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On Systems of Fuzzy Nonlinear Equations

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Abstract

In this paper, we suggest and analyze a new two-step iterative method for solving a system of fuzzy nonlinear equations by using the Midpoint quadrature rule. We prove that this method has quadratic convergence. The fuzzy quantities are presented in parametric form. Sever examples are given to illustrate the efficiency of the proposed method.

Mathematics Subject Classification: 03E72; 65H10

Keywords: Newton's method; Fixed point iteration; Fuzzy system of nonlinear equations

1 Introduction

In recent years much attention has been given to develop iterative type methods for solving systems of simultaneous nonlinear equations because of their important rule in various areas such as mathematics, statistics, engineering and social sciences.

The concept of fuzzy numbers and arithmetic operation with these numbers were first introduced and investigated by Zadeh [10]. One of the major applications of fuzzy number arithmetic is nonlinear systems whose parameters are all or partially represented by fuzzy numbers[2, 5, 7]. Standard analytical techniques presented by Buckley and Qu in [1], cannot be suitable for solving the equations such as

$$\begin{cases} AX^4 + BY^4 + CX^3 + DY^3 + EX^2Y^2 + F = G, \\ X - \cos(Y) = H, \end{cases}$$

where X, Y, A, B, C, D, E, F, G and H are fuzzy numbers. In this paper we have an adjustment on the classic Newton's method in order to accelerate the convergence or to reduce the number of operations and evaluations in each step of the iterative process. We suggest and analyze an iterative method by using the Trapezoidal rule. This method is an implicit-type method. To implement this, we use Newton's method as predictor method and then use this method as corrector method. Several examples are given to illustrate the efficiency and advantage of this two-steep method.

In Section 2, we bring some basic definitions and results on fuzzy numbers. In Section 3 we develop some modification on Newton's method to introduce Trapezoidal Newton's method for solving of the system of nonlinear real equations and quadratic convergence of this method has been proved. In Section 4 we apply the obtained results from Section 3 for solving of nonlinear fuzzy systems. The proposed algorithm is illustrated by some examples in Section 5 and a comparison with Classical Newton's method will be done, and conclusion is in Section 6.

2 Preliminaries

Definition 2.1 *A fuzzy number is set like* $U : \mathbb{R} \to I = [0, 1]$ *which satisfies,* [4, 10, 11],

1. U is upper semi-continuous,
2. $U(x) = 0$ outside some interval $[c, d]$,
3. There are real numbers a, b such that $c \leqslant a \leqslant b \leqslant d$ and
   - 3.1. $U(x)$ is monotonic increasing on $[c, a]$,
   - 3.2. $U(x)$ is monotonic decreasing on $[a, b]$,
   - 3.3. $U(x) = 1$, $a \leqslant x \leqslant b$.

An equivalent parametric is also given in [6] as follows.

Definition 2.2 *A fuzzy number* U *in parametric form is a pair* (U_1, U_2) *of functions* $U_1(r), U_2(r), 0 \leqslant r \leqslant 1$, *which satisfies the following requirements:*

1. $U_1(r)$ is a bounded monotonic increasing left continuous function,
2. $U_2(r)$ is a bounded monotonic decreasing left continuous function,
3. $U_1(r) \leqslant U_2(r)$, $0 \leqslant r \leqslant 1$.

A crisp number α is simply represented by $U_1(r) = U_2(r) = \alpha$, $0 \leqslant r \leqslant 1$.

A popular fuzzy number is triangular fuzzy number $U = (x_0, x_1, y_0, y_1)$ with interval defuzzifier $[x_1, y_0]$ where the membership function is

$$U(x) = \begin{cases} \frac{x-x_0}{x_1-x_0} & ; \quad x_0 \leqslant x \leqslant x_1, \\ 1 & ; \quad x \in [x_1, y_0], \\ \frac{y_1-x}{y_1-y_0} & ; \quad y_0 \leqslant x \leqslant y_1, \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

Its parametric form is

$$U_1(r) = (x_1 - x_0)r + x_0, \quad U_2(r) = y_1 - (y_1 - y_0)r.$$

If $x_1 = y_0$ then $U = (x_0, x_1, y_1)$is called triangular fuzzy number.

Let $TF(\mathbb{R})$ be the set of all trapezoidal fuzzy numbers. The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows.

