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Smoking Model with Cravings to Smoke

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Abstract

Among smokers, often the desire to quit smoking arise. A large number of smokers attempt to quit, but only a fraction of them are successful. While the health and monetary benefits are inarguable. The huge amount of stress associated with smoking, a combination of powerful cravings, sleep and mood disturbances and cognitive changes often stymie even the most motivated of individuals. In this paper, a mathematical model is proposed and analyzed in which the temporary quitters turn back to smoking by powerful cravings for smoking. We have discussed the qualitative behavior of the model and numerical simulations are carried out to support the analytical results.

Keywords: Smoking, Cravings, local stability, global stability

1 Introduction

Smoking has been becoming a crucial public health peril globally since the onset of tobacco in 6000 BC [1]. It is a widespread risk factor for many killing diseases like tuberculosis, chronic lung disease, emphysema, cardiovascular disease and stroke etc. Approximately, a fifth of total population of world smokes tobacco [2]. Smoking is renowned to be the immense cause of both treatable and premature worldwide. According to World Health Organization (WHO), smoking-associated diseases are cause of nearly 5 million deaths annually all over the world and this figure is expected to double by 2025 [3]. Despite of the fact that smoking is extensively recognized as a treatable cause of death and total number of new smokers rise, intrusive tobacco control can halt a large number of deaths from smoking. In order to suppress smoking especially at early ages, persistent teaching of health peril associated to smoking is recommended.

Mathematical models are used to interpret the increase in escalation of smoking and to anticipate the impact of smokers on the society so that the number of smokers can be cut down. In 2000, a simple mathematical model for giving up smoking by C. Castillo-Garsow [4]. Later, a generic epidemological model to elaborate the dynamics of tobacco use in which the effect of peer pressure, relapse, counseling and treatment were took under consideration. In their model, the total population was partitioned into three classes: Non-smokers, smokers and quitters. Further, Sharomi and Gumel [5] extended this model in which the class of smokers who temporarily quit smoking denoted by Q_t was added. They observed the global stability of SFE whenever the threshold parameter number is less than one. Zaman [6] presented and investigated a mathematical model with occasional smokers class. Din et al. [7] extended a mathematical model presented by Zeb et al. [8]. They analyzed the qualitative behavior of a mathematical model in which the total population is partitioned into four compartments. In 2014, Zainab Alkhudhari et al. [3] extended the model [4, 5], and investigated that mathematical model in which the total population was break down into four compartments: Potential smokers (P), smokers (S), temporary quitters (Q_t) and permanent quitters (Q_p). They studied the impact of smokers (S) on temporary quitters (Q_t).

The paper is arranged as follows. In section 2, we formulated the mathematical model along with its flow diagram. Smoking generation number and existence of SFE as well as SPE is investigated in section 3. In the next section, stability of equilibria is found by constructing the Lyapunov function and proving that the system has no homoclinic loops, periodic solutions, and oriented phase polygons in the interior of invariant region. In last section, discussions and simulations are carried out.

2 Formulation of the Model

Suppose the size of total population at time t be symbolized by N(t). We have divided N(t) into four compartments which are: potential smokers ($\breve{P}(t)$), Smokers ($\breve{S}(t)$), Smokers who temporarily quit smoking ($\breve{Q}_t$) and the smokers who permanently quit smoking ($\breve{Q}_p(t)$). Thus $N(t) = \breve{P}(t) + \breve{S}(t) + \breve{Q}_t(t) + \breve{Q}_p(t)$. The model is presented in the following flow diagram:

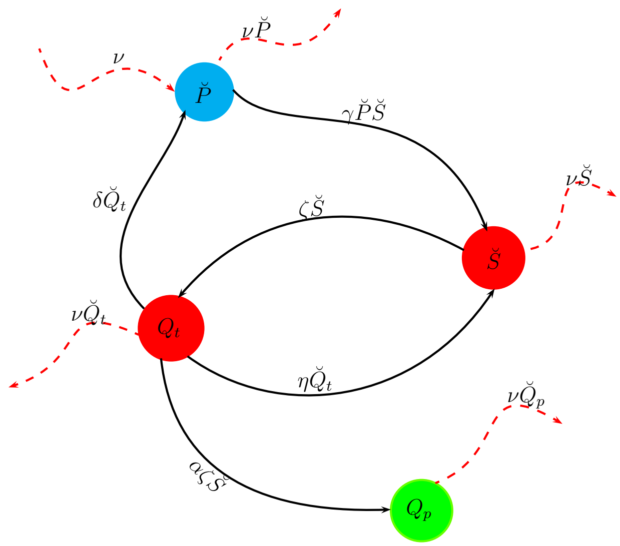


Figure 1: Flow diagram of Model

The above model is given by the following system of differential equations.

