Applied Mathematical Sciences, Vol. 3, 2009, no. 1, 11 - 25

Optimization in an Inventory Model with Reliability Consideration

P.K. Tripathy[1] and M. Pattnaik[*]

[1] P.G. Department of Statistics, Utkal University
Bhubaneswar-751004, Orissa, India
msccompsc@gmail.com

* Department of Business Administration, Utkal University
Bhubaneswar-751004, India
monalisha_1977@yahoo.com

Abstract

This article investigates an instantaneous production plan to obtain an optimal ordering policy wherein demand exceeds supply, all items are subjected to inspection and defective items are discarded. The unit cost of production is inversely related to both process reliability and demand rate. Under reasonable conditions, maximum positive cost savings are generated when the process reliability increases.

Mathematics Subject Classification: 90B05

Keywords: Inventory, Demand, Process Reliability

1. Introduction

The basic objective of inventory control is to reduce investment in inventories and at the same time, ensures that production process does not suffer. Extensive surveys of inventory research have been conducted by Silver(1981),

---

* This paper was presented at the seminar on Information Technology and its relevance on present Higher Education held during Feb 2007.

Urgeletti (1983), Wanger (1980) and Whitin (1954). Two major assumptions in the classical EOQ model are that demand is constant and deterministic and that the unit cost of production is independent of the production order quantity, Hadley et al. (1963) and Silver et al. (1985). Implicitly the model also assumes that items produced are of perfect quality. In reality however, product quality is never perfect, but is the function of the reliability of the production process and quality assurance established to monitor the product quality. Consequently investment in improving the reliability of production process is the key to achieve a high level of production quality. Two sophisticated models have been presented by Rosenblatt et al. (1986) to deal with the EOQ problem with imperfect production which deteriorate over time under different scenarios. Porteus (1986) has discussed the effect of process quality improvement and set up cost reduction on optimal lot sizing. Tapiero et al. (1987) have presented a theoretical framework to examine the tradeoffs between pricing, reliability, design and quality control issues in manufacturing operations. Cheng (1989) studied an inventory with an imperfect production process and quality dependent unit production cost.

This paper is related to more general way of Cheng (1991) and Tripathy et al. (2003). Cheng (1991) who assumed demand to be constant and unit cost of production to increase with increased reliability. Tripathy et al. (2003) considered that the unit cost of production is directly related to reliability and inversely related to the demand rate where demand exceeds supply. Cheng (1989) studied an EOQ model with an imperfect production process and quality dependent unit cost of production. We have studied that unit cost of production is inversely related to both process reliability and demand rate when demand exceeds supply. Inventory models with various quality considerations were considered by different authors.

Until now most attention in inventory theory has been paid to incorporating demand variability. Due to ever increasing demand resulted by better performance, most of these models have been able to produce efficient solutions for optimization problems. This demand has encouraged the researchers to look for a better method to optimize the total cost.

In recent years, different companies have started to recognise that a tradeoff exists between product variety interms of quality and performance of demand with the cost of production. In the absence of a proper quantitative model for the effect of product quality and demand on cost of production, these companies have mainly relied on qualitative judgement. In this paper, we develop and solve a model to analyse the effect of product quality and demand on cost of production.

Assumptions and notations

- Production is instantaneous, Back order is not allowed, Demand for the product exceeds supply.
- All items are subjected to inspection and defective items are discarded and
- The unit cost of production is inversely related to both process reliability and to demand rate and directly related to quality assurance.

According to this general power function is given by

$$P(r,\lambda) = a(1-r)^c \lambda^{-b} \tag{1}$$

where *a, b,* and *c* are non-negative real numbers to be chosen to provide the best fit of the estimated cost function.

S and *H* denote for Set-up cost per production run & Inventory carrying cost per item per unit time respectively.

2. Model

We study the optimal ordering policy with unit cost of production as *P(r,λ)*. The demand *λ* is deterministic and the unit cost of production depends on process reliability *r* and demand rate λ. Consider the case of a company manufacturing a single product for which the demand exceeds supply. The market is able to absorb virtually any quantity of the product rolled out from the production line. This situation is typical of a technologically advanced product entering the growth phase of its life cycle. Since the demand for the product is high, the manufacturer will increase production to meet it, which will result in lower unit cost of production because production overheads are spreaded over the items.

As for the manufacturer of the product, the company uses a production process with specified level of reliability which depends on the number of factors. However, high reliability can only be achieved with additional costs, both fixed and variable, which together will push up the unit cost of production. An example from practice will be given to explain the process reliability mechanism. An electronic company is confronted with the following inventory system. For Nokia Mobile, the company is confronted with perfect quality to reach the unexpected demand in the market. In this situation it is likely that decreasing unit cost of production occurs while process reliability and demand rate increases. Production cost is an important strategy for increasing profit.