For arbitrary $U = (U_1, U_2)$, $V = (V_1, V_2)$ and $k > 0$ we defined addition $U + V$ and multiplication by real number $k > 0$ as

$$(U + V)_1(r) = U_1(r) + V_1(r), \quad (U + V)_2(r) = U_2(r) + V_2(r),$$
$$(kU)_1(r) = kU_1(r), \quad (kU)_2(r) = kU_2(r).$$

3 Midpoint Newton's method

We consider the problem of finding a real zero of a function $F : D \subseteq \mathbb{R}^n \to \mathbb{R}^n$, that is, a real solution α, of the nonlinear equation system $F(x) = 0$, of n equations with n variables. This solution can be obtained as a fixed point of some function $G : \mathbb{R}^n \to \mathbb{R}^n$ by means of the fixed point iteration method

$$x_{k+1} = G(x_k), \quad k = 0, 1, \ldots,$$

where x_0 is the initial estimation. The best known fixed point method is the classical Newton's method, given by

$$x_{k+1} = x_k - J_F(x_k)^{-1}F(x_k), \quad k = 0, 1, 2, \ldots,$$

where $J_F(x_k)$ is the Jaccobian Matrix of the function F evaluated in x_k.

Let $F : D \subseteq \mathbb{R}^n \to \mathbb{R}^n$ be a sufficiently differentiable function and α be a zero of the system of nonlinear equations $F(x) = 0$. The following result will be used describe the Newton's method and Midpoint Newton's method; see its proof in [8, 9].

Lemma 3.1 *Let $F : D \subseteq \mathbb{R}^n \to \mathbb{R}^n$ be continuously differentiable on a convex set D. Then, for any $x, y \in D$, F satisfies*

$$F(y) - F(x) = \int_0^1 J_F(x + t(y - x))(y - x)dt. \quad (1)$$

Once the iterate x_k has been obtained, using (1):

$$F(y) = F(x_k) + \int_0^1 J_F(x_k + t(y - x_k))(y - x_k)dt. \quad (2)$$

If we estimate $J_F(x_k + t(y - x_k))$ in the interval $[0, 1]$ by its value in $t = 0$, that is by $J_F(x_k)$, and take $y = \alpha$, then

$$0 \approx F(x_k) + J_F(x_k)(\alpha - x_k),$$

is obtained, and a new approximation of α can be done by

$$x_{k+1} = x_k - J_F(x_k)^{-1}F(x_k),$$

what is the classical *Newton method* (CN) for $k = 0, 1, \dots$

If an estimation of (2) is made by means of the Midpoint rule and $y = \alpha$ is taken, then

$$0 \approx F(x_k) + J_F(\frac{x_k + \alpha}{2})(\alpha - x_k),$$

is obtained and a new approximation x_{k+1} of α is given by

$$x_{k+1} = x_k - J_F(\frac{x_k + x_{k+1}}{2})^{-1}F(x_k).$$

In order to avoid the implicit problem that this equation involves, we use the $(k + 1)$th iteration of Newton method in the right side. Then,

$$x_{k+1} = x_k - J_F(\frac{x_k + z_k}{2})^{-1}F(x_k), \quad k = 0, 1, \dots, \quad (3)$$

where

$$z_k = x_k - J_F(x_k)^{-1}F(x_k).$$

This method is called *Midpoint Newton's method* (MN).

The Midpoint Newton's method can be understood as a *substitution of* $J_F(x_k)$ *in Newton's method by* $J_F(\frac{x_k+z_k}{2})$. In the following, we bring the quadratic convergence of Midpoint Newton's method from [3].

Theorem 3.1 *Let* $F : \mathbb{R}^n \to \mathbb{R}^n$ *be differentiable at each point of an open neighborhood* D *of* $\alpha \in \mathbb{R}$*, that is a solution of the system* $F(x) = 0$*. Let us suppose that* $J_F(x)$ *is continuous and non singular in* α*. Then the sequence* $\{x_k\}_{k\geqslant 0}$ *obtained using the iterative expression (3) converges to* α *and*

$$\lim_{k\to\infty} \frac{\|x_{k+1} - \alpha\|}{\|x_k - \alpha\|} = 0.$$

Moreover, if there exists $\gamma > 0$ *such that*

$$\|J_F(x) - J_F(\alpha)\| \leqslant \gamma \|x - \alpha\|,$$

for any x *in* D*, then there exists a constant* $M > 0$ *such that*

$$\|x_{k+1} - \alpha\| \leqslant M \|x_k - \alpha\|^2, \quad \forall k \geqslant k_0,$$

where k_0 *depends on the initial estimation* x_0*.*

4 Midpoint Newton's method for fuzzy nonlinear system

Now our aim in this section is to obtain a solution for nonlinear system

$$\begin{cases} F(X,Y) = C, \\ G(X,Y) = D, \end{cases} \tag{4}$$

where X, Y, C and D are fuzzy numbers. The parametric form $\forall r \in [0,1]$, is as follows

$$\begin{cases} F_1(X_1, X_2, Y_1, Y_2; r) = C_1, \\ F_2(X_1, X_2, Y_1, Y_2; r) = C_2, \\ G_1(X_1, X_2, Y_1, Y_2; r) = D_1, \\ G_2(X_1, X_2, Y_1, Y_2; r) = D_2, \end{cases} \tag{5}$$

