**Applied Mathematical Sciences, Vol. 20, 2026, no. 4, 257 - 264**
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**https://doi.org/10.12988/ams.2026.919360**

# Morphological Diversity Controlled by a Parameter from a Non-Iterative Functional Building Block

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**Abstract**

We present several families of point distributions generated by deterministic, non-iterative functions constructed from a common elementary functional building block [1]. The general analytical structure remains unchanged, while one or only a few constants are modified.Despite these limited variations, the functions produce remarkably different spatial organizations, including banded structures, two-cluster configurations, symmetric four-lobed patterns, filled and annular domains, quasi-one-dimensional contours, as well as highly anisotropic shapes exhibiting cusps and caustic-like structures.In several cases, varying a single constant induces a gradual morphological transformation, such as the approach and eventual merging of two point clouds, the evolution of a filled domain into an increasingly thin annulus, or the transition from an annular structure to cusp-shaped and strongly anisotropic geometries.Some configurations exhibit visual similarities to structures encountered in mathematics and physics, including interference patterns [2], the spatial separation observed in the Stern–Gerlach experiment [3], and geometries associated with caustics and wavefront singularities [4,6]. These similarities are purely morphological and do not imply any equivalence with the underlying physical mechanisms.These results demonstrate that a relatively simple deterministic non-iterative functional construction can generate a wide diversity of geometric organizations through only very limited parametric variations.

## Introduction

The emergence of complex spatial structures from relatively simple mathematical rules is encountered in many areas of mathematics and physics. Classical examples include interference patterns [2], beam splitting in the Stern–Gerlach experiment [3], caustics and wavefront singularities [4,6], as well as a variety of general pattern formation phenomena [7].The approach developed here is fundamentally different. Rather than reproducing the physical equations responsible for these phenomena, our objective is to investigate the geometric diversity generated by deterministic non-iterative functions built from a common elementary functional building block. The overall functional architecture remains unchanged while only a small number of constants are modified. The resulting point-cloud morphologies are then analyzed.A first configuration exhibits a banded organization with varying density, visually reminiscent of certain interference patterns [1].A second family consists of two dense regions separated in space. By varying a single constant, their separation can be continuously controlled until they merge into a single point cloud. The initial configuration displays a purely visual analogy with the spatial separation of two components observed in the Stern–Gerlach experiment [3].A third configuration produces a symmetric four-lobed structure.Another one-parameter transformation progressively changes a filled two-dimensional domain into a configuration composed of a central region surrounded by a peripheral ring, then into a true annular domain with a central cavity, and finally into an extremely thin annulus approaching a quasi-one-dimensional support.Finally, another family evolves gradually from an annular geometry toward curved sectors, cusp-shaped structures, and highly anisotropic fan- or wedge-like configurations. Some of these geometries display visual similarities to caustics, singular envelopes, and wavefront structures described in the mathematical and optical literature [4,6].Throughout this work, these comparisons remain strictly morphological. No equivalence with interference phenomena, quantum processes, optical propagation, or any other physical mechanism is proposed.The sole objective is to demonstrate that a single deterministic, non-iterative functional construction can generate a broad variety of morphologies through the variation of only a very limited number of constants.

## 1. Banded Structure

```
x=1:1:0.55*10^5;
y1=0.5*acot(0.43*sin(x.^5));
y2=acot(1.5+cot(x.^25))+0.5*acot(0.1*cos(x.^35));
y3=acot(0.15*cos(x.^37));
y4=acot(0.35*cos(x.^3.7));
y=y1+y2+y3+y4;  plot(y,'.')
```

