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Soliton Perturbation Theory for the Splitted Regularized Long Wave Equation

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Abstract

The soliton perturbation theory is used to obtain adiabatic parameter dynamics of solitons due to the splitted regularized long wave equation in presence of perturbation terms. The adiabatic change of soliton velocity is also obtained in this paper.

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1 Introduction

The dimensionless form of the splitted regularized long wave equation (sRLW), that is going to be studied in this paper, is given by

$$q_t + q_x - aqq_x - bq_{xxt} = 0 \tag{1}$$

where a and b are positive parameters. This equation is used in the study of shallow water dynamics and is integrable. It is shown that for a= b = 1, equation (1) can be used as modified model for various phenomena represented by the KdV equation.

Many powerful methods such as the inverse scattering transform, Backlund transform, Wadati trace method, Hirota's bilinear forms, pseudo-spectral method, tanh-sech method, sine-cosine method and the Riccati equation expansion method were used to investigate these type of nonlinear evolution equations that are given by (1) [1-10]. However, the solitary wave solution of this sRLW equation first appeared in 2001 [9]. The 1-soliton solution of (1) is given by

$$q(x,t) = \frac{A}{\cosh^2 B\,(x-\bar{x})} \tag{2}$$

where

$$B = \sqrt{\frac{aA}{12bv}} \tag{3}$$

Here A represents the amplitude of the soliton that is arbitrary, while B is the inverse width of the soliton and $\bar{x}$ represents the center position of the soliton and therefore the velocity of the soliton is given by

$$v = \frac{d\bar{x}}{dt} \tag{4}$$

2 Mathematical Properties

Equation (1) has eaxctly three integrals of motion that are mass (W), linear momentum (M) and energy (E). These are respectively given by

$$W = \int_{-\infty}^{\infty} q\,dx = \frac{2A}{B} \tag{5}$$

$$M = \int_{-\infty}^{\infty} \left(q^2 + bq_x^2\right) dx = \frac{4A^2}{15B}\left(5 + 4B^2\right) \tag{6}$$

and

$$E = \int_{-\infty}^{\infty} \left(q^3 + 3q^2\right) dx = \frac{4A^2}{15B}\,(4A + 15) \tag{7}$$

These conserved quantities are calculated by using the 1-soliton solution given by (2). The center of the soliton $\bar{x}$ is given by the definition

$$\bar{x} = \frac{\int_{-\infty}^{\infty} xq dx}{\int_{-\infty}^{\infty} q dx} = \frac{\int_{-\infty}^{\infty} xq dx}{W} \tag{8}$$

where W is defined in (5). Thus, the velocity of the soliton is given by

$$v = \frac{d\bar{x}}{dt} = \frac{\int_{-\infty}^{\infty} xq_t dx}{\int_{-\infty}^{\infty} q dx} = \frac{\int_{-\infty}^{\infty} xq_t dx}{W} \tag{9}$$

On using (1) and (5), the velocity of the soliton reduces to

$$v = 1 - \frac{aA}{3} \tag{10}$$

3 Perturbation Terms

The perturbed sRLW equation that is going to be studied in this paper is given by

$$q_t + q_x + aqq_x - bq_{xxt} = \epsilon R \tag{11}$$

where, in (11), ϵ is the perturbation parameter and $0 < \epsilon \ll 1$ [1, 3], while R gives the perturbation terms. In presence of perturbation terms, the momentum and the energy of the soliton do not stay conserved. Instead, they undergo adiabatic changes that lead to the adiabatic deformation of the soliton amplitude, width and a slow change in the velocity [3, 4]. Using (7), the law of adiabatic deformation of the soliton energy is given by [1-4]

$$\frac{dE}{dt} = 3\epsilon \int_{-\infty}^{\infty} \left(q^3 R + 2qR \right) dx \tag{12}$$

while the adiabatic law of change of the velocity of the soliton is given by [1-4]

$$v = 1 - \frac{aA}{3} + \frac{\epsilon}{W} \int_{-\infty}^{\infty} xR dx \tag{13}$$

3.1 Examples

In this paper, the perturbation terms that are going to be considered are

$$\begin{aligned} R = {} & \alpha q + \beta q_{xx} + \gamma q_x q_{xx} + \delta q^m q_x + \lambda q q_{xxx} + \nu q q_x q_{xx} \\ & + \sigma q_x^3 + \xi q_x q_{xxxx} + \eta q_{xx} q_{xxx} + \rho q_{xxxx} + \psi q_{xxxxx} + \kappa q q_{xxxxx} \end{aligned} \tag{14}$$

So, the perturbed gKdV equation that is going to be considered in this paper is

$$q_t + q_x - aqq_x - bq_{xxt} = \epsilon\left[\alpha q + \beta q_{xx} + \gamma q_x q_{xx} + \delta q^m q_x + \lambda q q_{xxx} + \nu q q_x q_{xx} + \sigma q_x^3 + \xi q_x q_{xxxx} + \eta q_{xx} q_{xxx} + \rho q_{xxxx} + \psi q_{xxxxx} + \kappa q q_{xxxxx}\right] \quad (15)$$

The perturbation terms due to α and β are dissipative coefficients. The coefficient of δ is the higher nonlinear dispersion while the coefficient of ψ represents the higher spatial dispersion. In (14), m is a positive integer and $1 \leq m \leq 4$. The term with the coefficient of ρ will will provide the higher stabilizing term and must therefore be taken into account while ψ is the coefficient of higher order dispersion. The remaining coefficients appear in the context of Whitham hierarchy [7].

In presence of these perturbation terms, the adiabatic variation of the energy of the soliton is given by

$$\frac{dE}{dt} = \frac{8\epsilon}{385}\frac{A^2}{B}\left[11\left(35\alpha - 27\beta B^2 + 80\rho B^4\right) + 4\left(35\alpha A^2 - 44\beta B^2 + 624\rho A^2 B^4\right)\right] \quad (16)$$

The law of the change of velocity for the given perturbation terms in (14) is

$$v = 1 - \frac{aA}{3} - \epsilon\left[\frac{m\delta A^m}{(m+1)(2m+1)}\frac{\Gamma\left(\frac{1}{2}\right)\Gamma(m)}{\Gamma\left(m+\frac{1}{2}\right)} \frac{A}{315}\left\{3\left(7\gamma - 14\lambda - 15\xi + 5\eta + 25\kappa\right) + 2\nu A\right\}\right] \quad (17)$$

4 Conclusions

In this paper, soliton perturbation theory is used to study the peturbed sRLW equation. This theory is used to establish the adiabatic parameter dynamics of the soliton energy. Also, it is shown that the velocity undergoes a slow change due to these perturbation terms. In future, it is possible to extend these perturbation terms to include other perturbation terms that are non-local ones too. The quasi-stationary aspects of the perturbed soliton in presence of such perturbation terms will be studied and reported in future publications.

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