As with the classical inventory model, we assume that the production run repeats indefinitely throughout an infinite planning horizon. So we can base our analysis on one typical production run. A level of process reliability *r* means that all the items produced in a production run meet only *r%* of the stipulated quality standard and can be used to meet the demand. It is thus evident that a production run covers the demand over a period of length $\frac{\lambda q}{r}$.

3. Derivation of the Cost Function

$$\text{Total relevant cost per production run} = S + P(r,\lambda)q + \frac{H\lambda q^2}{2r^2} \tag{2}$$

Our objective is to minimize the total relevant cost per unit time subject to constraint that the process reliability of products found to be acceptable in a production run can not exceed 100%. So we wish to minimize

$$Z(r,\lambda,q) = \frac{Total\ \ relevant\ \ cost\ \ per\ \ production\ \ run}{\frac{\lambda q}{r}} \quad \text{for } 0 \le r \le 1 .$$

$$= \frac{r}{\lambda q}\left[S + P(r,\lambda)q + \frac{H\lambda q^2}{2r^2} \right] \tag{3}$$

4. Optimality of the Proposed Criteria

To solve the constrained optimization equation(3), the first partial derivative of *Z(r,λ,q)* with respect to *r,* λ and *q* are set to zero to obtain the necessary optimality conditions:

$$\frac{S}{\lambda q} + a\lambda^{-b-1}\left\{(1-r)^c - rc(1-r)^{c-1}\right\} - \frac{Hq}{2r^2} = 0 \tag{4}$$

$$\frac{-Sr}{\lambda^2 q} - ar(1-r)^c(b+1)\lambda^{-b-2} = 0 \tag{5}$$

$$\frac{-Sr}{\lambda q^2} + \frac{H}{2r} = 0 \tag{6}$$

from which

$$\Rightarrow q = \sqrt{\frac{2S}{H\lambda}} \times r\ ,\ r = \frac{1}{1+c} \text{ and } \lambda = \left[\frac{HS}{2a^2(b+1)^2 r^2 (1-r)^{2C}}\right]^{\frac{-1}{(2b+1)}} \quad (7)\ \&\ (8)$$

Hence,

$$Z(r,\lambda,q) = \sqrt{2SH}\, L^{\frac{1}{2(2b+1)}} + ac^c(1+c)^{-(1+c)} \times L^{\frac{(1+b)}{(2b+1)}} \tag{9}$$

where, $L = \dfrac{HS(1+c)^{2(1+c)}}{2a^2(b+1)^2 c^{2c}}$

Special Case

The results of our analysis can be used by decision makers to qualify how a change in product variety (quality) through demand affects cost of production coupled with the knowledge of how product quality affect sales.

After substituting the stationary point

$$r^* = \frac{1}{1+c}\ ,\ q^* = \sqrt{\frac{2S}{H\lambda^*}} \times \frac{1}{1+c} \text{ and } \lambda^* = \left[\frac{HS(1+c)^{2(1+c)}}{2a^2(b+1)^2 c^{2c}}\right]^{\frac{-1}{(2b+1)}} \quad (10),\ (11)\ \&\ (12)$$

Let

$$A = a\lambda^{*-(b+1)}\left\{-2c(1-r^*)^{c-1} + (c-1)cr^*(1-r^*)^{c-2}\right\} + \frac{Hq^*}{r^{*3}} \tag{13}$$

$$B = \frac{2Sr^*}{\lambda^{*3}q^*} + (b+1)(b+2)ar^*(1-r^*)^c\lambda^{*-(b+3)} \tag{14}$$

$$C = 2H\lambda^* q^* \tag{15}$$

$$D = \frac{-S}{\lambda^{*2}q^*} - a(b+1)\lambda^{*-(b+2)}\left\{(1-r^*)^c - r^*c(1-r^*)^{c-1}\right\} \tag{16}$$

$$E = 2a\lambda^{*-b}r^{*2}\left\{(1-r^*)^c - r^*c(1-r^*)^{c-1}\right\} - 2H\lambda^* q^* \tag{17}$$

$$F = \frac{Sr^*}{\lambda^{*2}q^{*2}} \tag{18}$$

$$G = S + a(b+1)\lambda^{*-b}q^*\left\{(1-r^*)^c - r^*c(1-r^*)^{c-1}\right\} \tag{19}$$

$$I = \frac{-S}{\lambda^* q^{*2}} - \frac{H}{2r^{*2}} \tag{20}$$

$$J = Hq^{*2} \tag{21}$$

Now the sufficient condition for stationary point (r^*, λ^*, q^*) is given by

$$\begin{vmatrix} f_{r^*r^*} & f_{r^*\lambda^*} & f_{r^*q^*} \\ f_{\lambda^*r^*} & f_{\lambda^*\lambda^*} & f_{\lambda^*q^*} \\ f_{q^*r^*} & f_{q^*\lambda^*} & f_{q^*q^*} \end{vmatrix} > 0 \Rightarrow \begin{vmatrix} A & D & E \\ G & B & F \\ I & J & C \end{vmatrix} > 0$$