$$\begin{aligned}
\frac{d\breve{P}}{dt} &= \nu - \nu\breve{P} - \gamma\breve{P}\breve{S} + \delta\breve{Q}_t, \\
\frac{d\breve{S}}{dt} &= -(\nu+\zeta)\breve{S} + \gamma\breve{P}\breve{S} + \eta\breve{Q}_t, \\
\frac{d\breve{Q}_t}{dt} &= -(\nu+\eta)\breve{Q}_t + \zeta(1-\alpha)\breve{S} - \delta\breve{Q}_t, \\
\frac{d\breve{Q}_P}{dt} &= -\nu\breve{Q}_P + \alpha\zeta\breve{S}, \qquad (2.1)
\end{aligned}$$

where γ is the transmission rate between $\breve{P}$ and $\breve{S}$, ν represents the rate of natural death, η shows the rate at which $\breve{Q}_t$ return to smoking class, ζ is the rate at which smokers quit smoking, the fraction of smokers who temporarily quit smoking (at a rate ζ) is denoted by $(1-\alpha)$ and the remaining fraction of smokers who permanently quit smoking (at rate ζ) is represented by α.
It is assumed that $\breve{P}$ is increased by the recruitment of individuals at a rate ν. In model (2.1), $N(t)$ is constant, so we assume that $\breve{P}(t)+\breve{S}(t)+\breve{Q}_t(t)+\breve{Q}_P(t) = 1$. As, the variable $\breve{Q}_p$ of model (2.1) is not in any of the first three equations, so we consider the following subsystem:

$$\begin{aligned}\frac{d\breve{P}}{dt} &= \nu - \nu\breve{P} - \gamma\breve{P}\breve{S} + \delta\breve{Q}_t,\\ \frac{d\breve{S}}{dt} &= -(\nu+\zeta)\breve{S} + \gamma\breve{P}\breve{S} + \eta\breve{Q}_t,\\ \frac{d\breve{Q}_t}{dt} &= -(\nu+\eta)\breve{Q}_t + \zeta(1-\alpha)\breve{S} - \delta\breve{Q}_t. \end{aligned} \tag{2.2}$$

The positive invariant region for system (2.2) is
$\Omega = \{(\breve{P}, \breve{S}, \breve{Q}_t) : \breve{P} + \breve{S} + \breve{Q}_t \leq 1, \breve{P} > 0, \breve{S} \geq 0, \breve{Q}_t \geq 0\}$.

3 Existence of Equilibria and Local Stability

The smoking-free equilibrium (SFE) of the system (2.2) is $E_0 = (1, 0, 0)$. Using the method of next generation matrix proposed by Van den Driessche and Watmough [9], smoking generation number R_0 is given by

$$R_0 = \frac{\gamma(\eta+\delta+\nu)}{\zeta\delta + \eta\nu + \zeta\nu + \delta\nu + \nu^2 + \eta\zeta\alpha}.$$

Thus we have following theorem

Theorem 3.1. *If $R_0 < 1$, then the smoke-free equilibrium (SFE) E_0 of the model (3.1) is (LAS) and if $R_0 > 1$, then SFE is unstable.*

By setting the equations of the system (2.2) equal to zero, we obtain smoke present equilibrium SPE $E^* = (\breve{P}^*, \breve{S}^*, \breve{Q}_t^*)$, where

$$\breve{P}^* = \frac{\nu + \delta\breve{Q}_t^*}{\nu + \gamma\breve{S}^*},\ \breve{Q}_t^* = \frac{\zeta(1-\alpha)\breve{S}^*}{\nu+\eta+\delta} \text{ and } \breve{S}^* = \frac{\nu(\eta+\nu+\delta)(R_\circ - 1)}{R_\circ(\nu^2 + \nu(\eta+\delta+\nu) + \zeta\alpha(\eta+\delta))}. \tag{3.1}$$

We note that whenever the smoking generation number is less than unity, there is no smoking-present equilibrium and when the smoking generation number exceeds unity, there exists unique SPE $E^* = (\breve{P}^*, \breve{S}^*, \breve{Q}_t^*)$, where $\breve{P}^*$, $\breve{S}^*$ and $\breve{Q}_t^*$ are given in (3.1).
We find the local stability of E^*. The lemma given below, stated and proved by McCluskey and Van den Driessche [10] to prove the local stability of E^*.