Suppose that $X = (\alpha_1, \alpha_2)$ and $Y = (\beta_1, \beta_2)$ are the solutions of (5), i.e.,$\forall r \in [0,1]$

$$\begin{cases} F_1(\alpha_1, \alpha_2, \beta_1, \beta_2; r) = C_1, \\ F_2(\alpha_1, \alpha_2, \beta_1, \beta_2; r) = C_2, \\ G_1(\alpha_1, \alpha_2, \beta_1, \beta_2; r) = D_1, \\ G_2(\alpha_1, \alpha_2, \beta_1, \beta_2; r) = D_2, \end{cases}$$

Therefore, if $X_0 = (X_{1_0}, X_{2_0})$ and $Y_0 = (Y_{1_0}, Y_{2_0})$ are approximation solutions for this system, then $\forall r \in [0,1]$, there are $h_i(r), k_i(r); i = 1,2$ such that

$$\begin{cases} \alpha_1(r) = X_{1_0}(r) + h_1(r), \\ \alpha_2(r) = X_{2_0}(r) + k_1(r), \\ \beta_1(r) = Y_{1_0}(r) + h_2(r), \\ \beta_2(r) = Y_{2_0}(r) + k_2(r). \end{cases}$$

Now by using of the Taylor series of F_1, F_2, G_1, G_2 about $(X_{1_0}, X_{2_0}, Y_{1_0}, Y_{2_0})$, then $\forall r \in [0,1]$,

$$\begin{cases} F_1(\alpha_1,\alpha_2,\beta_1,\beta_2;r) \\ \quad = F_1(\Delta_0) + h_1 F_{1_{X_1}}(\Delta_0) + k_1 F_{1_{X_2}}(\Delta_0) + h_2 F_{1_{Y_1}}(\Delta_0) + k_2 F_{1_{Y_2}}(\Delta_0) + O(\Gamma) = C_1, \\ F_2(\alpha_1,\alpha_2,\beta_1,\beta_2;r) \\ \quad = F_2(\Delta_0) + h_1 F_{2_{X_1}}(\Delta_0) + k_1 F_{2_{X_2}}(\Delta_0) + h_2 F_{2_{Y_1}}(\Delta_0) + k_2 F_{2_{Y_2}}(\Delta_0) + O(\Gamma) = C_2, \\ G_1(\alpha_1,\alpha_2,\beta_1,\beta_2;r) \\ \quad = G_1(\Delta_0) + h_1 G_{1_{X_1}}(\Delta_0) + k_1 G_{1_{X_2}}(\Delta_0) + h_2 G_{1_{Y_1}}(\Delta_0) + k_2 G_{1_{Y_2}}(\Delta_0) + O(\Gamma) = D_1, \\ G_2(\alpha_1,\alpha_2,\beta_1,\beta_2;r) \\ \quad = G_2(\Delta_0) + h_1 G_{2_{X_1}}(\Delta_0) + k_1 G_{2_{X_2}}(\Delta_0) + h_2 G_{2_{Y_1}}(\Delta_0) + k_2 G_{2_{Y_2}}(\Delta_0) + O(\Gamma) = D_2, \end{cases}$$

where $\Delta_0 = (X_{1_0}, X_{2_0}, Y_{1_0}, Y_{2_0}; r)$, $\Gamma = h_1^2 + h_2^2 + h_1k_1 + h_2k_2 + h_1k_2 + h_2k_1 + k_1^2 + k_2^2$ and if X_{1_0}, X_{2_0}, Y_{1_0} and Y_{2_0} are near to α_1, α_2, β_1 and β_2, respectively, then $h_i(r)$ and $k_i(r); i = 1,2$ are small. It is assumed, of course, that all needed partial derivative exists and bounded. Therefore for enough small $h_i(r)$ and $k_i(r); i = 1,2$ we have $\forall r \in [0,1]$,