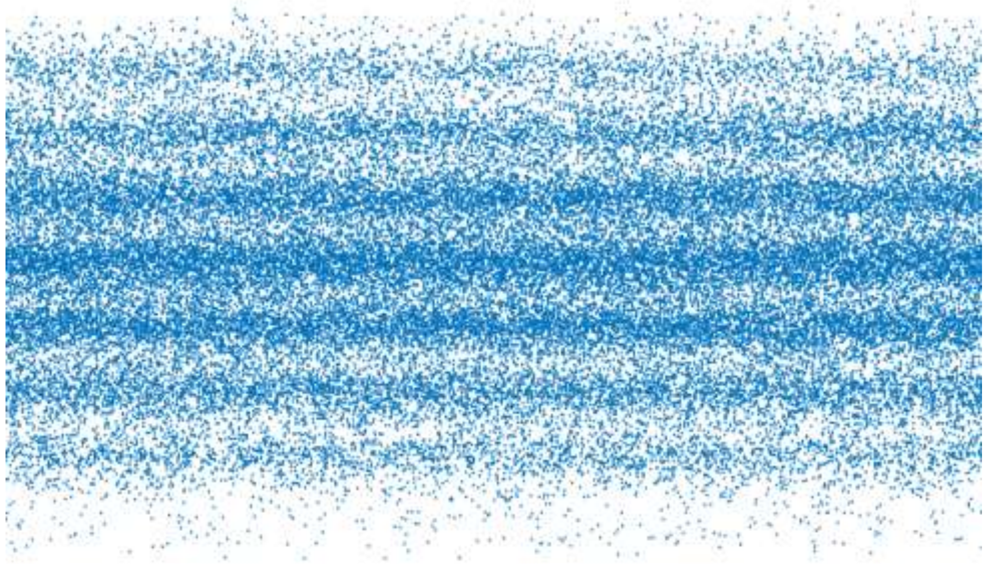


**Fig. 1.** Banded structure.

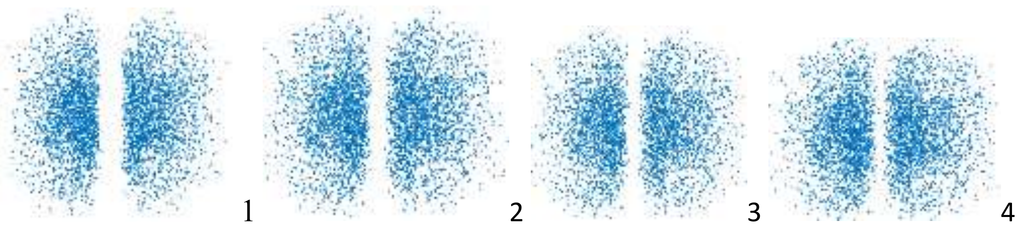
Point distribution generated by a non-iterative function constructed from the common functional building block. The organization into bands of varying density exhibits a morphological resemblance to certain interference patterns [2], without assuming the presence of any wave or interference mechanism.

## 2. Pattern Analogous to the Stern–Gerlach Experiment

The Stern–Gerlach experiment demonstrates the separation of an atomic beam into discrete components under the action of a non-uniform magnetic field [4,5].In the present model, two distinct point clouds are produced using the same elementary building blocks, without introducing either a magnetic field or a spin variable .

```
x=1:1:4*10^3;
 (8) k = 4; (7) k=5 ; (6) k=6 ; ;(5) k=7; (4) k=8 ; (3) k=9 ; (2) k=10 ;(1) k=15 ;

 ya = 2*acot(cos(x.^50))+k*acot(cos(x.^40))+acos(sin(x.^31))+asin(cos(x.^30));
 yb =atan(sin(x.^5))+atan(cos(x.^4))+50*asin(cos(x.^3.1))+2*acos(sin(x.^3));
 y1=3*ya.*(2+cos(x.^3.3));
 y2=yb.*(2+sin(x.^3.3));
 plot(y1,y2,'.');
```



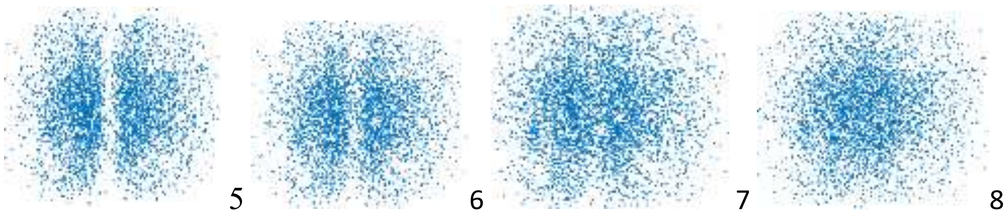



**Fig. 2.** Progressive merging of two point clouds.

Eight successive configurations obtained by varying a single constant while keeping all other parameters unchanged. Two initially separated dense regions gradually approach one another until they merge into a single point cloud. The initial configuration presents a visual analogy with the spatial separation of two components observed in the Stern–Gerlach experiment [3], without any physical implication..

