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Equilibriums and Permanence for an Autonomous Competitive System with Feedback Control*

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Abstract. In this paper, an autonomous competitive model with feedback control is studied. And the equilibriums of the system are discussed at first. Then the stability of equilibriums are obtained by ODE's stability theory and qualitative theory. Finally, some sufficient conditions for the permanence and the extinction of the system are obtained and some reasonable ecological explanations are given.

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§1 Introduction

For ecological system in the natural world, competition is very common among species in the same environment, which describes the dynamic relation among them (see [5]). On the other hand, it is unavoidable to be interfered by some external factors, which leads to the frequent changes of the parameters of the system. And this may finally bring influence to the permanence of the system. In view of the control theory, these interferences are often considered as certain controlled arguments. Therefore, it very meaningful and interesting to study ecological models with feedback control.

In fact, many researchers had done much work about it. For example, Chen etal.[2-4] studied the permanence, global attractivity and the almost periodic solution in two species competitive system with feedback control and delay or functional response. Si [6] studied the persistence for three species predator-prey system with feedback control. And Chen [7] discussed the asymptotic properties

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of a two-species competitive system with feedback controls. Tian [1] also studied an a nonautonomous prey-predator system with feed-back control and Holling IV functional response, in which some useful conclusions are obtained.

However, all of these papers are concerned about the nonautonomous ecological models, and it's very scarcely to see papers focus on the autonomous system with feedback control. Thus, in this paper, the author derived the following autonomous competitive system (1.1) with feedback control.

$$\begin{cases} \dfrac{dx_1}{dt} = x_1(r_1 - a_1x_1 - b_1x_2 + k_1s_1) \\ \dfrac{dx_2}{dt} = x_2(r_2 - a_2x_2 - b_2x_1 + k_2s_2) \\ \dfrac{ds_1}{dt} = a_3 - b_3s_1 - c_1x_1 \\ \dfrac{ds_2}{dt} = a_4 - b_4s_2 - c_2x_2 \end{cases} \tag{1.1}$$

Here, for the ecological reason, all the parameters in the system $a_i, b_i (i = 1,2,3,4)$ and $r_j, k_j, c_j (j = 1,2)$ are positive in the following paper, i. e,

$$r_1,\ r_2,\ a_1,\ a_2,\ b_1,\ b_2,\ k_1,\ k_2,\ a_3,\ a_4,\ b_3,\ b_4,\ c_1,\ c > 0 \tag{1.2}$$

§2 Equilibriums and their stability

In this section, for the realistic application, we are only concerned with the positive equilibriums (including the boundary equilibriums) of the system and their stability. First of all, we can obtain the following conclusion by direct calculation.

Lemma 2.1 If system (1.1) satisfies (1.2) and the following inequalities (2.1)

$$\begin{cases} a_2a_4 > c_2r_2, \quad b_2a_3 > c_1r_1 \\ c_2r_1 > b_1a_4, \quad c_1r_2 > b_2a_3 \\ a_2(r_1 + k_1) > b_1r_2 \\ a_1r_2 > b_2a_1 \\ a_1 > b_2, \qquad b_1 > a_2 \end{cases} \tag{2.1}$$

then system (1.1) has following four positive equilibriums:

$p_1^*(0, 0, \dfrac{a_3}{b_3}, \dfrac{a_4}{b_4})$; $p_2^*(0, \dfrac{b_4r_2 + k_2a_4}{b_4a_2 + k_2c_2}, \dfrac{a_3}{b_3}, \dfrac{a_2a_4 - c_2r_2}{c_2k_2 + a_2b_4})$; $p_3^*(\dfrac{b_3r_1 + k_1a_3}{a_1b_3 + k_1c_1}, 0, \dfrac{a_1a_3 - c_1r_1}{a_1b_3 + k_1c_1}, \dfrac{a_4}{b_4})$ and

