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Information Retrieval Using Near Fuzzy Sets

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Abstract

This article presents a concise overview of near fuzzy sets, a class of descriptively near sets developed within the framework of fuzzy theory. Near fuzzy sets were first introduced by fuzzy similarity relations and fuzzy membership functions. It is observed that each level value of a fuzzy similarity relation induces a corresponding partition of the underlying data set, which naturally leads to the problem of selecting an appropriate level value that yields a meaningful and effective partition. Subsequently, near fuzzy sets are reformulated using fuzzy membership functions interpreted as probe functions, providing an alternative and flexible construction. The theoretical concepts are supported and clarified through illustrative examples, highlighting the applicability and structural properties of near fuzzy sets.

Keywords: Fuzzy set, Near set, Near fuzzy set

1 Introduction

Information systems constitute a fundamental component of contemporary computational paradigms, particularly within the contexts of generative artificial intelligence and machine learning [1,2]. Formally, an information system may be represented as a tuple consisting of a universe of objects, associated attributes, and the relations that integrate these components [3]. Information retrieval and processing in such multimodal systems continue to pose substantial theoretical and practical challenges, thereby remaining a central area of research in computational intelligence and mathematical logic [4]. The remarkable advancements achieved in this field over the past century have been supported by the systematic development of diverse mathematical theories and formal models [3, 5].

Information retrieval in complex and uncertain environments presents considerable challenges, as traditional probability theory is often inadequate for modeling all forms of uncertainty and randomness [6]. The concept of *fuzzy sets*, introduced by L. A. Zadeh [5], offers a more robust and practical framework for addressing issues arising from non-specificity and imprecision in data representation. This paradigm established a novel approach to computation and knowledge acquisition, enabling the modeling of vagueness inherent in real-world information. Subsequently, the theory of *rough sets*, proposed by Z. Pawlak [3], provided an alternative mechanism of approximation within the domain of information systems, focusing on the indiscernibility of objects based on available attributes. Building upon these foundations, Dubois and Prade introduced the notions of *rough fuzzy sets* and *fuzzy rough sets* [7], thereby integrating the complementary strengths of fuzzy and rough set theories to enhance the handling of uncertainty and granularity in data analysis.

The concept of *near sets*, introduced through the *nearness relation* by J. F. Peters [8,9], extends the framework of perceptual information systems by providing a mathematical basis for describing perceptual similarity. In this context, perceptual objects are regarded as entities that exist in the real world and are perceivable by the human mind [10]. Near sets arise from the combination of nearness relations and *probe functions*, which act as mathematical sensors to capture and represent perceptual information [11]. Over the past two decades, *near set theory* has evolved into a well-established mathematical framework and is frequently regarded as an extension of *rough set theory* [12]. In our previous work [20], we introduced a fuzzy similarity relation to derive fuzzy equivalence classes, which were subsequently applied to medical image segmentation. It was observed that, for each value in the level set of the fuzzy similarity relation, a corresponding partition of the data could be obtained. This naturally raised the question of how to select the most appropriate value from the level set to achieve a suitable partition of the given set. This issue was further investigated in our subsequent work [20], where we proposed the concept of a near set derived from the fuzzy similarity relation. The principal objective of the present study is to demonstrate that the notion of near sets can be viewed as an extension of fuzzy sets, which we term near fuzzy sets.

2 Near sets through fuzzy similarity relation

The distinction between spatial intersection and descriptive intersection as introduced by Peters was further examined in [13]. In classical set theory, two sets are considered near if and only if their intersection is nonempty. In contrast, under descriptive intersection, it is not required that the sets share actual elements; rather, nearness may arise from the presence of matching descriptive attributes. The corresponding proximity relations defined on such descriptively similar sets were also investigated in the work of Peters and Naimpally [14].
In this section, we recall some basic notions of near fuzzy sets. For the purpose of this paper we confine ourselves the consideration on only the finite version of the universe of discourse. Let X be the universe of discourse and γ be a fuzzy binary relation defined on X as given in [20]. For each value of $\alpha \in \Lambda(\gamma)$, ${}^{\alpha}\gamma$ is a fuzzy equivalence relation and splits X into disjoint equivalence classes.Let $\mathcal{O}$ denoted the set of all disjoint equivalence classes obtained by each quotient set 'O/α_i'