## 3. Four-Lobed Structure

The third distribution exhibits four main regions of high density arranged symmetrically around a central area. Two approximately orthogonal separations divide the distribution into four components.Geometrically, this configuration may be described as a four-lobed structure. Its symmetry and spatial organization differ markedly from both the banded pattern and the two-cluster morphology.The existence of this third morphology is significant because it is obtained from the same functional building block as the previous configurations.

```
x=1:1:4*10^4;
ya=acot(cos(x.^7))+9*acot(cos(x.^8))+acos(sin(x.^31))+asin(cos(x.^3.7));

yb=atan(sin(x.^5))+atan(cot(x.^4))+50*acot(cos(x.^3.1))+4*acos(sin(x.^3));
y1=ya.*(2+cos(x.^3.3));
y2=0.5*yb.*(2+sin(x.^3.3));
plot(y1,y2,'.');
```

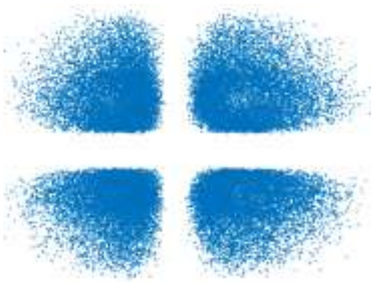


**Fig. 3.** Symmetric four-lobed structure.

Point distribution forming a four-lobed configuration exhibiting approximate symmetry with respect to two axes. This structure represents another geometric morphology generated by the same family of deterministic non-iterative functions; no specific physical interpretation is assigned to this quadrilobed pattern.

## 4. One-parameter controlled morphological transformation: from a full domain to a quasi-one-dimensional annular structure.

We present a morphological evolution obtained by varying a single constant. The distribution successively transitions from a two-dimensional filled domain to an arrangement comprising a dense central region surrounded by a crown, and then to an annular structure with a central cavity. By continuing to vary the same parameter, the thickness of the crown decreases sharply until it produces a seemingly quasi-one-dimensional support. The enlargement of the latter shows, however, that its contour consists of discrete points. From a strictly visual standpoint, this succession evokes certain density or intensity profiles encountered in confined physical systems, notably core-shell structures, annular distributions, and modes localized on a ring. This comparison is exclusively morphological and does not imply any identification with a particular physical mechanism.

```
x=1:1:1*10^4;
k=10^5; (4)   k=3;(3)    k=0.25;(2)      k=0; % (1)

y1=sin(sin(x.^3.3)).*(k+sin(x.^35));
y2=sin(cos(x.^3.3)).*(k+sin(x.^35));
plot(y1,y2,'.');
```

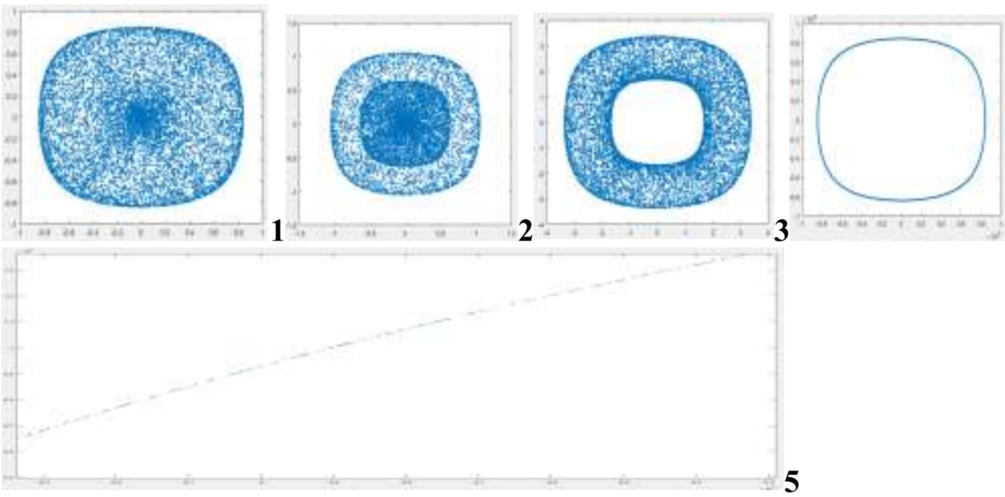



**Fig. 4.** Evolution of a point cloud controlled by the variation of a single constant : (1) filled domain; (2) structure composed of a dense central region and a peripheral crown; (3) annular distribution with a central cavity; (4) crown of extremely small

thickness forming a quasi-one-dimensional contour; (5) enlargement of a portion of configuration (4), revealing the discrete nature of the contour. The whole presents a visual resemblance with certain confined profiles observed in physics, without implying underlying physical correspondence.