$$p_4^*\Big(\frac{b_3b_1b_4r_2 - b_3c_2k_2r_1 + b_3k_2b_1a_4 - b_3a_2b_4r_1 - b_3a_2b_4k_1 - a_3c_2k_2k_1}{c_2k_2a_1b_3 - b_1b_4b_2b_3 + a_2b_4c_1k_1 + a_2b_4a_1b_3 + c_2k_2c_1k_1}, \frac{-b_4k_1b_2a_3 + b_4c_1k_1r_2 + b_4a_1b_3r_2 - b_4r_1b_2b_3 + a_4k_2a_1b_3 + a_4k_2c_1k_1}{c_2k_2a_1b_3 - b_1b_4b_2b_3 + a_2b_4c_1k_1 + a_2b_4a_1b_3 + c_2k_2c_1k_1},$$

$$\frac{b_1b_4c_1r_1 - c_2k_2c_1r_1 + c_2k_2a_1a_3 + k_2c_1b_1a_4 - b_1b_4b_2a_3 + a_2b_4a_1a_3 - a_2b_4c_1r_1}{c_2k_2a_1b_3 - b_1b_4b_2b_3 + a_2b_4c_1k_1 + a_2b_4a_1b_3 + c_2k_2c_1k_1}, \frac{c_2k_1b_2a_3 - c_2c_1k_1r_1 - c_2a_1b_3r_2 + c_2r_1b_2b_3 - b_1a_4b_2b_3 + c_1a_4a_2k_1 + a_4a_2a_1b_3}{c_2k_2a_1b_3 - b_1b_4b_2b_3 + a_2b_4c_1k_1 + a_2b_4a_1b_3 + c_2k_2c_1k_1}\Big).$$

At first, in order to discuss the stability of the equilibrium $p_1^*(0,0,\frac{a_3}{b_3},\frac{a_4}{b_4})$, we can consider the linear system at the point p_1^* as follows:

$$\begin{cases} \frac{dx_1}{dt} = (r_1 + k_1\frac{a_3}{b_3})x_1 - b_1x_2 \\ \frac{dx_2}{dt} = -b_2x_1 + (r_2 + k_2\frac{a_4}{b_4})x_2 \\ \frac{ds_1}{dt} = -c_1x_1 - b_3(s_1 - s_1^*) \\ \frac{ds_2}{dt} = -c_2x_2 - b_4(s_2 - s_2^*) \end{cases} \quad (2.2)$$

And it's easy to obtain its characteristic equation,

$$(\lambda - (r_1 + k_1\frac{a_3}{b_3}))(\lambda - (r_2 + k_2\frac{a_4}{b_4}))(\lambda + \mathrm{b}_3)(\lambda + \mathrm{b}_4) = 0 \quad (2.3)$$

Whose eigenvalues are $\lambda_1 = r_1 + k_1\frac{a_3}{b_3} > 0$, $\lambda_2 = r_2 + k_2\frac{a_4}{b_4} > 0$, $\lambda_3 = -\mathrm{b}_3 < 0$, $\lambda_4 = -\mathrm{b}_4 < 0$.

Therefore, the equilibrium p_1^* is unstable, that is,

Theorem 2.1 If all the parameters of the system (1.1) are positive(i.e. if the system (1.1) satisfies (1.2)), then the equilibrium p_1^* is unstable.

In the same way and Hurwitz's criteria(see [8]), we can obtain the stability theorem of the equilibrium $p_i^*(i = 2,3,4)$ as following Lemma 2.2—2.3.