Definition 2.1. Let $\mathcal{O}$ be the set of all perceptual objects and $\phi : \mathcal{O} \to \mathcal{R}$ is given by $\mathcal{O}(x) = \frac{|[x]|}{|\mathcal{O}/[x]|}$ where $|[x]|$ is the cardinality of the each equivalence classes and $|\mathcal{O}/[x]|$ denote the number of equivalence classes in each partition.

Hence by definition 1.1 we clearly observe that $< \mathcal{O}, F >$ be a perceptual objects with $\mathcal{O} = \{[x]_\alpha / \forall \alpha \in \Lambda(\gamma)\}$ and $F = \{\phi_i / \forall i \in I\}$.

Definition 2.2. A fuzzy perceptual system $< \mathcal{O}, F >$ consists of a non empty set $\mathcal{O}$ of perceptual objects as equivalence classes and a non empty set F of probe functions defined.

Definition 2.3. Let $< \mathcal{O}, F >$ be a fuzzy perceptual system. $\forall$,$B \subsetneq F$, the indiscernibility relation is defined as follows:

$$\omega_B = \{([x]_{\alpha_i}, [y]_{\alpha_i}) \in \mathcal{O} \times \mathcal{O} \mid \forall\, \mathcal{O}/\alpha_i, \mathcal{O}/\alpha_j,\ such\ that\, \phi_i(x) = \phi_j(y)\}$$

Definition 2.4. Let $< \mathcal{O}, F >$ be a fuzzy perceptual system. $\forall$,$B \subsetneq F$, the weak indiscernibility relation is defined as follows:

$$\hat{\omega_B} = \{([x]_{\alpha_i}, [y]_{\alpha_i}) \in \mathcal{O} \times \mathcal{O} \mid \exists\, \mathcal{O}/\alpha_i, \mathcal{O}/\alpha_j,\ such\ that\, \phi_i(x) = \phi_j(y)\}$$

Definition 2.5 (Weak near fuzzy relation)**.** Let $< \mathcal{O}, F >$ be a fuzzy perceptual system and $X, Y \subsetneq \mathcal{O}$. A set X is weakly near fuzzy $\left(\hat{\delta}\right)$ to set Y within the fuzzy perceptual system,

$$< \mathcal{O}, F > \Rightarrow \exists\, an\, x \in X\, and\, y \in Y\, such\, that\, x\hat{\omega_B}y\, for\, some\, B \subsetneq F$$

Definition 2.6 (Near fuzzy relation)**.** Let $< \mathcal{O}, F >$ be a fuzzy perceptual system and $X, Y \subsetneq \mathcal{O}$. A set X is weakly near fuzzy (δ) to set Y within the fuzzy perceptual system,

$$< \mathcal{O}, F > \Rightarrow \forall\ x \in X\, and\, y \in Y\, such\, that\, x\omega_F y$$

The above definitions are essentially the by-products of applying the probe functions to the set of equivalence classes. It has been observed that this type of set plays a particularly significant role in image-processing applications, where understanding how an image behaves under various equivalence relations provides meaningful structural insight.
In many practical scenarios, digital images exhibit features that can be grouped, compared, or transformed in ways that naturally lead to such classifications, making these equivalence classes a useful analytical tool. To further substantiate the definitions introduced earlier, we now consider an example presented in [15]. This example highlights how the same digital image can be classified differently for various values of $\alpha \in \Lambda(\gamma)$.
Example1 Consider the value of $\alpha = 0.02$ and $\alpha = 0.06$ as given in the **figure 1**. The quotient sets corresponding to each α is given below:

$$\mathcal{O}/0.02 = \{x_1, x_2, x_3, x_4\}\ where\, x_1 = \{1,2,3\}\, ; x_2 = \{6,7,10\}\, ; x_3 = \{8\}\ and\, x_4 = \{4,9\}$$