## 5. One-parameter deformation of caustic-like geometric patterns

The figure presents a series of nine patterns obtained by varying a single constant, all other parameters remaining unchanged. The transformation progressively evolves from an annular structure towards curved sector shapes, then towards cuspidal shapes and finally towards strongly anisotropic configurations in a fan or wedge shape. Some of these geometries present visual resemblances with structures encountered in mathematics and physics, notably annular sectors, caustics, or singular envelopes. These comparisons remain strictly morphological and do not constitute a physical interpretation.

```
k=3.5;  k=3.49;   k=3.47; k=3.43;  k=3.4; k=3;k=2.5 ; k=1.5;   k=1 ;
              x=1:1:10^4;
y1=cos(sin(x.^3)).*(cos(x.^7));
y2=cos(sin(x.^3)).*(sin(x.^7));
plot(y1.^k,y2.^k,'.');
```

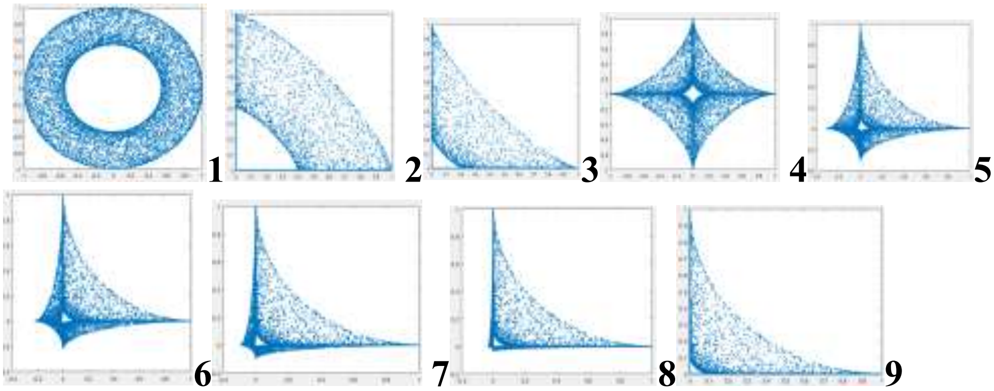



**Fig. 5.** Morphological evolution of nine patterns obtained by the variation of a single constant. The sequence shows a continuous transition from an annular structure towards sectorial, cuspidal, then strongly anisotropic forms, while all other parameters remain unchanged. The analogies with certain mathematical or physical structures are purely visual.

# Conclusion

The results presented here show that a single deterministic, non-iterative functional building block can generate several qualitatively different families of point distributions when only a small number of constants are modified. Banded structures, separated and subsequently merged point clouds, four-lobed configurations, filled and annular domains, extremely thin contours, as well as cusp-shaped and highly anisotropic forms can all be obtained without fundamentally altering the underlying functional architecture.A particularly noteworthy result is that varying a single constant can control significant—and sometimes gradual—morphological transformations, including the merging of two point clouds, the transition from a two-dimensional domain to a quasi-one-dimensional contour, and the evolution of an annular structure into strongly anisotropic configurations.Some of the generated shapes exhibit visual similarities to structures associated with interference phenomena [2], the Stern–Gerlach experiment [3], and the geometry of caustics and wavefronts [4–6]. These analogies, however, remain strictly morphological and should not be interpreted as identifying the corresponding physical mechanisms.The principal interest of these results therefore lies in the remarkable geometric diversity produced by a common and relatively simple functional structure.Future work based on quantitative measurements of symmetry, radial and angular density distributions, curvature, topology, and the stability of the generated structures as the number of points varies would provide a more rigorous characterization of these morphological transitions and their reproducibility.

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**Received: August 5, 2026; Published: August 30, 2026**