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Geometric Probabilities Problems for Social Network Analysis and Network Applications

Valeria Isgrò

University Mediterranea of Reggio Calabria, Italy

Sabrina Lo Bosco

Pegaso University, Italy

Michael Morabito

University of Messina, Italy

Abstract

In this work the authors investigate how tools from geometric probability can support the construction of classification criteria for Social Network Analysis, treating social, economic, and financial systems as networks embedded in a geometric framework. The study starts from the observation that contemporary markets generate vast quantities of data on prices, interactions, and decisions, which can be interpreted through network structures with nodes, arcs, and weighted relationships reflecting different roles and constraints. Within this perspective, both "material" networks, such as mobility and logistics infrastructures, and "immaterial" networks, such as financial systems or online social platforms, are modeled using regular lattices and random test bodies whose intersections represent potential interactions or interferences between flows and obstacles. The paper places particular emphasis on the historical development of geometric probability, recalling classic problems such as Buffon's needle and subsequent generalizations that introduced obstacles and more complex tessellations, then adapts these ideas to modern network applications.

Keywords: Social Network Analysis, Geometric Probability, Stochastic Geometry, Network Analysis

1 Introduction

The enormous availability of data that characterizes today's socio-economic and financial markets represents an extraordinary resource for understanding how economists and sector operators formulate effective decisions, grounding them in systematic processing of information and statistical analyses capable of distilling macroscopic dynamics from individual or institutional behaviors [17]. This informational richness, amplified by the pervasiveness of digital platforms and real-time interactions, enables mapping complex relationships emerging from flows of transactions, shares, and exchanges, generating reticular structures that capture the essence of contemporary social and economic dynamics. Unlike mobility and logistics networks or molecular models in statistical physics, financial and social systems exhibit distinctive traits such as relational asymmetry and contextual influence that necessitate tailored modeling approaches, while still allowing the use of shared mathematical toolkits to represent node configurations with differentiated weights and multiple constraints [11]. Whereas mathematics applied to economics and finance has historically favored a deductive method, the network analysis proposed here establishes a virtuous circle between an inductive approach rooted in the examination of vast observational datasets—and formal deduction, enriching the understanding of complex interactions [2]. In a landscape of growing global interdependencies, these elements assume urgency: local perturbations can spread exponentially, compromising systemic stability and testing structural resilience [2]. More broadly, in immaterial networks, graphical representations of concepts or theories in the social sciences privilege a specific dimension of social reality, employing node and edge structures to generate complex graphs [8] revisited in [6]. Social networks, arising from interpersonal constraints and opportunities, produce graphs analogous to origin-destination (OD) matrices in transportation systems, with nodes embodying individual actors and edges conveying differentiated input/output stresses [1, 9].

Geometric probability, a discipline that quantifies probabilities related to spatial arrangements of objects, traces its origins to Buffon's needle problem, conceived by the French naturalist in the eighteenth century while observing a game with sticks on tiled floors [10]. This enigma persists due to the elegance of its solutions, from Poincaré [12] to Stoka [15]. In the late 1990s and early

2000s, Stoka's team innovated by introducing obstacles into Buffon-Laplace problems on lattices [16]. These formulations have permeated optimization in transportation engineering via geometric probability on tessellations [13] and applications span GIS to neural networks via probabilistic intersections [14]. In SNA, geometric approaches model relational proximity through Euclidean embeddings, where node positions determine edge probabilities [5]. Data abundance reveals small-world properties balancing clustering and path lengths, alongside scale-free topologies from preferential attachment generating hubs [1]. Expanding to financial SNA, Minimum Spanning Trees from correlations cluster economic sectors, revealing banking hubs via k-correlations [17]. Shock propagation models quantify systemic risk through default cascades in networked agents [7, 4].