Lemma 3.2. *Consider a real matrix M of order 3×3. If $tr(M)$, $det(M)$ and $det(M^{[2]})$ are less than zero, then all the eigenvalues of M have negative real parts.*

Linearizing the model (2.2) at E^*, we obtain

$$J(E^*) = \begin{bmatrix} -(\nu+\gamma\breve{S}^*) & \gamma\breve{P}^* & \delta \\ \gamma\breve{S}^* & \frac{-\eta\breve{Q}_t^*}{\breve{S}^*} & \eta \\ 0 & \zeta(1-\alpha) & -(\nu+\eta+\delta) \end{bmatrix}.$$

The second additive compound matrix is given by

$$J^{[2]}(E^*) = \begin{bmatrix} -(\nu+\gamma\breve{S}^*)+\frac{\eta\breve{Q}_t^*}{\breve{S}^*} & \eta & -\delta \\ \zeta(1-\alpha) & -(2\nu+\gamma\breve{S}^*+\eta+\delta) & -\gamma\breve{P}^* \\ 0 & \gamma\breve{S}^* & -(\frac{\eta\breve{Q}_t{}^*}{\breve{S}^*}+\nu+\eta+\delta) \end{bmatrix}.$$

$$\begin{aligned}
tr(J(E^*)) &= -(\nu+\gamma\breve{S}^*)-\frac{\eta\breve{Q}_t^*}{\breve{S}^*}-(\nu+\eta+\delta) \\
&< 0. \\
det(J(E^*)) &= -\frac{\nu\eta\breve{Q}_t^*}{\breve{S}^*}(\nu+\eta+\delta)+\nu\eta\zeta(1-\alpha)-\gamma\eta\breve{Q}_t^*(\nu+\eta+\delta)+\gamma\breve{S}^*\eta\zeta(1-\alpha) \\
&\quad -\gamma^2\breve{P}^*\breve{S}^*(\nu+\eta+\delta)+\delta\gamma\breve{S}^*\zeta(1-\alpha) \\
&= \gamma(\nu+\eta)(-\gamma\breve{P}^*\breve{S}^*+\delta\breve{Q}_t^*)+\gamma\delta(-\gamma\breve{P}^*\breve{S}^*+\delta\breve{Q}_t^*) \\
&= \gamma\nu(\nu+\eta+\delta)(-1+\breve{P}^*) \\
&< 0. \\
det(J^{[2]}(E^*)) &= (\frac{\eta\breve{Q}_t^*}{\breve{S}^*}+(\nu+\eta+\delta))[-(\nu+\gamma\breve{S}^*+\frac{\eta\breve{Q}_t^*}{\breve{S}^*})(2\nu+\gamma\breve{S}^*+\eta+\delta)+\zeta\eta(1-\alpha)] \\
&\quad -\zeta\gamma\delta\breve{S}^*(1-\alpha)-\gamma^2\breve{P}^*\breve{S}^*(\nu+\gamma\breve{S}^*+\frac{\eta\breve{Q}_t^*}{\breve{S}^*}) \\
&= -(\frac{\eta\breve{Q}_t^*}{\breve{S}^*}+(\nu+\eta+\delta))[(2\nu+\delta+\eta)(\nu+\gamma\breve{S}^*)(\frac{\eta\breve{Q}_t^*}{\breve{S}^*})+(\nu+\gamma\breve{S}^*)\gamma\breve{S}^* \\
&+ \eta\gamma\breve{Q}_t^*+\eta\zeta(1-\alpha)+\zeta\gamma\delta\breve{S}^*(1-\alpha)]-((\nu+\gamma\breve{S}^*+\frac{\eta\breve{Q}_t^*}{\breve{S}^*})\gamma^2\breve{P}^*\breve{S}^*) \\
&< 0.
\end{aligned}$$

Hence, by above lemma E^* is LAS .