5. Numerical Example

Example - 1
a=10, b=1, c=1.00, 0.75, 0.50, 0.25, 0.10, 0.05, H=10 and S=100

Example - 2
a=10, b=1, c=1.00, 0.75, 0.50, 0.25, 0.10, 0.05, H=5 and S=100
The tables given below present the optimal values of $r, \lambda, P(r,\lambda), q$ and total cost $Z(r,\lambda,q)$. Also the effect of *a, b* and *c* on $P(r,\lambda)$, q and r are obtained by assuming different values of c as 1.00, 0.75, 0.50, 0.25, 0.10 and 0.05 respectively.

Table 1.1

r	λ	P(r, λ)	q	z(r, λ,q)
c=1.00				
0.1	0.18643	48.27365	1.03573	129.46111
0.2	0.27359	29.23979	1.70997	106.87313
0.3	0.32798	21.34274	2.34267	97.61121
0.4	0.35851	16.73565	2.98758	93.36183
0.5*	0.36840*	13.57207*	3.68402*	92.10071*
0.6	0.35854	11.15607	4.48117	93.35493
0.7	0.32798	9.14689	5.46624	97.61153
0.8	0.27359	7.30994	6.83985	106.87238
0.9	0.18643	5.36382	9.32168	129.46784
0.99	0.04280	2.33632	21.40014	270.20123
c=0.75				
0.1	0.18973	48.70010	1.02668	128.33609
0.2	0.28396	29.78895	1.67847	104.90441
0.3	0.34806	21.98665	2.27407	94.75266
0.4	0.39037	17.46361	2.86309	89.47160
0.5	0.41352	14.37899	3.47724	86.93095
0.6*	0.41766*	12.04251*	4.15195*	86.49897*
0.7	0.40085	10.11232	4.94445	88.29376
0.8	0.35777	8.35924	5.98139	93.45923
0.9	0.27365	6.49834	7.69411	106.86233
0.99	0.09221	3.42935	14.57995	184.09025

Table 1.2

r	λ	P(r, λ)	q	z(r, λ,q)
c=0.50				
0.1	0.19309	49.12957	1.01771	127.21419
0.2	0.29472	30.34810	1.64754	102.97180
0.3	0.36938	22.65018	2.20748	91.97848
0.4	0.42506	18.22306	2.74377	85.74283
0.5	0.46415	15.23414	3.28209	82.05242
0.6	0.48657	12.99806	3.84672	80.14000
0.7*	0.48993*	11.17955*	4.47244*	79.86507*
0.8	0.46784	9.55904	5.23063	81.72873
0.9	0.40166	7.87301	6.35079	88.20547
0.99	0.19866	5.03360	9.93321	125.41930
c=0.25				
0.1	0.19651	49.56283	1.00881	126.10217
0.2	0.30589	30.91768	1.61719	101.07461
0.3	0.39201	23.33332	2.14282	89.28424
0.4	0.46284	19.01542	2.62941	82.16901
0.5	0.52100	16.13991	3.09787	77.44681
0.6	0.56686	14.02937	3.56391	74.24820
0.7	0.59880	12.35941	4.04541	72.24099
0.8*	0.61178*	10.93101*	4.57410*	71.47033*
0.9	0.58955	09.53833	5.24196	72.80491
0.99	0.42800	7.38833	6.76742	85.44728

Table 1.3

r	λ	P(r, λ)	q	z(r, λ,q)
c=0.10				
0.1	0.19860	49.82462	1.00351	125.43961
0.2	0.31279	31.26424	1.59924	99.95234
0.3	0.40624	23.75337	2.10496	87.70668
0.4	0.48709	19.50749	2.56311	80.09733
0.5	0.55839	16.70919	2.99236	74.80905
0.6	0.62125	14.68715	3.40433	70.92359
0.7	0.67541	13.12628	3.80914	68.02050
0.8	0.71860	11.84706	4.22045	65.94454
0.9*	0.74220*	10.70224*	4.67191*	64.88770*
0.99	0.68039	9.27334	5.36745	67.70968
c=0.05				
0.1	0.19929	49.91219	1.00175	125.21953
0.2	0.31512	31.38106	1.59331	99.58213
0.3	0.41110	23.89500	2.09248	87.18684
0.4	0.49546	19.67428	2.54138	79.41829
0.5	0.57144	16.90334	2.95799	73.94979
0.6	0.64052	14.91317	3.35273	69.84871
0.7	0.70307	13.39234	3.73347	66.66919
0.8	0.75821	12.16915	4.10874	64.19915
0.9*	0.80141*	11.12091*	4.49601*	62.44462*
0.99	0.79089	10.04335	4.97839	62.85851

