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Overview of the Algorithms for Solving the P-Median Facility Location Problems

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Abstract

The *p*-median problem is defined as an optimization problem that is well known in the OR literature and has been extensively applied to, facility location. This paper reviews summarize of the literature on solution algorithm for the p-median problem. The concentrate is on the different proposed algorithms as well as exact algorithms, and heuristic or metaheuristic algorithms.

Keyword: p-median problem; exact algorithms; heuristic; metaheuristic

1. Introduction

The *p*-median problem is one of several problems considered fundamental for the field of combinatorial optimization. From a computational perspective, many approaches have been suggested to solve the P-Median Facility Location Problems **(**PMFLPs). In this study, briefly are explained some algorithms that they used for solving PMFLPs**.**

2. Exact Methods

Ceselli [1] presented two exact algorithms, branch-and-bound and branch-and-price technique for solving the capacitated variation of the p-median problem. In this algorithm, the Lagrangian relaxation and subgradient optimization are utilized for branch-and-bound technique and column generation is used for the branch-and-price algorithm. The author analysed and compared some performance details, showed how a fine tuning can improve the performances of their technique.

Canos et al.[2] proposed an exact algorithm for the fuzzy p-median problem. They considered a fuzzy set of restrictions such that the decisionmaker can select partially feasible solutions which partially cover the demands as they extensively decrease the cost. The researchers, proposed an alternative enumeration algorithm for solving the problem which is based on Hakimi's seminal papers [3, 4], and it was appropriate when the number of vertices is not too large. Neebe [5] considered a branch and bound algorithm for the p-median transportation problems and for providing lower bound the Lagrangian relaxation is used.

Järvinen et al. [6] constructed a branch-and-bound algorithm for seeking the PMFLPs. The authors showed how the vertex-substitution technique can lead to premature convergence and consequently local optimum, and give a heuristic method for finding a good initial solution for this technique. In addition, they studied four techniques namely, branch-and-bound, branch-and-bound without backtracking, substitution so that the initial solution was formed by the first p vertex heuristic, and substitution with a heuristic initial solution. Consequently Järvinen et al. [6] concluded when $p \rightarrow n$ increased the competences of the branch-and-bound with backtracking, substitution, and substitution with a heuristic initial solution, but the heuristic technique itself behaved in the opposite way.

3. Heuristic Methods

Hribar and [7] proposed a heuristic restricts the size of the state space of a dynamic programming for the *p*-median problem. In this algorithm a cross between the polynomial time greedy algorithm for the *p*-median problem and a non-polynomial time dynamic programming algorithm is provided. Several solution found by this polynomial-time technique and determined how often particular points were used in solutions. Pizzolato [8] considered a heuristic algorithm for the p-median problem designed for large weighted graphs. They started their algorithm with an initial collection of *p* trees and then reshape them iteratively through a root interchange technique. These processes were fast and provided a near optimal solution. In addition they applied some techniques for improving the final solution.

Taillard [9] utilized clustering technique to solving *p*-median problem. They applied candidate list search (CLS), local optimization (LOPT) and decomposition/recombination (DEC). The CLS starting by the alternate heuristic technique where introduced by Maranzana [10] and obtaining a locally optimal solution. Regarding the solutions, in the CLS clustering technique, the solution is concerned by elimination a vertex and adding another, similar to vertex substitution. In this case, the new solution is selected only if it is better than the initial one. In the LOPT clustering technique, a median and some nearby medians are chosen and then are generated and solved corresponding sub problems. Also the author utilized LOPT technique in DEC clustering method for finding a good solution in the overall problem.

Resende and Werneck [11] introduced a multistart hybrid heuristic that combined elements of several metaheuristics as Greedy Randomized

Adaptive Search Procedure (GRASP). In this process for each generation, a greedy randomized algorithm is applied by a local search technique. The author utilized the idea of path-relinking from tabu search and scatter search for storing a group of the best solutions of previous generation. This algorithm was useful from strategies that improve diversity: selecting solutions from the pool in a biased way, returning a local minimum in the path if no improving solution is found, and applying local search to the solution returned.

4. Metaheuristic Methods

The Population Based Hybrid Metaheuristic (PBS) algorithm uses a Genetic Algorithm (GA) based metaheuristic, principally based on cut and paste crossover operators, to generate new chromosomes for a hybrid local search. The PBS algorithm is based on the fundamental concept of increasing a population based metaheuristic with a Hybrid Local Search (HLS). The metaheuristic algorithm is applied for providing a wide range of chromosomes for the HLS while the HLS is utilized to effectively explore regions that are relatively close to these chromosomes.

Rosing et al. [12] introduced the modelling context of the earlier Heuristic Concentration (HC) research by focusing on its application to the p-median problem. This algorithm worked as follow. In stage one of HC, systematic or random starts of an interchange heuristic are used to develop a data base of desirable facility sites. The elements of this data base, consist of a collection of the solution components identified in the multiple heuristic runs of stage one. Stage two of HC limits the set of potential facility locations to this set of good sites and resolves the model.