4 Global Stability

The global stability of E_0 has been proved under the condition $\gamma \leq \zeta\alpha$. It is noted that $\breve{P} < 1$. Suppose the Lyapunov function as under:

$$\begin{aligned} L &= \breve{S} + \breve{Q}_t, \\ \frac{dL}{dt} &= -(\nu+\zeta)\breve{S} + \gamma\breve{P}\breve{S} + \eta\breve{Q}_t - (\nu+\eta)\breve{Q}_t + \zeta(1-\alpha)\breve{S} - \delta\breve{Q}_t \\ &= -\nu\breve{S} + \gamma\breve{P}\breve{S} - \nu\breve{Q}_t - \zeta\alpha\breve{S} - \delta\breve{Q}_t \\ &< -\nu(\breve{S} + \breve{Q}_t) + \breve{S}(\gamma - \zeta\alpha) - \delta\breve{Q}_t \leq 0. \end{aligned}$$

We have $\frac{dL}{dt} < 0$ for $\gamma \leq \zeta\alpha$ whereas, $\frac{dL}{dt} = 0$ only if $\breve{S} = 0$ and $\breve{Q}_t = 0$. Hence, by Laselle's Invariance Principle [11], all the roots to the equations of the system (2.2) having initial conditions in Ω approaches E_0 as t approaches infinity. Hence, we have the result given below:

Theorem 4.1. *The smoke-free equilibrium E_0 is GAS in Ω, provided $\gamma \leq \zeta\alpha$.*

Now, we determine the global stability of SPE E^* by applying the following theorem given in [12, 13] by proving that inside the invariant region, model (2.2) has no homoclinic loops, periodic solutions and oriented phase polygons.

Theorem 4.2. *Consider a piece-wise smooth vector field $\mathbf{g}(\breve{P}, \breve{S}, \breve{Q}_t) = \{g_1(\breve{P}, \breve{S}, \breve{Q}_t), g_2(\breve{P}, \breve{S}, \breve{Q}_t), g_3(\breve{P}, \breve{S}, \breve{Q}_t)\}$ on Ω^* that satisfies the conditions $(Curl\ \mathbf{g}).\overrightarrow{n} < 0$, $\mathbf{g}.\mathbf{f} = 0$ inside Ω^*, where $\mathbf{f} = (\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3)$ is a Lipschitz continuous field inside Ω^*, $\overrightarrow{n}$ is the normal vector to Ω^* and $(Curl\ \mathbf{g}) = (\frac{\partial g_3}{\partial \breve{S}} - \frac{\partial g_2}{\partial \breve{Q}_t})\hat{i} - (\frac{\partial g_3}{\partial \breve{P}} - \frac{\partial g_1}{\partial \breve{Q}_t})\hat{j} + (\frac{\partial g_2}{\partial \breve{P}} - \frac{\partial g_1}{\partial \breve{S}})\hat{k}$. Then, the system of differential equations $\breve{P} = \mathrm{f}_1, \breve{S} = \mathrm{f}_2, \breve{Q}_t = \mathrm{f}_3$ has no homoclinic loops, periodic solutions and oriented phase polygons inside Ω^*.*

Proof: Let $\Omega^* = \{(\breve{P}, \breve{S}, \breve{Q}_t) : \breve{P} + (\frac{nu+\zeta\alpha}{\nu})\breve{S} + \breve{Q}_t = 1, \breve{P} > 0, \breve{S} \geq 0, \breve{Q}_t \geq 0\}$. It can easily proved that Ω^* is a subset of Ω, Ω^* is positively invariant and endemic equilibrium E^* belongs to Ω^*. Let f_1 , f_2 and f_3 represent the right hand sides of equations in the model (2.2) respectively. Using $\breve{P} + (\frac{nu+\zeta\alpha}{\nu})\breve{S} + \breve{Q}_t = 1$ to write $\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3$ in the equivalent forms, we get

$$\begin{aligned}
\mathrm{f}_1(\breve{P},\breve{S}) &= \nu-\nu\breve{P}-\gamma\breve{P}\breve{S}+\delta\breve{Q}_t,\\
\mathrm{f}_1(\breve{P},\breve{Q}_t) &= \nu-\nu\breve{P}-\gamma\breve{P}[\frac{\nu}{\nu+\zeta\alpha}(1-\breve{P}-\breve{Q}_t)]+\delta\breve{Q}_t,\\
\mathrm{f}_2(\breve{P},\breve{S}) &= -(\nu+\zeta)\breve{S}+\gamma\breve{P}\breve{S}+\eta\breve{Q}_t\\
&= -(\nu+\zeta)\breve{S}+\gamma\breve{P}\breve{S}+\eta[1-\breve{P}-(\frac{\nu+\zeta\alpha}{\nu})\breve{S}],\\
\mathrm{f}_2(\breve{S},\breve{Q}_t) &= -(\nu+\zeta)\breve{S}+\gamma[1-\breve{Q}_t-\frac{\nu}{(\nu+\zeta\alpha)}\breve{S}]\breve{S}+\eta\breve{Q}_t,\\
\mathrm{f}_3(\breve{P},\breve{Q}_t) &= -(\nu+\eta)\breve{Q}_t+\zeta(1-\alpha)[\frac{\nu}{(\nu+\zeta\alpha)}(1-\breve{P}-\breve{Q}_t)]-\delta\breve{Q}_t,\\
\mathrm{f}_3(\breve{S},\breve{Q}_t) &= -(\nu+\eta)\breve{Q}_t+\zeta(1-\alpha)\breve{S}-\delta\breve{Q}_t.
\end{aligned}$$