Table 2.1

r	λ	P(r, λ)	q	z(r, λ,q)
c=1.00				
0.1	0.23489	38.31541	1.30495	81.55960
0.2	0.34470	23.20791	2.15443	67.32602
0.3	0.41322	16.93992	2.95160	61.49171
0.4	0.45169	13.28322	3.76414	58.81471
0.5*	0.46416*	10.77209*	4.64157*	58.01950*
0.6	0.45170	8.85541	5.64619	58.81437
0.7	0.41322	7.25990	6.88704	61.49132
0.8	0.34471	5.80192	8.61769	67.32551
0.9	0.23489	4.25726	11.74459	81.55964
0.99	0.05392	1.85434	26.96251	170.21642
c=0.75				
0.1	0.23905	38.65330	1.29354	80.84668
0.2	0.35777	23.64351	2.11474	66.08564
0.3	0.43853	17.45098	2.86516	59.69090
0.4	0.49184	13.86079	3.60725	56.36329
0.5	0.52100	11.41271	4.38107	54.76345
0.6*	0.52622*	9.55809*	5.23111*	54.49069*
0.7	0.50504	8.02616	6.22962	55.62159
0.8	0.45076	6.63469	7.53605	58.87529
0.9	0.34477	5.15775	9.69399	67.31917
0.99	0.11617	2.72187	18.36959	115.96961

Table 2.2

r	λ	P(r, λ)	q	z(r, λ,q)
c=0.50				
0.1	0.24328	38.99418	1.28223	80.13995
0.2	0.37132	24.08728	2.07578	64.86812
0.3	0.46539	17.97746	2.78125	57.94282
0.4	0.53554	14.46358	3.45692	54.01436
0.5	0.58480	12.09134	4.13518	51.68979
0.6	0.61305	10.31653	4.84655	50.48484
0.7*	0.61727*	8.87322*	5.63492*	50.17409*
0.8	0.58944	7.24495	6.59019	51.02163
0.9	0.50605	6.24882	8.00150	55.56598
0.99	0.25030	3.99517	12.51506	79.00992
c=0.25				
0.1	0.24760	39.33756	1.27102	79.43860
0.2	0.38539	24.53938	2.03753	63.67303
0.3	0.49389	18.51983	2.69980	56.24594
0.4	0.58313	15.09265	3.31286	51.76353
0.5	0.65642	12.81029	3.90308	48.78852
0.6	0.71419	11.13518	4.49026	46.77363
0.7	0.75444	9.80967	5.09700	45.50899
0.8*	0.77079*	8.67599*	5.76302*	45.02366*
0.9	0.74279	7.57062	6.60447	45.86441
0.99	0.53926	5.86408	8.52639	53.82815

Table 2.3

r	λ	P(r, λ)	q	z(r, λ,q)
c=0.10				
0.1	0.25022	29.54584	1.26435	79.02201
0.2	0.39409	24.81473	2.01493	62.96661
0.3	0.51183	18.85308	2.65208	55.25178
0.4	0.61370	15.48310	3.22932	50.45816
0.5	0.70353	13.26209	3.77014	47.12675
0.6	0.78272	11.65720	4.28919	44.67908
0.7	0.85096	10.41834	4.79922	42.85023
0.8	0.90538	9.40302	5.31743	41.54247
0.9*	0.93512*	8.49437*	5.88624*	40.87670*
0.99	0.85466	7.38249	6.77277	42.75742
c=0.05				
0.1	0.25110	39.61534	1.26211	78.69943
0.2	0.39703	24.90696	2.00744	62.73241
0.3	0.51795	18.96548	2.63636	54.92428
0.4	0.62424	15.15730	3.20193	49.73668
0.5	0.71997	13.41619	3.72683	46.58546
0.6	0.80700	11.83659	4.22418	44.00194
0.7	0.88581	10.62951	4.70388	41.99896
0.8	0.95528	9.65866	5.17669	40.44295
0.9*	1.00972*	8.82669*	5.66463*	39.33771*
0.99	0.99646	7.97435	6.27239	39.60136