Lemma 2.2 If the system (1.1) satisfies (1.2) and the following inequalities (2.4),

$$\begin{cases} b_4 > r_2 + k_2\frac{a_2a_4 - c_2r_2}{c_2k_2 + a_2b_4} - 2a_2\frac{b_4r_2 + k_2a_4}{b_4a_2 + k_2c_2} \\ b_1b_3(b_4r_2 + k_2a_4) > (b_3 + k_1a_3)(b_4a_2 + k_2c_2) \\ b_4r_2 < 2a_2b_4\frac{b_4r_2 + k_2a_4}{b_4a_2 + k_2c_2} - k_2b_4\frac{a_2a_4 - c_2r_2}{c_2k_2 + a_2b_4} + c_2k_2\frac{b_4r_2 + k_2a_4}{b_4a_2 + k_2c_2} \end{cases} \quad (2.4)$$

then the equilibrium $p_2^*(0,\frac{b_4r_2 + k_2a_4}{b_4a_2 + k_2c_2},\frac{a_3}{b_3},\frac{a_2a_4 - c_2r_2}{c_2k_2 + a_2b_4})$ is local asymptotic stable.

If we suppose x_1 specie is not exist, then the system can be simplified as following system,

$$\begin{cases} \frac{dx_2}{dt} = x_2(r_2 - a_2x_2 + k_2s_2) \\ \frac{ds_1}{dt} = a_3 - b_3s_1 \\ \frac{ds_2}{dt} = a_4 - b_4s_2 - c_2x_2 \end{cases} \quad (2.5)$$

It is easy to obtain system (2.5) have two equilibriums $\overline{p}_1(0,\frac{a_3}{b_3},\frac{a_4}{b_4})$ and $\overline{p}_2(\overline{x}_2,\overline{s}_1,\overline{s}_2)$.

Where, $\bar{x}_2 = \dfrac{b_4 r_2 + k_2 a_4}{b_4 a_2 + k_2 c_2}$, $\bar{s}_1 = \dfrac{a_3}{b_3}$, $\bar{s}_2 = \dfrac{a_2 a_4 - c_2 r_2}{c_2 k_2 + a_2 b_4}$

In the same way of the Theorem 2.1, we can declaim $\bar{p}_1(0, \frac{a_3}{b_3}, \frac{a_4}{b_4})$ is unstable for its corresponding characteristic equation has a positive eigenvalue $\lambda = r_2 + k_2 \frac{a_4}{b_4} > 0$. Also, we can obtain the linear system at the equilibrium $\bar{p}_2$,whose characteristic equation is as follow:

$$(\lambda + b_3)[\lambda^2 + (b_4 + 2a_2\bar{x}_2 - k_2\bar{s}_2 - r_2)]\lambda + (k_2 c_2 \bar{x}_2 + 2a_2 b_4 \bar{x}_2 - b_4 k_2 \bar{s}_2 - b_4 r_2) = 0 \qquad (2.6)$$

Obviously, $\bar{p}_2$ is local asymptotic stable if

$$b_4 + 2a_2\bar{x}_2 > k_2\bar{s}_2 + r_2, k_2 c_2 \bar{x}_2 + 2a_2 b_4 \bar{x}_2 > b_4 k_2 \bar{s}_2 + b_4 r_2 \qquad (2.7)$$

On the other hand, if we constrcut following Liapunov function,

$$V(x_2, s_1, s_2) = \lambda_1 (x_2 - \bar{x}_2 - \bar{x}_2 \ln \frac{x_2}{\bar{x}_2}) + \frac{\lambda_2}{2}(s_1 - \bar{s}_1)^2 + \frac{\lambda_3}{2}(s_2 - \bar{s}_2)^2$$

Calculating the upper right derivation $V(x_2, s_1, s_2)$ along the system (1.1) we obtain,

$$D^+V(x_2, s_1, s_2) = \lambda_1 \frac{x_2 - \bar{x}_2}{x_2}\frac{dx_2}{dt} + \lambda_2 (s_1 - \bar{s}_1)\frac{ds_1}{dt} + \lambda_3 (s_2 - \bar{s}_2)\frac{ds_2}{dt}$$

$$= -\lambda_1 (x_2 - \bar{x}_2)[a_2(x_2 - \bar{x}_2) - k_2(s_2 - \bar{s}_2)] - \lambda_2 b_3 (s_1 - \bar{s})^2 - \lambda_3 (s_2 - \bar{s}_2)[b_4(s_2 - s_2^*) + c_2(x_2 - x_2^*)]$$

$$= -\lambda_1 a_2 (x_2 - \bar{x}_2)^2 - \lambda_2 b_3 (s_1 - \bar{s})^2 - \lambda_3 b_4 (s_2 - \bar{s}_2)^2 + (\lambda_1 k_2 - \lambda_3 c_2)(x_2 - \bar{x}_2)(s_2 - \bar{s}_2)$$