Suppose, $\mathbf{g}=(g_1,g_2,g_3)$ be a vector field such that:

$$\begin{aligned}
g_1 &= \frac{f_3(\breve{P},\breve{Q}_t)}{\breve{P}\breve{Q}_t}-\frac{\mathrm{f}_2(\breve{P},\breve{S})}{\breve{P}\breve{S}}\\
&= \frac{-(\nu+\eta)\breve{Q}_t}{\breve{P}\breve{Q}_t}+\frac{\zeta(1-\alpha)}{\breve{P}\breve{Q}_t}[\frac{\nu}{((\nu+\zeta\alpha))}(1-\breve{P}-\breve{Q}_t)]\\
&- \frac{\delta\breve{Q}_t}{\breve{P}\breve{Q}_t}+\frac{(\nu+\zeta)\breve{S}}{\breve{P}\breve{S}}-\frac{\gamma\breve{P}\breve{S}}{\breve{P}\breve{S}}-\frac{\eta}{\breve{P}\breve{S}}[1-\breve{P}-\frac{(\nu+\zeta\alpha)}{\nu}\breve{S}]\\
&= -\frac{\eta}{\breve{P}}+\frac{\zeta\nu(1-\alpha)}{\breve{P}\breve{Q}_t(\nu+\zeta\alpha)}-\frac{\nu\zeta(1-\alpha)}{\breve{Q}_t(\nu+\zeta\alpha)}-\frac{\zeta(1-\alpha)\nu}{\breve{P}(\nu+\zeta\alpha)}\\
&- \frac{\delta}{\breve{P}}+\frac{\zeta}{\breve{P}}-\gamma-\frac{\eta}{\breve{P}\breve{S}}+\frac{\eta}{\breve{S}}+\frac{\eta(\nu+\zeta\alpha)}{\breve{P}\nu},\\
g_2 &= \frac{\mathrm{f}_1(\breve{P},\breve{S})}{\breve{P}\breve{S}}-\frac{\mathrm{f}_3(\breve{S},\breve{Q}_t)}{\breve{S}\breve{Q}_t}\\
&= \frac{\nu}{\breve{P}\breve{S}}-\gamma+\frac{\delta\breve{Q}_t}{\breve{P}\breve{S}}+\frac{\eta}{\breve{S}}-\frac{\zeta(1-\alpha)}{\breve{Q}_t}+\frac{\delta}{\breve{S}},\\
g_3 &= \frac{\mathrm{f}_2(\breve{S},\breve{Q}_t)}{\breve{S}\breve{Q}_t}-\frac{\mathrm{f}_1(\breve{P},\breve{Q}_t)}{\breve{P}\breve{Q}_t}\\
&= -\frac{\zeta}{\breve{Q}_t}+\frac{\gamma}{\breve{Q}_t}-\gamma-\frac{\gamma\breve{S}(\nu+\zeta\alpha)}{\breve{Q}_t\nu}+\frac{\eta}{\breve{S}}-\frac{\nu}{\breve{P}\breve{Q}_t}\\
&+ \frac{\gamma\nu}{\breve{Q}_t(\nu+\zeta\alpha)}-\frac{\gamma\breve{P}\nu}{(\nu+\zeta\alpha)\breve{Q}_t}-\frac{\gamma\nu}{\nu+\zeta\alpha}-\frac{\delta}{\breve{P}}.
\end{aligned}$$