$$\begin{cases} F_1(\Delta_0) + h_1 F_{1_{X_1}}(\Delta_0) + k_1 F_{1_{X_2}}(\Delta_0) + h_2 F_{1_{Y_1}}(\Delta_0) + k_2 F_{1_{Y_2}}(\Delta_0) \simeq C_1, \\ F_2(\Delta_0) + h_1 F_{2_{X_1}}(\Delta_0) + k_1 F_{2_{X_2}}(\Delta_0) + h_2 F_{2_{Y_1}}(\Delta_0) + k_2 F_{2_{Y_2}}(\Delta_0) \simeq C_2, \\ G_1(\Delta_0) + h_1 G_{1_{X_1}}(\Delta_0) + k_1 G_{1_{X_2}}(\Delta_0) + h_2 G_{1_{Y_1}}(\Delta_0) + k_2 G_{1_{Y_2}}(\Delta_0) \simeq D_1, \\ G_2(\Delta_0) + h_1 G_{2_{X_1}}(\Delta_0) + k_1 G_{2_{X_2}}(\Delta_0) + h_2 G_{2_{Y_1}}(\Delta_0) + k_2 G_{2_{Y_2}}(\Delta_0) \simeq D_2, \end{cases}$$

and hence $h_i(r)$ and $k_i(r); i = 1,2$ are unknown quantities which can be obtained by solving the following equations, $\forall r \in [0,1]$,

$$J(\Delta_0) \begin{bmatrix} h_1(r) \\ k_1(r) \\ h_2(r) \\ k_2(r) \end{bmatrix} = \begin{bmatrix} C_1(r) - F_1(\Delta_0) \\ C_2(r) - F_2(\Delta_0) \\ D_1(r) - G_1(\Delta_0) \\ D_2(r) - G_2(\Delta_0) \end{bmatrix}, \tag{6}$$

where

$$J(\Delta_0) = \begin{bmatrix} F_{1_{X_1}}(\Delta_0) & F_{1_{X_2}}(\Delta_0) & F_{1_{Y_1}}(\Delta_0) & F_{1_{Y_2}}(\Delta_0) \\ F_{2_{X_1}}(\Delta_0) & F_{2_{X_2}}(\Delta_0) & F_{2_{Y_1}}(\Delta_0) & F_{2_{Y_2}}(\Delta_0) \\ G_{1_{X_1}}(\Delta_0) & G_{1_{X_2}}(\Delta_0) & G_{1_{Y_1}}(\Delta_0) & G_{1_{Y_2}}(\Delta_0) \\ G_{2_{X_1}}(\Delta_0) & G_{2_{X_2}}(\Delta_0) & G_{2_{Y_1}}(\Delta_0) & G_{2_{Y_2}}(\Delta_0) \end{bmatrix}.$$

is the Jaccobian Matrix evaluated in $\Delta_0 = (X_{10}, X_{20}, Y_{10}, Y_{20}; r)$. Hence, the next approximations for $X_1(r)$, $X_2(r)$, $Y_1(r)$ and $Y_2(r)$ are as follows

$$\begin{cases} X_{1_1}(r) = X_{1_0}(r) + h_1(r), \\ X_{2_1}(r) = X_{2_0}(r) + k_1(r), \\ Y_{1_1}(r) = Y_{1_0}(r) + h_2(r), \\ Y_{2_1}(r) = Y_{2_0}(r) + k_2(r), \end{cases}$$

for all $r \in [0,1]$.

We can obtain approximated solution, $\forall r \in [0,1]$, by using the recursive scheme

$$\begin{cases} X_{1_{n+1}}(r) = X_{1_n}(r) + h_{1,n}(r), \\ X_{2_{n+1}}(r) = X_{2_n}(r) + k_{1,n}(r), \\ Y_{1_{n+1}}(r) = Y_{1_n}(r) + h_{2,n}(r), \\ Y_{2_{n+1}}(r) = Y_{2_n}(r) + k_{2,n}(r), \end{cases} \tag{7}$$

where $h_{i,0}(r) = h_i$ and $k_{i,0}(r) = k_i(r)$; $i = 1,2$ for $n = 0,1,2,\ldots$ Analogous to (6) $\forall r \in [0,1]$,

$$J(\Delta_n) \begin{bmatrix} h_{1,n}(r) \\ k_{1,n}(r) \\ h_{2,n}(r) \\ k_{2,n}(r) \end{bmatrix} = \begin{bmatrix} C_1(r) - F_1(\Delta_n) \\ C_2(r) - F_2(\Delta_n) \\ D_1(r) - G_1(\Delta_n) \\ D_2(r) - G_2(\Delta_n) \end{bmatrix}, \tag{8}$$