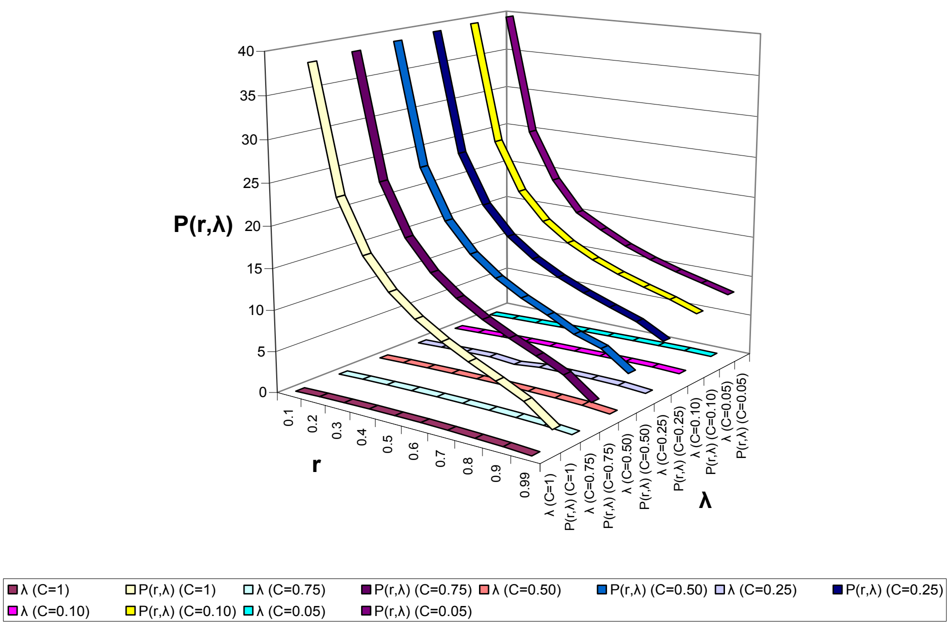

Fig. 1: Mesh plot of a typical unit cost function.

Results and Discussion

It is evident from equation (1) that as $r \rightarrow 1$, $P(r, \lambda) \rightarrow 0$, which substantiates that the process acceptable reliability can never be 100% or that it will be prohibitively not expensive to attain perfect production. Notice that a typical unit cost function $P(r, \lambda)$ displayed in Fig.1 is convex.

The optimal values as found from table 1.1, 1.2, 1.3 and table 2.1, 2.2, 2.3 are as follows:

For $c=1$, $r^*=0.5$, $\lambda^*=0.368403517$, $P^*(r, \lambda)=13.5720$, $q^*=3.684029$ and total cost $Z^*(r, \lambda, q)=92.100714$.

For $c=2$, $r^*=0.5$, $\lambda^*=0.464162$, $P^*(r, \lambda)=10.772090$, $q^*=4.641571$ and total cost $Z^*(r, \lambda, q)=58.0195042$.

Table 3: Sensitive analysis of λ, P(r,λ),q and Z(r,λ,q) w.r.t. a,b &c when H=10

System parameter	% change in the System parameter	% change in λ^*	% change in $P^*(r,\lambda)$	% change in q^*	% change in $Z^*(r,\lambda,q)$
a	+60	0.262074174	11.44714092	4.367902036	109.1975456
	+40	0.200000032	9.9999984	4.9999996	124.999984
	+20	0.125992131	7.937003621	6.299604599	157.4901052
	-20	0.125992131	-7.937003621	6.299604599	94.49407874
	-40	0.200000032	-9.9999984	4.9999996	75.000000
	-60	0.262074174	-11.44714092	4.367902036	65.74118434
b	+60	0.209181235	12.78369412	4.889040566	128.3373144
	+40	0.12737737	11.40074136	6.26525661	170.0569643
	+20	0.056723574	8.875999866	9.388651287	266.0121002
	-20	0.00031999	0.999999375	125.0001953	4062.507813
	-40	$1.84528125 \times 10^{-12}$	0.00010125	1646090.535	60356652.95
	-60	3.125×10^{13}	62500000	0.0000004	0.000018
c	+60	0.443198248	14.88620405	3.35881432	83.97035379
	+40	0.486110905	15.59023415	3.207135583	80.17838787
	+20	0.533178599	16.32756074	3.062306497	76.55766099
	-20	0.641427167	17.90847682	2.791973729	69.79934226
	-40	0.70353333	18.75544269	2.665892689	66.64731657
	-60	0.771652927	19.64246507	2.545505257	63.63763093