As, the alternate forms of $\mathrm{f}_1,\mathrm{f}_2$ and f_3 are equivalent in Ω^*, so

$$\mathbf{g}.\mathbf{f}=g_1\mathrm{f}_1+g_2\mathrm{f}_2+g_3\mathrm{f}_3=0.$$

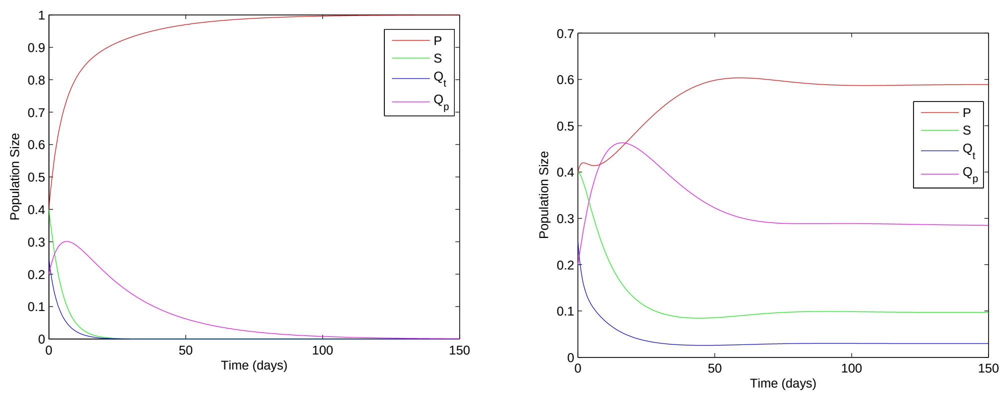
Now using normal vector $\overrightarrow{n}$, where $\overrightarrow{n} = (1, \frac{\nu+\zeta\alpha}{nu}, 1)$ to Ω^*, we have

$$
\begin{aligned}
(Curl\ \mathbf{g}) &= -\frac{\gamma\zeta\alpha}{\nu\breve{Q}_t} - \frac{\delta}{\breve{P}\breve{S}} - \frac{\nu+\zeta\alpha}{\breve{P}^2\breve{Q}_t} - \frac{\delta(\nu+\zeta\alpha)}{\breve{P}^2\nu} - \frac{\zeta(1-\alpha)}{\breve{P}\breve{Q}_t^2} - \frac{\nu}{\breve{P}^2\breve{S}} - \frac{\delta\breve{Q}_t}{\breve{P}^2\breve{S}} - \frac{\eta}{\breve{P}\breve{S}^2} \\
&< 0.
\end{aligned}
$$

So, the system (2.2) has no homoclinic loops, periodic solutions and oriented phase polygons in the interior of Ω^*. Consequently, E^* is GAS in the interior of Ω^*.

5 Conclusion

This paper deals with the mathematical model of smoking to investigate the smoking dynamics in a population with the effect of powerful craving for smoking on temporary quitters $\breve{Q}_t$. Our main results present the global dynamics of the nonlinear system. It is observed that when the threshold parameter (smoking generation number) is less than unity then smoking-free equilibrium E_0 is globally asymptotically stable under the condition $\gamma \leq \zeta\alpha$ and the smoking present equilibrium is globally asymptotically stable whenever the threshold parameter exceeds unity. Numerical simulations carried out for system (2.2) show that the disease "dies out" when $R_0 < 1$ and the disease persists at an "endemic" level when $R_0 > 1$, figs.(2a,2b). Here the question arise that if the inequality $\gamma \leq \zeta\alpha$ is disobeyed, what will be the dynamics of system (2.2)? Fig. 5 numerically shows that with different initial values the disease still can "die out" even if the above conditions are not satisfied by the parameters, which shows that for the global asymptotic stability of the "disease-free" equilibrium E_0 there are weaker conditions.



(a) The Population approaches disease - free equilibrium for $R_0 < 1$ whenever $\gamma = 0.002 < \zeta\alpha = 0.12$

(b) The Population approaches EE for $R_0 > 1$

Figure 2: The Population approaches disease-free equilibrium and endemic equilibrium.

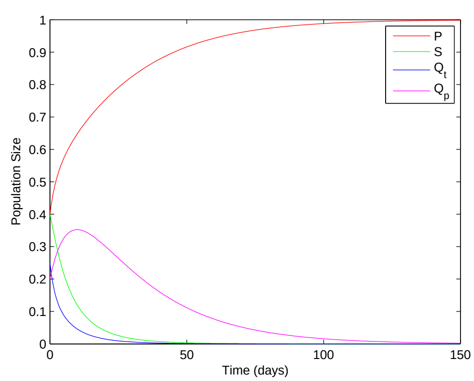


Figure 3: The Population approaches disease-free equilibrium for $R_0 < 1$ whenever $\gamma = 0.45 > \zeta\alpha = 0.12$

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