where $\Delta_n = (X_{1_n}, X_{2_n}, Y_{1_n}, Y_{2_n}; r)$. Now, we assume $J(\Delta_n)$ be nonsingular, then from (7) recursive scheme of Classical Newton's method (CN) is obtained as follows $\forall r \in [0,1]$,

$$\begin{bmatrix} X_{1_{n+1}}(r) \\ X_{2_{n+1}}(r) \\ Y_{1_{n+1}}(r) \\ Y_{2_{n+1}}(r) \end{bmatrix} = \begin{bmatrix} X_{1_n}(r) \\ X_{2_n}(r) \\ Y_{1_n}(r) \\ Y_{2_n}(r) \end{bmatrix} - J(\Delta_n)^{-1} \begin{bmatrix} F_1(\Delta_n) \\ F_2(\Delta_n) \\ G_1(\Delta_n) \\ G_2(\Delta_n) \end{bmatrix}, \tag{9}$$

where $n = 0,1,2,\ldots$

From *Midpoint Newton's method* (MN) in Section 3, by substitution of $J(\Delta_n)$ in (8) by $J(\frac{\Delta_n+\Delta'_n}{2})$, where $\Delta'_n = (X'_{1_n}, X'_{2_n}, Y'_{1_n}, Y'_{2_n}; r)$ and

$$\begin{bmatrix} X'_{1_n} \\ X'_{2_n} \\ Y'_{1_n} \\ Y'_{2_n} \end{bmatrix} = \begin{bmatrix} X_{1_n} \\ X_{2_n} \\ Y_{1_n} \\ Y_{2_n} \end{bmatrix} - J(\Delta_n)^{-1} \begin{bmatrix} F_1(\Delta_n) \\ F_2(\Delta_n) \\ G_1(\Delta_n) \\ G_2(\Delta_n) \end{bmatrix},$$

then recursive scheme for Midpoint Newton's method is obtained as follows

$$\begin{bmatrix} X_{1_{n+1}}(r) \\ X_{2_{n+1}}(r) \\ Y_{1_{n+1}}(r) \\ Y_{2_{n+1}}(r) \end{bmatrix} = \begin{bmatrix} X_{1_n}(r) \\ X_{2_n}(r) \\ Y_{1_n}(r) \\ Y_{2_n}(r) \end{bmatrix} - J(\frac{\Delta_n + \Delta'_n}{2})^{-1} \begin{bmatrix} F_1(\Delta_n) \\ F_2(\Delta_n) \\ G_1(\Delta_n) \\ G_2(\Delta_n) \end{bmatrix}, \tag{10}$$

where $n = 0,1,2,\ldots$ For initial guess, one can use the fuzzy number

$$\begin{cases} X_0 = (X_1(0), X_1(1), X_2(1), X_2(0)), \\ Y_0 = (Y_1(0), Y_1(1), Y_2(1), Y_2(0)), \end{cases}$$

and in parametric form

$$\begin{cases} X_{1_0}(r) = X_1(1) + (X_1(1) - X_1(0))(r-1), \\ X_{2_0}(r) = X_2(1) + (X_2(0) - X_2(1))(r-1), \\ Y_{1_0}(r) = Y_1(1) + (Y_1(1) - Y_1(0))(r-1), \\ Y_{2_0}(r) = Y_2(1) + (Y_2(0) - Y_2(1))(r-1) \end{cases}$$

when $X_1(0) \leqslant X_1(1) \leqslant X_2(1) \leqslant X_2(0)$ and $Y_1(0) \leqslant Y_1(1) \leqslant Y_2(1) \leqslant Y_2(0)$.
Remark 1. Sequence $\{(X_{1_n}, X_{2_n})\}_{n=0}^{\infty}$ and $\{(Y_{1_n}, Y_{2_n})\}_{n=0}^{\infty}$ convergent to (α_1, α_2) and (β_1, β_2), respectively, iff $\forall r \in [0,1], \lim_{n\to\infty} X_{1_n}(r) = \alpha_1(r)$, $\lim_{n\to\infty} X_{2_n}(r) = \alpha_1(r)$, $\lim_{n\to\infty} Y_{1_n} = \beta_1(r)$ and $\lim_{n\to\infty} Y_{2_n} = \beta_2(r)$.

