

thus

$$\frac{\mathbf{n} - x}{1 + \beta x\mathbf{n}} = \mathbf{n}$$

from this we obtain $\beta = -\frac{1}{\mathbf{n}^2}\cdot$ Therefore the final form of f is

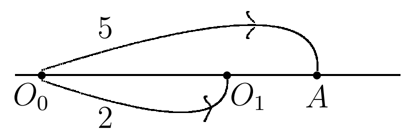
$$f(x, y) = \frac{y - x}{1 - \frac{xy}{\mathbf{n}^2}}\cdot \tag{15}$$

Remark. For $n \to \pm\infty$ relation (15) is the ordinary directed distance of x and y.

4 Examples

In this section we present some examples. In these examples we obtain relative distance of two particular points.

Example 1. Consider three points $O_0,\ O_1$ and A like the following figure



what is $(O_1 \mapsto A)$? From our notation we have

$$\left.\begin{array}{l}(O_0 \mapsto O_1) = 2\\(O_0 \mapsto A) = 5\end{array}\right\} \Rightarrow (O_1 \mapsto A) = f(2,5)$$

thus

$$(O_1 \mapsto A) = \frac{5-2}{1-\frac{10}{\mathbf{n}^2}} = \frac{3\mathbf{n}^2}{\mathbf{n}^2-10}.$$

And so

$$\left.\begin{array}{l}(O_1 \mapsto O_0) = -2\\(O_1 \mapsto A) = \frac{3\mathbf{n}^2}{\mathbf{n}^2-10}\end{array}\right\} \Rightarrow (O_0 \mapsto A) = f(-2, \frac{3\mathbf{n}^2}{\mathbf{n}^2-10})$$

or

$$(O_0 \mapsto A) = \frac{\frac{3\mathbf{n}^2}{\mathbf{n}^2-10}+2}{1+\frac{6\mathbf{n}^2}{\mathbf{n}^2(\mathbf{n}^2-10)}} = \frac{5(\mathbf{n}^2-4)}{\mathbf{n}^2-4} = 5$$

such that we expect.

Example 2. Suppose that N_1 is a point near N, e.g. for O_0 we have $(O_0 \mapsto N_1) = \mathbf{n} - \varepsilon$, for any positive and small ε. Does Theorem 1 hold? In other words does $(N_1 \mapsto N) = \mathbf{n}$? We have

$$\left.\begin{array}{l}(O_0 \mapsto N_1) = \mathbf{n} - \varepsilon\\(O_0 \mapsto N) = \mathbf{n}\end{array}\right\} \Rightarrow (N_1 \mapsto N) = f(\mathbf{n}-\varepsilon, \mathbf{n})$$

hence

$$(N_1 \mapsto N) = \frac{\mathbf{n}-\mathbf{n}+\varepsilon}{1-\frac{\mathbf{n}(\mathbf{n}-\varepsilon)}{\mathbf{n}^2}} = \mathbf{n}$$

that is a surprising result.

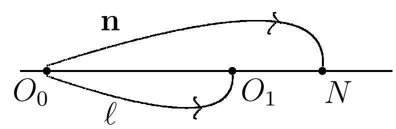
Example 3. What is point view of N from N? In this situation we have an ambiguity. On the one hand, the distance of N from itself is zero and on the other hand this is $\mathbf{n}$. Now by our notations

$$\left.\begin{array}{l}(O_0 \mapsto N) = \mathbf{n}\\(O_0 \mapsto N) = \mathbf{n}\end{array}\right\} \Rightarrow (N \mapsto N) = f(\mathbf{n}, \mathbf{n})$$

or

$$(N \mapsto N) = \frac{\mathbf{n}-\mathbf{n}}{1-\frac{\mathbf{n}^2}{\mathbf{n}^2}} = \frac{0}{0}$$

which we forecasted it by our ambiguity.
In the last example we show distance for N has no sense.
Example 4. Consider three points O_0, O_1 and N as follows



what are $(N \mapsto O_0)$ and $(N \mapsto O_1)$? we have

$$\left.\begin{array}{l}(O_0 \mapsto N) = \mathbf{n} \\ (O_0 \mapsto O_0) = 0\end{array}\right\} \Rightarrow (N \mapsto O_0) = f(\mathbf{n}, 0)$$

hence

$$(N \mapsto O_0) = \frac{0-\mathbf{n}}{1-0} = -\mathbf{n}$$

and so

$$\left.\begin{array}{l}(O_0 \mapsto N) = \mathbf{n} \\ (O_0 \mapsto O_1) = \ell\end{array}\right\} \Rightarrow (N \mapsto O_1) = f(\mathbf{n}, \ell)$$

thus

$$(N \mapsto O_1) = \frac{\ell-\mathbf{n}}{1-\frac{\mathbf{n}\ell}{\mathbf{n}^2}} = -\mathbf{n}.$$

Therefore point view of N from distance of O_0 and O_1 is the difference of $(N \mapsto O_0)$ and $(N \mapsto O_1)$, or

$$-\mathbf{n} + \mathbf{n} = 0.$$

Consequently distance for N has no sense.

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