Simulated Annealing (SA) is other method that was used for solving PMFLPs. A basic SA heuristic for p-median location problem has been proposed by Murray and Church [13]. This SA begins by initializing input parameters, generating a random *p* facility solution, and calculating the objective value, O-old. Following this, a facility not currently in the p-facility solution is randomly selected and a facility in the current p-facility solution is also randomly selected. The facility site is exchanged to create a trial solution for which the objective function value of this solution is calculated, O-new, and the parameter Count is incremented by one. If the objective value for the trial solution is an improvement over the previous solution's objective value, the new solution is selected as the current solution. That is, the facility exchange is accepted and Count is reset to zero. If the new solution is not an improvement, Count is evaluated. If Count exceeds a specified limit, N, the temperature, T, is cooled by a factor *p* and Count is reset to zero. If Count is less then N, the new solution objective value (O-new) is evaluated as a function of the previous solution and the temperature (T).

Chiyoshi and Galṽao [14] presented a SA for PMFLPs. This algorithm uses combines elements of the vertex substitution method of Teitz and Bart with the general methodology of SA. The cooling schedule adopted incorporates the notion of temperature adjustments rather than just temperature reductions.

Hansen et al. [15] presented the Variable Neighbourhood Search (VNS) for solving PMFLPS. The VNS is a metaheuristic algorithm which combines local search with systematic changes of neighbourhood in the descent and escape from local optimum phases. The Variable Neighbourhood Decomposition Search (VNDS) technique follows a basic VNS method within a consecutive approximations decomposition technique. In regarding to finding solution *x*, all but *k* variables are fixed in the local search phase and then all possible such fixations define a neighbourhood $N_k(x)$. The VNDS algorithm is started with a random solution $x' \in N_1(x)$ as in VNS. In the local search implement the whole solution space S with x' are considered as a starting point whereas the VNDS solve a one-dimensional problem in the space of the unfixed variable that has been chosen at random and then return a new value for this variable into the solution and compute the objective function value [15].

Approximation algorithms and metaheuristics are the dominant methods studied as the literature of the past few years indicates. One of the most popular metaheuristics is the GA. The first application of the GAs to the *p*-median problem was furnished by Hosage and Goodchild [16]. The researchers realized weaknesses and strengths of this application, indicating that it has the potential to be captured in local optimum while resolving location problems.

Chiou and Lan [17] employed cluster seed points to work out the *p*-median problem. A GA was employed for selecting the most appropriate cluster seeds (medians) while the rest vertices were allotted to clusters based on their resemblance with the cluster seeds or their capability of improving the objective function.

Bozkaya et al. [18] created and introduced a GA which models solutions using chromosomes. In this model, each chromosome gene is index of a *p*-median vertex. Three crossover operators were introduced and examined. Alp et al. [19] described some simple and rapid GA which modelled vertex indices in the solution as chromosome genes with the fitness function serving as the objective function. While the traditional genetic techniques utilize crossover approaches, this technique created a coalition/blend of the chromosomes of the parents producing an unfeasible solution with $m > p$ genes. A greedy deletion heuristic was used to reduce the gene numbers until p genes remained and no mutation operator was utilized.

Correa et al. [20] built a GA for the capacitated p-median problem. The algorithm assigned vertices to the median of closest proximity that was not filled already. And a novel genetic operator, known as heuristic hypermutation, was introduced. This operator improved fitness of a specific percentage of genes. Results of the algorithm computations, with and without use of heuristic hypermutation, were then compared with the outputs of a tabu search heuristic. A Consistency measure that assists GAs in making better conjectures when selecting solutions for the crossover stage was presented by Dvorett [21]. This technique considers applicant parents together rather than selecting the two parents separately.

Lim and Xu [22] used the fixed-length subset GA as candidate solution. However, substantial computational results were not presented. Nonetheless, they claimed that their technique outperforms the traditional GA and that it is capable of finding solutions very close to optimal for most problems.

A constructive GA based on a dynamic population was presented by Lorena and [23]. In this algorithm, two different fitness functions were considered and the clustering problems were formulated as bi-objective optimization problems.

A new GA that represents solutions by a collection of indices of the demand points instead of binary string solution chromosome was established and implemented by Moreno et al. [24]. This algorithm was structured as an evaluative algorithm implemented in parallel by using software to generate a parallel virtual machine on a local network.

5. Conclusion

A combinatorial optimisation problem can be considered as the search for a feasible solution with the best objective function value from a finite set of feasible solutions that optimises a given objective function. The PMPs are the important areas of such problems which arise in many diverse practical problems. The heuristic methods are suitable technique for solving these problems because these problems are NP-hard.

The PMFLP is combinatorial optimization problem that discussed in this study. In this problem locates p facilities among n demand points and allocates the demand points to the facilities. The objective is to minimize the total demand-weighted distance between the demand points and the facilities. A brief review of the research on PMFLPs, are presented in this paper.

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