Thus, if we choose $\lambda_1 = c_2\lambda, \lambda_2 = \lambda, \lambda_3 = k_2\lambda$ $(\lambda > 0)$, then we have,

$$D^+V(t) = -\lambda[a_2 c_2 (x_2 - \bar{x}_2)^2 + b_3(s_1 - \bar{s}_1)^2 + b_4 k_2 (s_2 - \bar{s}_2)^2] < 0,$$

Therefore, we have the following Theorem 2.2 according to the Liapunov Theorem.

Theorem 2.2 If the system (1.1) satisfies (1.2) and the following inequalities (2.4),(2.7), then the equilibrium p_2^* is global asymptotic stable.

In the same way, we can obtain Theorem 2.3 as follow:

Theorem 2.3 If the system (1.1) satisfies (1.2) and the following inequalities(2.8),

$$\begin{cases} b_3 > r_1 - 2a_1 \dfrac{b_3 r_1 + k_1 a_3}{a_1 b_3 + k_1 c_1} + k_1 \dfrac{a_1 a_3 - c_1 r_1}{k_1 c_1 + a_1 b_3} \\ b_2 b_4 (b_3 r_1 + k_1 a_3) > (r_2 b_4 + k_2 a_4)(a_1 b_3 + k_1 c_1) \\ b_3 r_1 < k_1 b_3 \dfrac{a_1 a_3 - c_1 r_1}{k_1 c_1 + a_1 b_3} - 2a_1 b_3 \dfrac{b_3 r_1 + k_1 a_3}{a_1 b_3 + k_1 c_1} + c_1 k_1 \dfrac{b_3 r_1 + k_1 a_3}{a_1 b_3 + k_1 c_1} \\ b_3 + 2a_1 \bar{x}_1 > k_1 \bar{s}_1 + r_1, k_1 c_1 \bar{x}_1 + 2a_1 b_3 \bar{x}_1 > b_3 k_1 \bar{s}_1 + b_3 r_1 \end{cases} \qquad (2.8)$$

Where, $\bar{x}_1 = \dfrac{b_3 r_1 + k_1 a_3}{b_3 a_1 + k_1 c_1}$, $\bar{s}_1 = \dfrac{a_1 a_3 - c_1 r_1}{b_3 a_1 + k_1 c_1}$, $\bar{s}_2 = \dfrac{a_4}{b_4}$ (2.9) then the equilibrium $p_3^*(\frac{b_3r_1+k_1a_3}{a_1b_3+k_1c_1},0,\frac{a_1a_3-c_1r_1}{k_1c_1+a_1b_3},\frac{a_4}{b_4})$ is global asymptotic stable.

For the convenience to discuss the stability of the equilibrium p_4^*, we set $p_4^*(x_1^*,x_2^*,s_1^*,s_2^*)$ for short and denote:

$X_1^*=r_1-2a_1x_1^*-b_1x_2^*+k_1s_1^*, X_2^*=r_2-2a_2x_2^*-b_2x_1^*+k_2s_2^*$. Then we obtain

Lemma 2.3 If the system (1.1) satisfies (1.2) and the following inequalities (2.10)—(2.13),

$$b_3+b_4>X_1^*+X_2^* \tag{2.10}$$

$$(b_3-X_1^*)(b_4-X_2^*)-b_3X_1^*-b_4X_2^*+c_1k_1x_1^*+c_2k_2x_2^*-b_1b_2x_1^*x_2^*>0 \tag{2.11}$$

$$(b_3-X_1^*)(c_2k_2x_2^*-b_4X_2^*)+(b_4-X_2^*)(c_1k_1x_1^*-b_3X_1^*)>b_1b_2(b_3+b_4)x_1^*x_2^* \tag{2.12}$$

$$b_3b_4X_1^*X_2^*+(c_1c_2k_1k_2-b_1b_2b_3b_4)x_1^*x_2^*>b_3c_2k_2x_2^*X_1^*+b_4c_1k_1x_1^*X_2^* \tag{2.13}$$

then the equilibrium p_4^* is local asymptotic stable.