Similarly,

$$\mathcal{O}/0.06 = \{x_5, x_6, x_7, x_8\}\ where\, x_5 = \{1,6,8\}\, ; x_6 = \{3,7\}\, ; x_7 = \{2,5\}\ and\, x_8 = \{4,9,10\}$$

The probe functions given in the **Definition 1.1** is calculated for this example and summarized in **Table 1**
Hence, a weak near fuzzy set obtained through the probe function is as follows:

O/F	ϕ_1	ϕ_2	ϕ_3
x_1	$\frac{3}{4}$	0	0
x_2	$\frac{3}{4}$	0	0
x_3	0	$\frac{1}{4}$	0
x_4	0	0	$\frac{1}{2}$
x_5	0	$\frac{1}{4}$	0
x_6	$\frac{3}{4}$	0	0
x_7	0		$\frac{1}{2}$
x_8	$\frac{3}{4}$	0	0

Table 1: Probe functions: Definition 1.1

$$WeakNearfuzzyset,\ \mathsf{WNF} = \{(x_1, x_2, x_5, x_8)\, , x_3, (x_4, x_6, x_7))\}$$

3 Near fuzzy sets

Let X be a non empty set with finite universe of discourse and $\mathcal{F}(\mathcal{X})$ is a fuzzy set defined on X. Let $\mathcal{P}(\mathcal{X})$ be the power set of $\mathcal{F}(\mathcal{X})$. Then we've the following definitions:

Definition 3.1. Let

$$A(x),\ B(x) \in \mathcal{P}(X),$$

and let

$$\mu : A(x) \to [0,1], \qquad \theta : B(x) \to [0,1],$$

and suppose there exists some $x \in X$ such that

$$\mu_A(x) = \theta_B(x).$$

In this context, the proposed definition suggests that for certain values within the given set X, the membership grades of corresponding subsets of X are established as equal. This equivalence condition is the mechanism by which the definition produces certain near sets, which we termed as **near fuzzy sets.** Hence our definition of the near fuzzy set is not based on an indiscernibility relation while it's evolved through the equivalence of fuzzy membership function. The fuzzy membership function is going to act as a probe function.

Definition 3.2 (Near fuzzy set)**.** Let

$$A(x),\ B(x) \in \mathcal{P}(X),$$

then weak near fuzzy set is given as:

$$\{x \in X/\exists x \in X, \mu_A(x) = \theta_B(x)\}$$

Proposition 3.3. *Near fuzzy set in the above definition is a descriptively near set.*

Proof. Let

$$x \in \Psi \Rightarrow \{x \in X/\exists x \in X, \mu_A(x) = \mu_B(x)\} \Rightarrow$$

It's a collection of elements of x such that it have some matching descriptions. Hence it's a descriptive near set. □

Proposition 3.4. *Near fuzzy set is not a fuzzy set.*

Proof. It's clear from the definition. □

Proposition 3.5. *Near fuzzy set is a weak near set.*

Proof. It follows from the definition □

A near fuzzy set,Ψ is defined with respect to an ordinary fuzzy set. However, it can be observed that only a descriptively weak near fuzzy set can be derived from an ordinary fuzzy set. The relevance of near fuzzy sets can be illustrated through the following example.
Example 1: Let X denote a digital image and let $\mathbf{F}(\mathbf{X})$ be a fuzzy set of X. In an image segmentation problem, the information what we are looking for in the 2^k set of $\mathbf{F}(\mathbf{X})$. An image analyst seeks to extract as much relevant information as possible from this set. By employing different classification methods, the image can be partitioned into several disjoint classes. Although these classes are obtained independently, there may exist partial or approximate correspondences among them. Such correspondences motivate the development of near fuzzy sets, which provide a framework for establishing new relationships among disjoint sets based on the associated fuzzy membership functions.
Based on the above definition, a question can naturally arise. What happens if the fuzzy set is not an ordinary fuzzy set. This item is addressed for the Atanassov intutionistic fuzzy set.If the fuzzy set is an Atanassov intutionistic fuzzy set .