2 Geometric probabilities with disc body test

In this variation we consider a tile $\Re(a, b, \alpha)$ composed by regular fundamental cells C_0 represented as in fig. 1

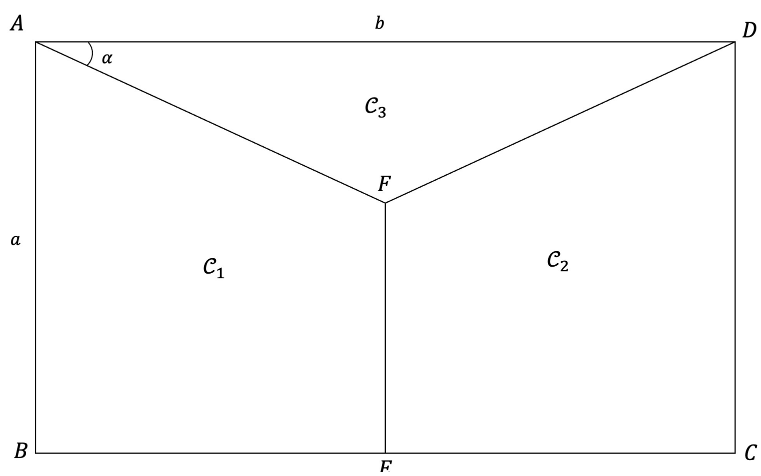

fig.1

where $a \leq b$ and $\alpha \in \left]0, \frac{\pi}{4}\right]$. We have that

$$areaC_0 = ab. \tag{1}$$

We consider a random disc d of constant radius r, with $r < min(a, \frac{b}{2})$. Our aim is to compute the probability P_{int} that this disc intersects a side of the lattice $\Re$. To be more specific, we denote by 0 the center of d, the disc position is determined by 0. In order to compute the probability P_{int} we consider the limiting positions of d, in each cell C_1, C_2 and C_3. We have the fig. 2

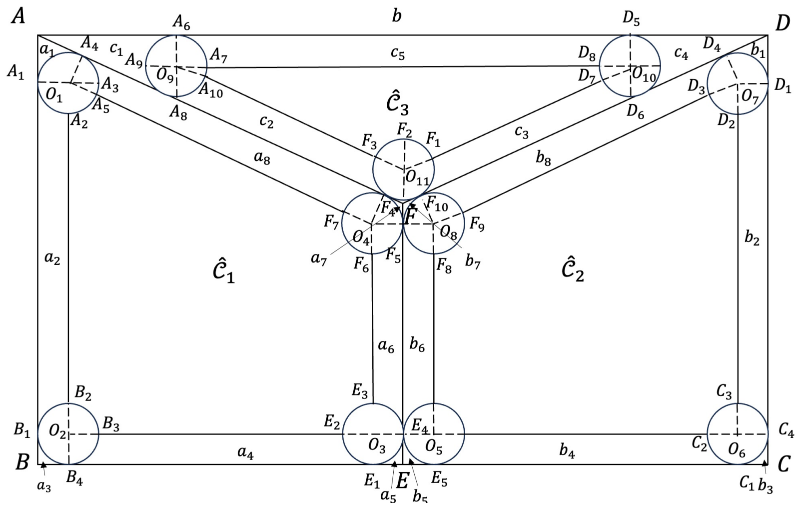

fig.2

and the relations

$$area\widehat{C}_1 = area\widehat{C}_2 = areaC_1 - \sum_{j=1}^{8} areaa_i - 4\pi r^2, \qquad (2)$$

$$area\widehat{C}_3 = areaC_3 - \sum_{j=1}^{5} areac_i - 3\pi r^2. \qquad (3)$$

Denoting by $M_i, (i = 1, 2, 3)$, the set of all discs d that their center in C_i. Denoting likewise by N_i the set of all discs d completely contained in C_i, we have that [2]:

$$P = 1 - \frac{2\mu(N_1) + \mu(N_3)}{2\mu(M_1) + \mu(M_3)} \qquad (4)$$

where μ denotes the Lebesgue measure in the Euclidean plane.

To compute the above measures we use the Poincaré kinematic measure [12]:

$$dK = dx \wedge dy.$$

We have

$$\mu(M_1) = \iint_{\{(x,y)\in C_1\}} dxdy = areaC_1,$$

$$\mu(M_3) = \iint_{\{(x,y)\in C_3\}} dxdy = areaC_3.$$

Then

$$2\mu(M_1) + \mu(M_3) = areaC_0 = ab. \qquad (5)$$

In the same way

$$\mu(N_1) = \iint\limits_{\{(x,y)\in\widehat{C}_1\}} dxdy = area\widehat{C}_1 = areaC_1 - \sum_{j=1}^{8} areaa_i - 4\pi r^2, \tag{6}$$

$$\mu(N_3) = \iint\limits_{\{(x,y)\in\widehat{C}_3\}} dxdy = area\widehat{C}_3 = areaC_3 - \sum_{j=1}^{5} areac_i - 3\pi r^2. \tag{7}$$