And we construct following Liapunov function,

$$V(x_1,x_2,s_1,s_2)=w_1(x_1-x_1^*-x_1^*\ln\frac{x_1}{x_1^*})+w_2(x_2-x_2^*-x_2^*\ln\frac{x_2}{x_2^*})+\frac{w_3}{2}(s_1-s_1^*)^2+\frac{w_4}{2}(s_2-s_2^*)^2$$

Calculating the upper right derivation $V(x_1,x_2,s_1,s_2)$ along the system (1.1) we obtain,

$$\begin{aligned}D^+V(x_1,x_2,s_1,s_2)&=w_1\frac{x_1-x_1^*}{x_1}\frac{dx_1}{dt}+w_2\frac{x_2-x_2^*}{x_2}\frac{dx_2}{dt}+w_3(s_1-s_1^*)\frac{ds_1}{dt}+w_4(s_2-s_2^*)\frac{ds_2}{dt}\\&=-w_1(x_1-x_1^*)[a_1(x_1-x_1^*)+b_1(x_2-x_2^*)-k_1(s_1-s_1^*)]\\&\quad-w_2(x_2-x_2^*)[a_2(x_2-x_2^*)+b_2(x_1-x_1^*)-k_2(s_2-s_2^*)]\\&\quad-w_3(s_1-s_1^*)[b_3(s_1-s_1^*)+c_1(x_1-x_1^*)]-w_4(s_2-s_2^*)[b_4(s_2-s_2^*)+c_2(x_2-x_2^*)]\end{aligned}$$

$$=-[a_1w_1(x_1-x_1^*)^2+a_2w_2(x_2-x_2^*)^2+b_3w_3(s_1-s_1^*)^2+b_4w_4(s_2-s_2^*)^2]$$
$$-(b_1w_1+b_2w_2)(x_1-x_1^*)(x_2-x_2^*)+(k_1w_1-c_1w_3)(x_1-x_1^*)(s_1-s_1^*)+(k_2w_2-c_2w_4)(x_2-x_2^*)(s_2-s_2^*)$$

Thus, if we choose $w_1=c_1w,w_2=c_2w,w_3=k_1w,w_4=k_2w,(w>0)$, then we have,

$$D^+V(t)=-w[a_1(x_1-x_1^*)^2+(b_1+b_2)(x_1-x_1^*)(x_2-x_2^*)+a_2(x_2-x_2^*)^2+b_3(s_1-s_1^*)^2+b_4(s_2-s_2^*)^2]$$

It is obvious to show that $D^+V(t)<0$ when the parameters $a_i,b_i(i=1,2)$ of the system (1.1) satisfies the inequality $(b_1+b_2)^2\le 4a_1a_2$ (2.14)

Therefore, we have the following Theorem 2.4 according to the Liapunov Theorem.

Theorem 2.4 If the system (1.1) satisfies (1.2) and the following inequalities

(2.10)—(2.14), then the equilibrium p_4^* is global asymptotic stable.

§3 Permanence

Lemma 3.1 $R_4^+ = \{(x_1, x_2, s_1, s_2) \mid x_i(t) > 0, s_i(t) > 0 (i = 1, 2)\}$ is the positively invariant set of the system (1.1).

Suppose $(x_1(t), x_2(t), s_1(t), s_2(t))$ is the random positive solution of the system (1.1) with positive initial values, then we can obtain the subsequent several lemams.