Definition 3.6 (Atanassov Weak near fuzzy set Type 1)**.** Let

$$A(x),\ B(x) \in \mathcal{P}(X),$$

$\Rightarrow$ membership functions $\mu_{A(x)},\ \mu_{B(x)}$ and non membership functions $\nu_{A(x)}$, $\nu_{B(x)}$ then Atanassov weak near fuzzy set is given as:

$$\{x \in X/\exists x \in X, \mu_A(x) = \mu_B(x)\ and\ \nu_A(x) = \nu_B(x)\ \}$$

Definition 3.7 (Atanassov weak near fuzzy set Type 2)**.** Let

$$A(x),\ B(x) \in \mathcal{P}(X),$$

$\Rightarrow$ membership functions $\mu_{A(x)}, \mu_{B(x)}$ and non membership functions $\nu_{A(x)}$, $\nu_{B(x)}$ then Atanassov weak near fuzzy set is given as:

$$\{x \in X/\exists x \in X, \mu_A(x) = \mu_B(x)\ and\ \nu_A(x) \neq \nu_B(x)\}$$

Definition 3.8 (Atanassov weak near fuzzy set Type 3)**.** Let

$$A(x),\ B(x) \in \mathcal{P}(X),$$

$\Rightarrow$ membership functions $\mu_{A(x)}, \mu_{B(x)}$ and non membership functions $\nu_{A(x)}$, $\nu_{B(x)}$ then Atanassov weak near fuzzy set is given as:

$$\{x \in X/\exists x \in X, \mu_A(x) \neq \mu_B(x)\ and\ \nu_A(x) = \nu_B(x)\}$$

Definition 3.9 (Atanassov near fuzzy set)**.** Let

$$A(x),\ B(x) \in \mathcal{P}(X),$$

$\exists$ membership functions $\mu_{A(x)}\ \mu_{B(x)}$ and non membership functions $\nu_{A(x)}\ \ \nu_{B(x)}$ then Atanassov weak near fuzzy set is given as:

$$\{x \in X/\forall x \in X, \mu_A(x) = \mu_B(x) \ and\ \nu_A(x) = \nu_B(x)\}$$

The above definition highlights the significant role of near fuzzy sets within an information system. Near fuzzy sets can be employed as a defuzzification criterion for fuzzy sets, enabling the extraction of representative information from fuzzy membership functions. By matching the descriptive properties of fuzzy membership values, relevant information is retrieved and utilized to define a new form of clustering.

Furthermore, near fuzzy sets derived from Atanassov's intuitionistic fuzzy sets can be interpreted within a lattice-theoretic framework, where the resulting structures correspond to concept lattices originating from Formal Concept Analysis,(FCA) [16] a mathematical framework rooted in order theory and lattice theory for the systematic analysis of binary relations between objects and attributes. An FCA context is formally defined as a triple (G, M, I), where G denotes a finite set of objects, M denotes a finite set of attributes, and $I \subseteq G \times M$ represents the incidence relation specifying which objects possess which attributes. A formal concept is defined as a pair (A, B), where $A \subseteq G$ and $B \subseteq M$, such that $A' = B$ and $B' = A$, with derivation operators mapping object sets to their common attributes and vice versa. The collection of all formal concepts derived from a context, ordered by extent inclusion (or dually by reverse intent inclusion), forms a complete lattice known as the concept lattice, in which the meet and join operations correspond to the greatest common specialization and least common generalization of concepts, respectively. [17]

The application of near fuzzy concept lattices in machine learning and data analytics is proposed as a direction for future research. This paper introduces the concept of near fuzzy sets derived from established fuzzy set frameworks. While conventional clustering algorithms typically produce a single partition of the data, concept lattice–based methods rely on object–attribute relationships to generate a richer, more structured representation. Near fuzzy concept lattices therefore offer promising potential for future development, particularly in the context of explainable AI and rule-extraction models.