Lemma 4.1 *Let*

$$\begin{cases} F(\alpha_1, \alpha_2) = (C_1, C_2), \\ G(\beta_1, \beta_2) = (D_1, D_2) \end{cases}$$

and if the sequence $\{(X_{1_n}, X_{2_n})\}_{n=0}^{\infty}$ *and* $\{(Y_{1_n}, Y_{2_n})\}_{n=0}^{\infty}$ *convergent to* (α_1, α_2) *and* (β_1, β_2)*, respectively, according to Trapezoidal Newton's method, then*

$$\lim_{n\to\infty} P_n = 0,$$

where

$$P_n = \sup_{0\leqslant r\leqslant 1} \max\{h_{1,n}(r), k_{1,n}(r), h_{2,n}(r), k_{2,n}(r)\}.$$

Proof. For $\forall r \in [0,1]$ in convergent case for $i = 1, 2$, we have

$$\lim_{n\to\infty} h_{i,n}(r) = \lim_{n\to\infty} k_{i,n}(r) = 0,$$

which completes the proof. □

Finally, under certain conditions of Theorem 3.1, for $n = 4$, can be assured that Midpoint Newton's method (MN) for fuzzy system (4) is convergent and that this convergence is quadratic.

5 Numerical application

In this section we will check the effectiveness of Midpoint Newton's method.

Example 5.1 *Consider the fuzzy nonlinear system*

$$\begin{cases} X^2 + Y^2 = (4.4, 5, 7), \\ X^2 + Y^3 + (1, 2, 3) = (8.6, 11, 17.2). \end{cases}$$

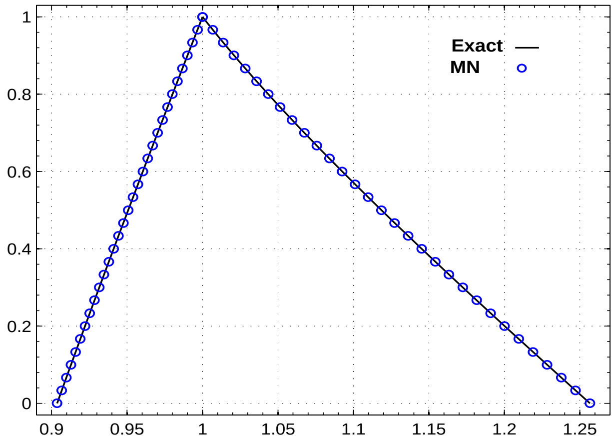

Figure 1: Solution of X

Without any loss of generality, assume that X and Y are positive, then the parametric form of this system is as follows:

$$\begin{cases} X_1^2(r) + Y_1^2(r) = (4.4 + 0.6r), \\ X_2^2(r) + Y_2^2(r) = (7 - 2r), \\ X_1^2(r) + Y_1^3(r) + (1 + r) = (8.6 + 2.4r), \\ X_2^2(r) + Y_2^3(r) + (3 - r) = (17.2 - 6.2r). \end{cases}$$

To obtain initial guess we solve above system for $r = 0$ and $r = 1$, therefore

$$\begin{cases} X_1^2(0) + Y_1^2(0) = 4.4, \\ X_2^2(0) + Y_2^2(0) = 7, \\ X_1^2(0) + Y_1^3(0) = 7.6, \\ X_2^2(0) + Y_2^3(0) = 14.2, \end{cases} \qquad \begin{cases} X_1^2(1) + Y_1^2(1) = 5, \\ X_2^2(1) + Y_2^2(1) = 5, \\ X_1^2(1) + Y_1^3(1) = 9, \\ X_2^2(1) + Y_2^3(1) = 9. \end{cases}$$

Consequently $X_1(0) = 0.903638$, $X_2(0) = 1.2567$, $Y_1(0) = 1.892997$, $Y_2(0) = 2.328240$, $X_1(1) = X_2(1) = 1$ and $Y_1(1) = y_2(1) = 2$. Therefore we obtain the initial guess $X_0 = (0.903638, 1, 1.2567)$ and $Y_0 = (1.892997, 2, 2.328240)$. After two iterations, we obtain the solution of X and Y by Midpoint Newton's method with the maximum error less than 10^{-6}, respectively, and by classical Newton's method after two iterations the maximum error would be less than 10^{-2}. For more details see Figures 1-2.