Lemma 3.2 There exists a $T_1^* > 0$, such that $0 < x_i(t) \le M_i, 0 < s_i(t) \le N_i (i = 1, 2)$ when $t > T_1^*$,where, $M_1 = \dfrac{r_1 b_3 + k_1 a_3}{a_1 b_3}$; $M_2 = \dfrac{r_2 b_4 + k_2 a_4}{a_2 b_4}$; $N_1 = \dfrac{a_3}{b_3}$; $N_2 = \dfrac{a_4}{b_4}$.

Proof. According to the last two equations of the system (1.1), we have

$\dfrac{ds_1}{dt} = a_3 - b_3 s_1 - c_1 x_1 \le a_3 - b_3 s_1 \Rightarrow \left.\dfrac{ds_1}{dt}\right|_{s_1 = N_1} \le 0$ and $\dfrac{ds_2}{dt} = a_4 - b_4 s_2 - c_2 x_2 \le a_4 - b_4 s_2 \Rightarrow \left.\dfrac{ds_2}{dt}\right|_{s_2 = N_2} \le 0 \cdot$

Then there exists a $T_1 > 0$ and a $T_2 > 0$, such that $0 < s_1(t) \le N_1$ when $t > T_1$ and $0 < s_2(t) \le N_2$ when $t > T_2$. (3.1)

On the other hand, according to the first two equations of the system (1.1) and (3.1), we can obtain

$$\frac{dx_1}{dt} = x_1(r_1 - a_1 x_1 - b_1 x_2 + k_1 s_1) \le x_1(r_1 - a_1 x_1 + k_1 s_1) \quad (3.2)$$

$$\frac{dx_2}{dt} = x_2(r_2 - a_2 x_2 - b_2 x_1 + k_2 s_2) \le x_2(r_2 - a_2 x_2 + k_2 s_2) \quad (3.3)$$

Via the formula (3.1)—(3.3) we have

$\dfrac{dx_1}{dt} \le x_1(r_1 - a_1 x_1 + k_1 N_1) \Rightarrow \left.\dfrac{dx_1}{dt}\right|_{x_1 = M_1} \le x_1(r_1 - a_1 M_1 + k_1 \dfrac{a_3}{b_3}) = 0$, when $t > T_1$.

$\dfrac{dx_2}{dt} \le x_2(r_2 - a_2 x_2 + k_2 N_2) \Rightarrow \left.\dfrac{dx_2}{dt}\right|_{x_2 = M_2} \le x_1(r_2 - a_2 M_2 + k_2 \dfrac{a_4}{b_4}) = 0$, when $t > T_2$.

Therefore, there exists a $T_3 (> T_1)$ and $T_4 (> T_2)$, such that

$$0 < x_1(t) \le M_1,\ when\ t > T_3 \text{ and } 0 < x_2(t) \le M_2,\ when\ t > T_4 \quad (3.4)$$

If we choose $T_1^* = \max\{T_3, T_4\}$, then $0 < x_i(t) \le M_i, 0 < s_i(t) \le N_i (i = 1, 2)$ when $t > T_1^*$.□

Lemma 3.3 If the system (1.1) satisfies initial condition (1.2) and the following inequalities (3.5)

$$\begin{cases} k_1a_3 + r_1b_3 > b_1b_3M_2 + k_1c_1M_1 \\ k_2a_4 + r_2b_4 > b_2b_4M_1 + k_2c_2M_2 \\ a_3 > c_1M_1, \quad a_2 > c_2M_2 \end{cases} \tag{3.5}$$

then there exists a $T_2^* > 0$, such that $x_i(t) \ge m_i, s_i(t) \ge n_i (i = 1,2)$ when $t > T_2^*$,

where, $m_1 = \dfrac{r_1b_3 - b_1b_3M_2 + k_1(a_3 - c_1M_1)}{a_1b_3}$, $n_1 = \dfrac{a_3 - c_1M_1}{b_3}$;

$$m_2 = \frac{r_2b_4 - b_2b_4M_1 + k_2(a_4 - c_2M_2)}{a_2b_4}, n_2 = \frac{a_4 - c_2M_2}{b_4}.$$