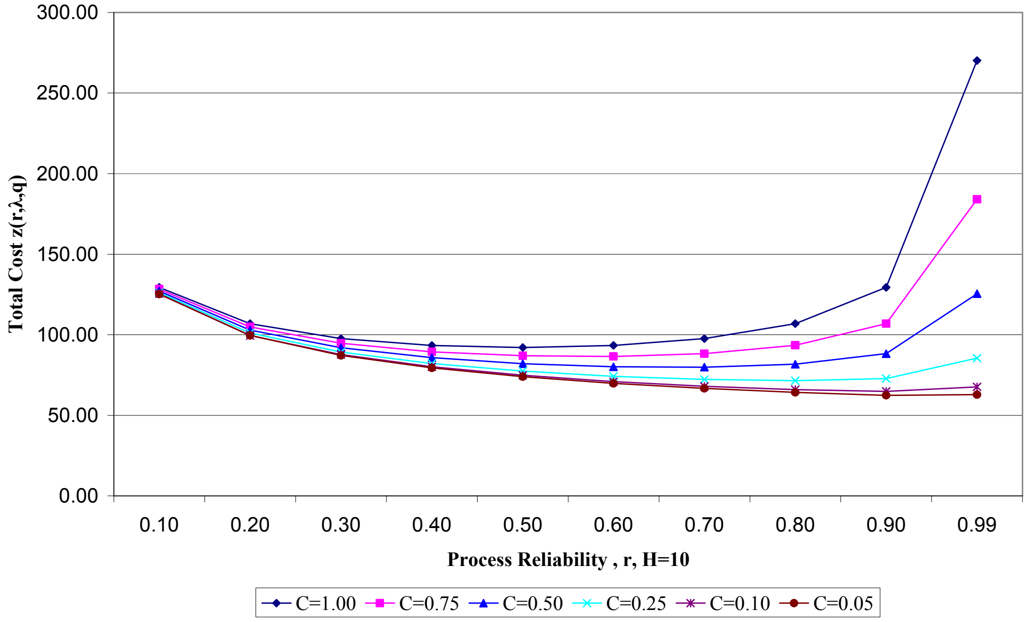

Fig. 2: Graph of the Process Reliability r and Total Cost Z(r,λ,q) when H=10

Table 4: Sensitive analysis of λ, P(r,λ),q and Z(r,λ,q) w.r.t. a,b &c when H=5

System parameter	% change in the System parameter	% change in λ^*	% change in $P^*(r,\lambda)$	% change in q^*	% change in $Z^*(r,\lambda,q)$
a	+60	0.330192728	9.0856028	5.50321205	68.7901505
	+40	0.251984244	7.93700418	6.29960482	78.7450571
	+20	0.158740134	6.2996041	7.93370045	99.2125582
	-20	0.158740134	-6.2996041	7.93700454	59.5275391
	-40	0.251984213	-7.937005	6.29960521	47.2470393
	-60	0.33019276	-9.0856019	5.50321178	41.273383
b	+60	0.28665170	10.581744	5.9063983	77.52147815
	+40	0.18721091	9.773216	7.3086038	99.18819412
	+20	0.09306523	8.039193	10.3658817	189.351031
	-20	0.00322539	1.587400	55.68117755	802.8901598
	-40	5.9049×10^{-11}	0.000405	411522.6337	7544581.619
	-60	9.76562×10^{11}	17021204.5	0.0000032	0.00004071
c	+60	0.55839476	11.815188	4.2318409	52.8980111
	+40	0.6124616	12.373971	4.0407367	50.509202
	+20	0.6717631	12.959189	3.8582637	48.228291
	-20	0.8081477	14.213965	3.5176661	43.970863
	-40	0.8863965	14.886208	3.3588141	41.985176
	-60	0.9722217	15.59023	3.207135	40.08919