Now let us suppose X and Y are negative, then we have the following system,

$$\begin{cases} X_2^2(r) + Y_2^2(r) = (4.4 + 0.6r), \\ X_1^2(r) + Y_1^2(r) = (7 - 2r), \\ X_2^2(r) + Y_1^3(r) + (1 + r) = (8.6 + 2.4r), \\ X_1^2(r) + Y_2^3(r) + (3 - r) = 17.2 - 6.2r). \end{cases}$$

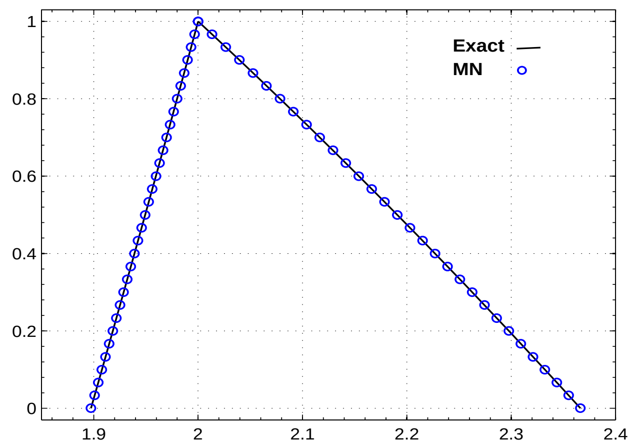

Figure 2: Solution of Y

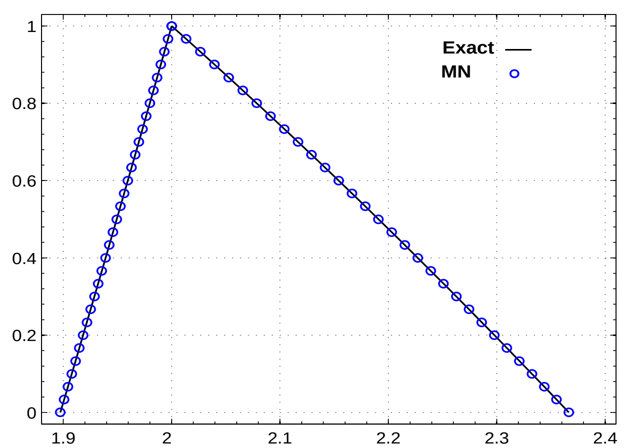

Figure 3: Solution of X

For $r = 0$, we have $Y_{01} = 2.018856$ and $Y_{02} = 2.242412$, therefore there are not negative roots.

Example 5.2 *Consider the fuzzy nonlinear system*

$$\begin{cases} X^2 + Y^2 = (12, 13, 17.6), \\ X^2 - \frac{1}{4}Y^2 = (0.6, 1.75, 3.5). \end{cases}$$

Without any loss of generality, assume that X and Y are positive, then the parametric form of this system is as follows:

$$\begin{cases} X_1^2(r) + Y_1^2(r) = (12 + r), \\ X_2^2(r) + Y_2^2(r) = (17.6 - 4.6r), \\ X_1^2(r) - \frac{1}{4}Y_2^2(r) = (0.6 + 1.15r), \\ X_2^2(r) - \frac{1}{4}Y_1^2(r) = (3.5 - 1.75r). \end{cases}$$

By solving above system for $r = 0$ and $r = 1$, the values $X_1(0) = 1.897366$, $X_2(0) = 2.366431$, $Y_1(0) = 2.898275$, $Y_2(0) = 3.464101$, $X_1(1) = X_2(1) = 2$ and $Y_1(1) = Y_2(1) = 3$ is concluded. Then we obtain the initial guess $X_0 = (1.897366, 2, 2.366431)$ and $Y_0 = (2.898275, 3, 3.464101)$. By applying Midpoint Newton's method, after two iterations, the maximum error for the obtained solution of X and Y would be less than 10^{-9}, and by classical Newton's method after two iterations the maximum error is less than 10^{-3}. For more details see Figures 3-4. Now suppose X and Y are negative, we have

$$\begin{cases} X_2^2(r) + Y_2^2(r) = (12 + r), \\ X_1^2(r) + Y_1^2(r) = (17.6 - 4.6r), \\ X_2^2(r) - \frac{1}{4}Y_1^2(r) = (0.6 + 1.15r), \\ X_1^2(r) - \frac{1}{4}Y_2^2(r) = (3.5 - 1.75r). \end{cases}$$

By solving the above system for $r = 0$ and $r = 1$, we obtain the initial guess $X_0 = (-2.366431, -2, -1.897366)$ and $Y_0 = (-3.464101, -3, -2.898275)$. If we apply two iterations from Midpoint Newton's method, the maximum error would be less than 10^{-9}, and by classical Newton's method after two iterations the maximum error is less than 10^{-3}, see Figures 5-6.