Substituting in (4) we obtain:

$$\begin{aligned} P_{int} = {} & \frac{1}{ab}\left(4a + \frac{2 - \sin\alpha + 3\cos\alpha}{\cos\alpha} b\right) r \\ & - \left[2\tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) - 4\cot\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) + 8\cot\alpha \right. \\ & \left. - \frac{\alpha}{2} + \frac{11\pi}{4}\right] r^2. \end{aligned} \tag{8}$$

3 Geometric probabilities with rectangle body test

In this variation we consider a tile $\Re(a, b, \alpha)$ composed by regular fundamental cells C_0 represented as in fig. 3

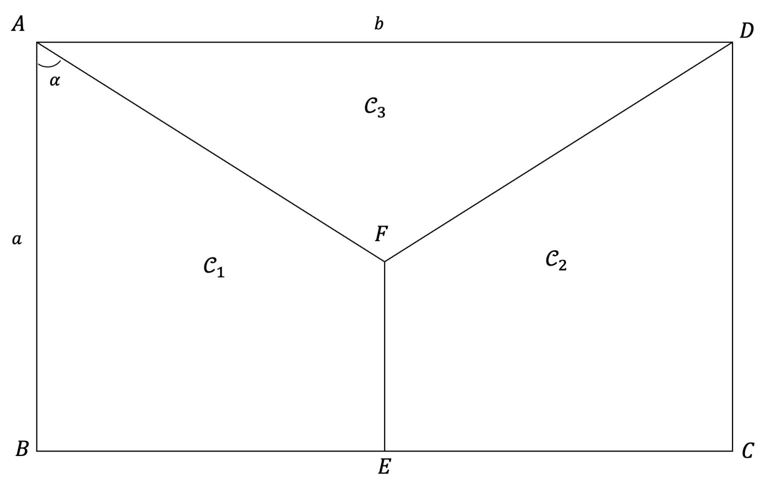

fig.3

where $\frac{a}{2} \leq b \leq a$ and $\alpha \leq \frac{\pi}{4}$. We have that

$$areaC_0 = ab. \tag{9}$$

Let r be a rectangle with side l and m with $0 \leq m < l$.

Our aim is to compute the probability P_{int} that this rectangle intersect a side of the lattice $\Re$. The position of r is determined by the center and by the angle ϕ. In order to compute the probability P_{int} we consider the limiting positions of r, in each cell C_1, C_2 and C_3 for a φ fixed. We have that fig. 4

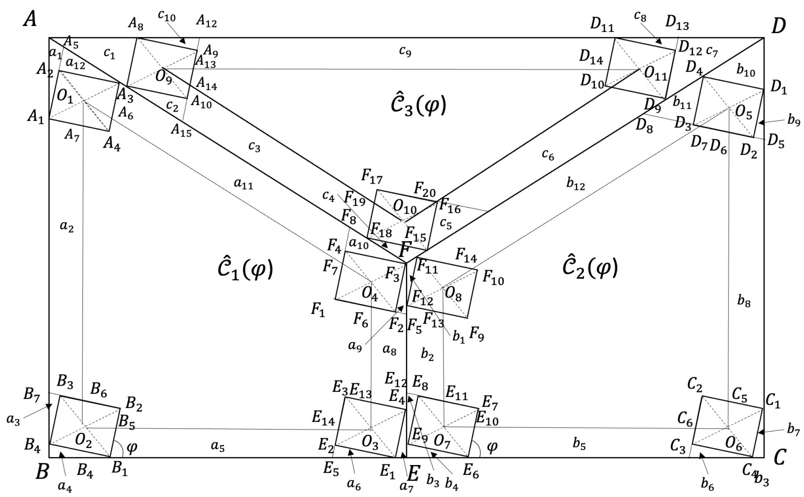

fig.4

and the relations

$$area\widehat{C}_{01} = areaC_{01} - -4lm - \sum_{i=1}^{12} areaa_i\left(\varphi\right),$$

$$area\widehat{C}_{02} = areaC_{02} - 4lm - \sum_{i=1}^{12} areab_i\left(\varphi\right),$$

$$area\widehat{C}_{03} = areaC_{03} - \sum_{i=1}^{10} areac_i\left(\varphi\right). \tag{10}$$

Denoting by $M_i, (i = 1, 2, 3)$ the set of all rectangles r that their center in C_{0i}. Denoting likewise by N_i the set of all rectangles r completely contained in C_{0i},

$$P_{int} = 1 - \frac{\sum_{i=1}^{3} \mu\left(N_i\right)}{\sum_{i=1}^{3} \mu\left(M_i\right)}, \tag{11}$$

where μ denotes the Lebesgue measure in the Euclidean plane.