Proof. According to the last two equations of the system (1.1) and utilizing the conclusion of Lemma 3.2, it is very simple to obtain the following formulas when $t > T_1^*$, $\dfrac{ds_1}{dt} = a_3 - b_3s_1 - c_1x_1 \ge a_3 - b_3s_1 - c_1M_1 \Rightarrow \left.\dfrac{ds_1}{dt}\right|_{s_1=n_1} \ge 0$

and $\dfrac{ds_2}{dt} = a_4 - b_4s_2 - c_2x_2 \ge a_4 - b_4s_2 - c_2M_2 \Rightarrow \left.\dfrac{ds_2}{dt}\right|_{s_2=n_2} \ge 0$

then there exists a $T_5(> T_1^*)$ and a $T_6(> T_1^*)$, such that

$$s_1(t) \ge n_1 \text{ when } t > T_5 \text{ and } s_2(t) \ge n_2 \text{ when } t > T_6 \tag{3.6}$$

On the other hand, according to the first two equations of system (1.1) and formula (3.6), also considering the conclusion of Lemma 3.2, we can obtain

$$\frac{dx_1}{dt} = x_1(r_1 - a_1x_1 - b_1x_2 + k_1s_1) \ge x_1(r_1 - a_1x_1 - b_1M_2 + k_1n_1) \Rightarrow \left.\frac{dx_1}{dt}\right|_{x_1=m_1} \ge 0, \text{when } t > T_5 \tag{3.7}$$

$$\frac{dx_2}{dt} = x_2(r_2 - a_2x_2 - b_2x_1 + k_2s_2) \ge x_2(r_2 - a_2x_2 - b_2M_1 + k_2n_2) \Rightarrow \left.\frac{dx_2}{dt}\right|_{x_2=m_2} \ge 0, \text{when } t > T_6. \tag{3.8}$$

Therefore, there exists a $T_7(> T_5)$ and $T_8(> T_5)$, such that

$$x_1(t) \ge m_1, \textit{ when } t > T_7 \text{ and } x_2(t) \ge m_2, \textit{ when } t > T_8 \tag{3.9}$$

If we choose $T_2^* = \max\{T_7, T_8\}$, then $x_i(t) \ge m_i, s_i(t) \ge n_i (i = 1,2)$ when $t > T_2^*$.□

Deriving from Lemma 3.2 and Lemma 3.3,

$K = \{(x_1, x_2, s_1, s_2) \mid m_i \le x_i \le M_i, n_i \le s_i \le N_i (i = 1,2)\}$ is the ultimately bounded set of the system (1.1).That is to say, each solution of the system (1.1) with positive initial values will be enter the compact region K and remain in it finally. Thus, we have the following persistence Theorem3.1 of the system (1.1).

Theorem 3.1 Under the condition of Lemma 3.1, system (1.1) is permanent.

§4 Extinction and Explanations

When two species compete for a certain food resource in the same living environment, the familiar result is the extinction of the weaker specie, and the population of the stronger specie reach to the carrying capacity of the environment. According to the conclusions in this paper, when the equilibrium p_2^*, p_3^* is gloabal asymptotic stable, either one of the two will be extinct or the other.

Theorem 4.1 Under the condition of Theorem 2.2, if system (1.1) also satisfies $a_2a_4 > c_2r_2$, then the population x_2 will be extinct finally.

This means it is good for the population x_1 in this circumstance, and the competitive capacity of the population x_1 is stronger then that of the population x_2.

Theorem 4.2 Under the condition of Theorem 2.3, if system (1.1) also satisfies $a_1a_3 > c_1r_1$, then the population x_1 will be extinct finally.

This means it is good for the population x_2 in this circumstance, and the competitive capacity of the population x_2 is stronger then that of the population x_1.

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