6 Conclusion

In this paper, we suggested numerical solving method for fuzzy nonlinear systems. This method is an implicit-type method. To implement this, we use Newton's method as predictor method and then use this method as corrector method. The method is discussed in detail. Several examples are given to illustrate the efficiency and advantage of this two-steep method.

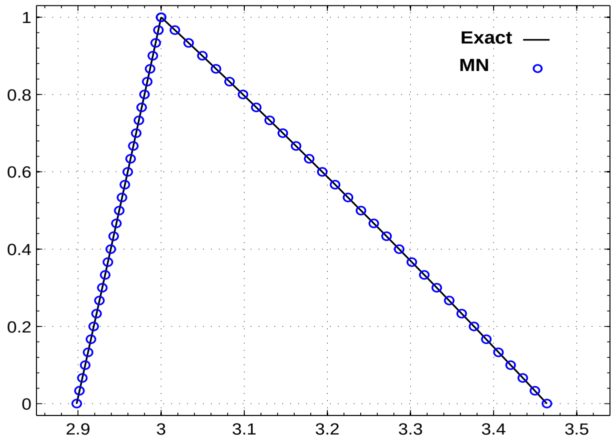

Figure 4: Solution of Y

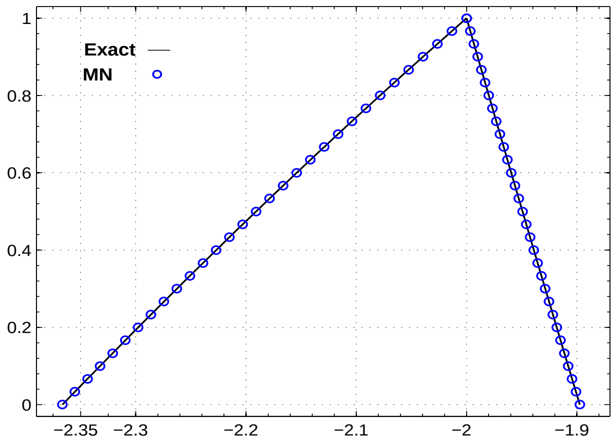

Figure 5: Solution of X

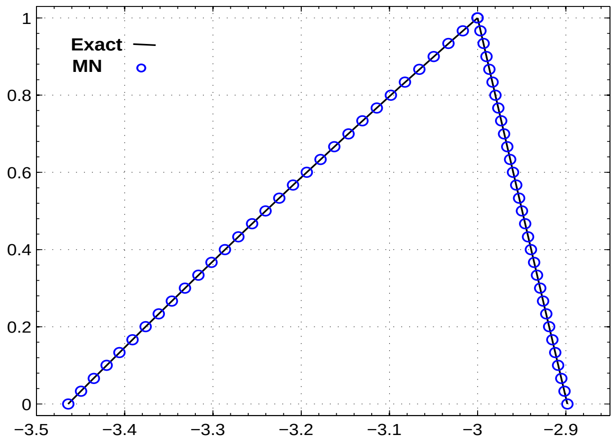

Figure 6: Solution of Y

References

[1] J. J. Buckley, Y. Qu, Solving fuzzy equations: A new solution concept, Fuzzy Set Syst. **39** (1991) 291-301.

[2] Y.J. cho, N.J. Huang, S.M. Kang, Nonlinear equations for fuzzy mapping in probabilistic normed spaces, Fuzzy Sets Syst. **110** (2000), 115-122.

[3] A. Corder, J.R. Torregrosa, Variant of Newton's method for functions of several variables, Appl. Math. Comput. **183** (2006), 199-208.

[4] D. Dubois, H. Prade, Fuzzy Sets and Systems: Theory and Application, Academic Press, new York, 1980.

[5] J. Fang, On nonlinear equations for fuzzy mapping in probabilistic normed spaces, Fuzzy Sets Syst. **131** (2002) 357-364.

[6] R. Goeschel, W. Voxman, Elementary caculus, Fuzzy Sets Syst. **18** (1986) 31-43.

[7] J. Ma, G. Feng, An approach to H_∞ control of fuzzy dynamic systems, Fuzzy Sets Syst. **137** (2003) 367-386.

[8] J.M. Ortega, Numerical Analysis, A Second course, SIAM, 1990.

[9] J.M. Ortega, W.C. Rheinboldt, Iterative Solution of Nonlinear Equations in Several Variables. Academic Press, Inc., 1970.

[10] L.A. Zadeh, Fuzzy sets, Inform. Contr. **8** (1965) 338-353.

[11] L.A. Zadeh, The concept of a linguistic variable and its application to approximation reasoning, Inform. Sci. **3** (1975) 199-249.

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