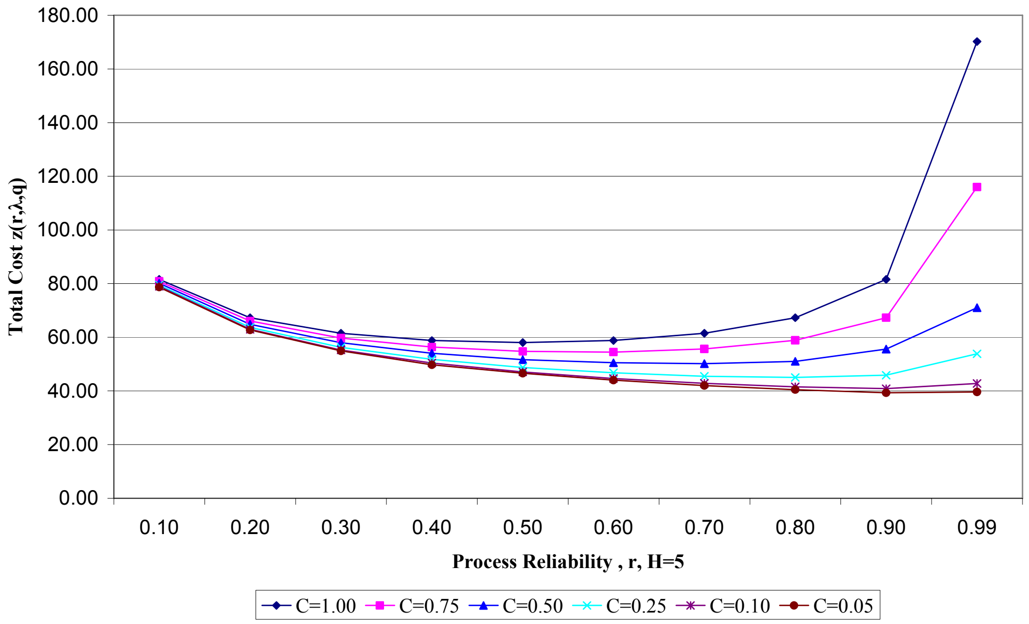

Fig. 3: Graph of the Process Reliability r and Total Cost Z(r,λ,q) when H=5

Interpretation of the Results

Using the two numerical examples the sensitivity of the decision variables λ, $P(r, \lambda)$, q and $z(r, \lambda, q)$ to change in each of the three parameters *a, b,* and *c* from table 3 and table 4 are computed. For any EOQ model, it is usually possible to estimate the setup cost, *S* and inventory holding cost, *H* using historical accounting and production data. However, the cost parameter *a, b,* and *c* used to define the unit cost of production, production quantity and the total cost in our EOQ model have to be largely estimated subjectively.
As a result

- $P^*(r, \lambda)$ is highly sensitive to '*a*' but $Z^*(r, \lambda, q)$ is moderately sensitive to *a*.
- $P^*(r, \lambda)$ and $Z^*(r, \lambda, q)$ are both highly sensitive to '*b*'.
- $P^*(r, \lambda)$ and $Z^*(r, \lambda, q)$ are both insensitive to '*c*'.

Table 5: Sensitive analysis of Z(r,λ,q) w.r.t. a,b &c when H=10 and H=5

Changing parameter	% change in the parameter	Example – 1		Example – 2	
		(1) % change in TC=Z(r,λ,q)	(2) TC	(3) % change in TC=Z(r,λ,q)	(4) TC
a	+60	109.197545	TC_{1A}	68.7901505	TC_{1A}
	+40	124.999984	TC_{1A}	78.74505711	TC_{1A}
	+20	157.490105	TC_{1A}	99.2125582	TC_{1A}
	-20	94.494078	TC_{1B}	59.5275391	TC_{1B}
	-40	75.000000	TC_1	47.2470393	TC_1
	-60	65.741184	TC_1	41.273383	TC_1
b	+60	128.337314	TC_{1A}	77.521478	TC_{1A}
	+40	170.05696	TC_{1A}	99.188194	TC_{1A}
	+20	266.012100	TC_{1B}	189.35103	TC_{1B}
	-20	4062.5078	TC_1	802.89015	TC_1
	-40	60356652.9	TC_1	7544581.61	TC_1
	-60	0.000018	TC_1	0.00004071	TC_1
c	+60	83.97035	TC_1	52.8980118	TC_1
	+40	80.178387	TC_1	50.5092024	TC_1
	+20	76.55766	TC_{1B}	48.2282913	TC_{1B}
	-20	69.79934	TC_{1A}	43.970863	TC_{1A}
	-40	66.647316	TC_{1A}	41.985176	TC_{1A}
	-60	63.63763	TC_{1A}	40.089195	TC_{1A}

Results and Discussion

The first two numerical examples are considered to study the effect of changes of the system parameters *a, b,* and *c* on the total cost. Whenever one parameter is changing by some percentage all other parameters are kept at their original values. Investigation has been done for positive and negative changes of these three parameters. The results obtained are discussed in table 5. A sensitivity analysis has also been conducted to reflect the effects of varied system inputs on the system cost and its robustness as well as satisfaction degrees of system performance. It is evident that the parameter *c* is comparatively less sensitive than the other two parameters *a* and *b*. The parameter *a* is more sensitive for positive changes than negative changes, but the parameter *b* and *c* are more sensitive for negative changes than the positive changes. So we can say after a negative change in *b* it becomes highly sensitive but this is not existing in case of *a*. It is observed that the change in the value of *TC* takes place in the order TC_{1A},TC_{1B},TC_1 when '*a*' decreases.

For parameter '*a*'

$TC= TC_{1A}$ if $a > 0$ $TC= TC_{1B}$ if $-2 \leq a < 0$ $TC= TC_1$ if $a < -2$

For parameter *'b'*

$TC= TC_{1A}$ if $b > 0$ $TC= TC_{1B}$ if $0 < b \leq 0.2$ $TC= TC_1$ if $b < 0$

and for parameter *'c'*

$TC= TC_1$ if $c > 0.2$ $TC= TC_{1B}$ if $0 < c \leq 0.2$ $TC= TC_{1A}$ if $c < 0$

If '*b*' decreases then the changes in *TC* takes place in the same order TC_{1A}, TC_{1B}, TC_1. So the order in which *TC* is changing is same for decrease of *a* and *b*. But when *c* decreases the order is just reversed. Further it can examine various policy scenarios that are associated with different levels of economic inputs.