4 Conclusion

In this paper, we introduced the concept of near fuzzy sets, which naturally emerges from the established fuzzy set framework. We further demonstrated that a near fuzzy set can be interpreted as an alternative defuzzification approach for fuzzy sets. The potential applications of this framework in explainable AI and concept lattice theory suggest promising directions for future research. In particular, exploring how near fuzzy sets can enhance interpretability and knowledge representation may yield valuable insights across a range of intelligent systems.

References

[1] S. Russell and P. Norvig, *Artificial Intelligence: A Modern Approach*, 4th ed., Pearson, 2021.

[2] I. Goodfellow, Y. Bengio, and A. Courville, *Deep Learning*, MIT Press, 2016.

[3] Z. Pawlak, Rough sets, *International Journal of Computer & Information Sciences*, **11** (1982), no. 5, 341–356. https://doi.org/10.1007/bf01001956

[4] D. C. Blair and G. Salton, *Information Retrieval: Data Structures and Algorithms*, Springer, 2017.

[5] L. A. Zadeh, Fuzzy sets, *Information and Control*, **8** (1965), no. 3, 338–353. https://doi.org/10.1016/s0019-9958(65)90241-x

[6] L. A. Zadeh, Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets and Systems*, **1** (1978), no. 1, 3–28. https://doi.org/10.1016/0165-0114(78)90029-5

[7] D. Dubois and H. Prade, Rough fuzzy sets and fuzzy rough sets, *International Journal of General Systems*, **17** (1990), no. 2–3, 191–209. https://doi.org/10.1080/03081079008935107

[8] J. F. Peters, Near sets: General theory about nearness of objects, *Applied Mathematical Sciences*, **1** (2007), no. 53, 2609–2629.

[9] J. F. Peters, Foundations of near sets, *Information Sciences*, **245** (2009), 57–74. https://doi.org/10.1016/j.ins.2009.04.018

[10] J. F. Peters, Perceptual objects, near sets, and information granules, in *Advances in Intelligent and Soft Computing*, vol. 57, Springer, 23–34, 2009.

[11] J. F. Peters, Near sets: An introduction, *Mathematics in Computer Science*, **5** (2013), no. 1, 29–57. https://doi.org/10.1007/s11786-013-0149-6

[12] J. F. Peters and A. Skowron, Rough and near sets in the foundations of granular computing, *Information Sciences*, **293** (2015), 97–114.

[13] J.F. Peters, Near Sets: An Introduction, *Mathematics in Computer Science*, **7** (2013), 3-9. https://doi.org/10.1007/s11786-013-0149-6

[14] Jim Peters and Som Naimpally, Application of near sets, *Notice of the AMS*, **59** (4) (2012). https://doi.org/10.1090/noti817

[15] Rogi Jacob, Sunny Kuriakose A., Near sets through fuzzy similarity relation, *Applied Mathematical Sciences*, **8** (2014), no. 41, 2035-2040. https://doi.org/10.12988/ams.2014.42104

[16] B. Ganter and R. Wille, *Formal Concept Analysis: Mathematical Foundations*, Springer, Berlin, 1999. ttps://doi.org/10.1007/978-3-031-63422-2

[17] R. Wille, Restructuring lattice theory: An approach based on hierarchies of concepts, in *Ordered Sets*, I. Rival (Ed.), Springer, Dordrecht, 1982, 445–470. https://doi.org/10.1007/978-94-009-7798-3_15

[18] S. O. Kuznetsov, Machine learning and Formal Concept Analysis, *Information Sciences*, **134** (2011), no. 1–4, 3–21.

[19] Rogi Jacob, Sunny Kuriakose A., Medical Image Segmentation Using Fuzzy Binary Relation, *International Journal of Fuzzy Mathematics & Systems*, **2** (1) (2012).

[20] Rogi Jacob, Sunny Kuriakose A., Near sets through fuzzy similarity relation, *Applied Mathematical Sciences*, **8** (2014), no. 41, 2035-2040. https://doi.org/10.12988/ams.2014.42104

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