To compute the above measures we use the Poincaré kinematic measure [12]:

$$dK = dx \wedge dy \wedge d\varphi.$$

We have

$$\begin{aligned}
\mu\left(M_i\right) &= \int\limits_{\{(x,y)\in C_{01}\}} dx\,dy = \int\limits_{\alpha}^{\frac{\pi}{2}} \left(areaC_{0i}\,d\varphi\right) \\
&= \left(\frac{\pi}{2}-\alpha\right) areaC_{0i}, \\
\sum_{i=1}^{3}\mu\left(M_i\right) &= \left(\frac{\pi}{2}-\alpha\right)\sum_{i=1}^{3} areaC_{0i} = \left(\frac{\pi}{2}-\alpha\right) ab.
\end{aligned} \tag{12}$$

In the same way

$$\mu\left(N_i\right) = \iint\limits_{\left\{(x,y)\in\widehat{C}_{0i}(\varphi)\right\}} dx\,dy = \int\limits_{\alpha}^{\frac{\pi}{2}} \left[area\widehat{C}_{0i}\left(\varphi\right)\right] d\varphi \tag{13}$$

Computing and substituting in (4) we obtain:

$$\begin{aligned}
P_{int} &= \frac{1}{ab}\Bigg\{\left(4a + \frac{2-\sin\alpha+3\cos\alpha}{\cos\alpha}b\right) r \\
&- \left[2\tan\left(\frac{\pi}{4}-\frac{\alpha}{2}\right) - 4\cot\left(\frac{\pi}{4}-\frac{\alpha}{2}\right) + 8\cot\alpha\right. \\
&\left. -\frac{\alpha}{2} + \frac{11\pi}{4}\right] r^2\Bigg\}.
\end{aligned} \tag{14}$$

4 Results and conclusion

The mathematical model proposed in this study, grounded in the application of geometric probability to regular networks with obstacles, unveils significant potential for analyzing complex systems both material and immaterial with particular emphasis on social, economic, and financial networks. By examining specific geometric configurations, such as regular tilings of fundamental cells with random test bodies (disks and rectangles), it becomes possible to quantify the probability of intersections between network flows and structural impediments, thereby providing a probabilistic framework for assessing resilience, vulnerability, and propagation dynamics. This approach not only generalizes classic problems like Buffon's needle but adapts them to contemporary contexts, where networks represent human interactions, financial transactions, or cognitive pathways, integrating kinematic and Lebesgue measures for precise and applicable calculations. The implications for Social Network Analysis

(SNA) are manifold and interdisciplinary. Firstly, in economic and financial systems modeled as directed graphs with agent nodes and edges representing price correlations or investment portfolios, such geometric probabilities serve as indicators of systemic risk. For instance, the intersection between a test body and the sides of a lattice can be simulated as a propagating ”collision” through the network, enabling estimation of the probability of cascading effects, such as liquidity crises or speculative bubbles. This tool supports strategic planning in banking, where interconnections among financial operators create increasingly dense networks: by quantifying ”impedance” analogous to electrical resistance in dynamic circuits post-crisis recovery processes can be optimized, resource allocation maximized, and sustainable development policies promoted, including job creation in interconnected sectors. Extending the analysis to immaterial networks, the proposed framework lends itself to modeling human psychological and decisional dynamics. Here emerges the concept of ”mental maps” or ”conceptual maps,” hierarchical radial structures representing the nonlinear flow of ideas in the human brain. In these configurations, nodes symbolize concepts or input/output stimuli, while edges indicate logical associations, with possible ”obstacles” reflecting cognitive barriers such as biases or informational gaps. In conclusion, the methodological contribution outlined here elevates geometric probability to a bridge between rigorous modeling and practical decisional tools for SNA and beyond. It not only quantifies interactions and interferences in multidimensional spaces but inspires innovations across heterogeneous fields, from economic resilience to cybersecurity, via cognitive psychology and logistical optimization.

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