Table 6: Comparison of Optimal Strategy and Strategy - I and Strategy – II

c	r	λ	$P(r, \lambda)$	q	z	r_1	λ_1	$P(r_1, \lambda_1)$	q_1	z_1	Relative difference = $(Z_1-Z)/Z_1$
1	0.5	1.4736	9.2101	10.8576	81.4325	0.5	0.3684	13.5720	3.6840	92.1007	0.115831827
0.75	0.6	1.2998	11.7679	8.4977	76.4796	0.6	0.4176	12.0425	4.1519	86.4989	0.115831837
0.50	0.7	1.1080	14.8697	6.7251	70.6141	0.7	0.4899	11.1795	4.4724	79.8650	0.115832115
0.25	0.8	0.8873	18.9899	5.2659	63.1919	0.8	0.6117	10.9310	4.5741	71.4703	0.115829091
C	r	λ	$P(r, \lambda)$	q	z	r_2	λ_2	$P(r_2, \lambda_2)$	q_2	z_2	Relative difference = $(Z-Z_2)/Z_2$
1	0.5	1.8566	5.8020	17.2354	65.2579	0.5	0.4641	10.7720	4.6415	58.0195	0.124759007
0.75	0.6	1.6376	7.4133	13.4893	61.6018	0.6	0.5262	9.5580	5.2311	54.4906	0.130502089
0.50	0.7	1.3960	9.3672	10.6755	56.0464	0.7	0.6172	8.8732	5.6349	50.1740	0.117039813
0.25	0.8	1.1180	11.9628	8.3592	51.7554	0.8	0.7707	8.6759	5.7630	45.0236	0.149516813

Discussion for the Relative Error

Comparing the minimum total cost $Z^*(r, \lambda, q)$ for c=1.00, 0.75, 0.50, and 0.25 from example 1 and 2 of the model of Tripathy et al. (2003) we observe that when the parameter *c* decreases, the relative difference in example (1) almost remain same whereas the relative difference of example 2 moderately changes.

9. Conclusions

In this paper an inventory model for optimal ordering policies in response to reliability is developed where the unit cost of production is inversely related to process reliability and demand rate where demand exceeds supply. Several aspects of managerial importance are also cited. The model developed here can help the manager in managing any major problem when unit cost of production decreases with high demand. In consequence, when the process reliability increases, the optimum total cost decreases proportionately. The generalization that will be considered in a future research mainly depends on the relevancy to a practical inventory situation.

The procedure is generically applicable to all production and inventory policy decisions. Our results show how an increase in product quality and demand may lead to decreased unit cost of production firms with large demand for the special type of items where 100% inspection is desired like costly electronic and electrical gadgets like Freeze, Air Conditioner, washing machine, plazma T.V., Video recorder and digital camera. Should hence used a large performance quality of the product in terms of reliability to account for the increased risk of cash flows. This complements in containsious of many practiceners that cost of quality of reliability and typically understood as under stated thinking. The opportunity cost of capital will be larger for product that have larger demand and larger reliability.

Acknowledgement. The authors are extremely grateful to Professor K.S. Chaudhury of Jadavpur University, Kolkata, and Professor M. Basu, Secretary, OR Society of India for their godly blessings which facilititates to intrude into the domain of this research area.

References

[1] C.S. Tapiero, P.H. Ritchken, A. Reisman, Reliability, Pricing and Quality Control. European Journal Operational Research, **31** (1987), 37-45.

[2] E.A. Silver, Operation Research in Inventory Management. A Review and Critique. Operations Research, **29** (1981), 628-645.

[3] E.A. Silver, R. Peterson. Decision Systems for Inventory Management and Production Planning. John Wiley and Sons, New York 1985.

[4] E.L. Porteus, Optimal Lot Sizing, Process Quality Improvement and Set up Cost Reduction. Operations Research, **34** (1986), 137-144.

[5] G. Hadley, T.M. Whitin, Analysis of Inventory Systems. Prentice-Hall, Englewood Cliffs, NewZaeland(1963).

[6] H.M. Wagner, Research Portfolio for Inventory management and Production Planning Systems. Operations Research, **28** (1980), 225-475.

[7] M.J. Rosenblatt, H.L. Lee, Economic Production Cycles with imperfect Production Processes. International Industrial Engineers Transactions, **14** (1986), 48-55.

[8] P.K. Tripathy, W.M. Wee, P.R. Majhi, An EOQ Model with Process Reliability Consideration. Journal of Operational Research Society, **54** (2003), 549-554.

[9] T.C.E. Cheng, An Economic Production Quantity Model with Flexibility and Reliability Consideration. European J. of Operational Research, **39** (1989), 174-179.

[10] T.C.E. Cheng, EPQ with Process Reliability and Quality Assurance Considerations. J. of Operational Research Society, **42** (1991), 713-720.

[11] T.G. Urgeleitti, Inventory Control Models and Problems. European Journal of Operational Research, **14** (1983), 1-12.

[12] T.M. Whitin, Inventory Control Research. A Survey Management Science, **1** (1954), 32-40.

Received: September 9, 2008