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Non-Iterative Architecture Generating U-Shaped Forms and Controlled Multimodal Transitions: Extension Toward Maxwell–Boltzmann-Type Profiles

Jelloul Elmesbahi

j.elmesbahi@ensem.ac.ma

Abstract

We present a family of non-iterative functions (NIF) in which the controlled variation of a limited number of constants makes it possible to obtain :

- Four distinct U-shaped distributions,
- Twelve morphologically different distributions generated from a single parameter,
- Multimodal series with a variable number of modes, generated from a single parameter,
- Profiles close to physical distributions.

The MB-like profile results from a transformation applied to a distribution derived from a modified Gaussian Unitary Ensemble (GUE), establishing a structural link between random matrix theory and thermodynamic distributions. Two constants emerge: the coefficient of variation (Ct1), and a second constant (Ct2), defined as the product of the mean gas value and the maximum peak value of the distribution (PIC).

The observed transitions are continuous, reproducible, and reversible under inverse parametric variation. Calculations are performed over 10^7 points, with

computation times of around one second without specific optimization on an HP G9.

Introduction

U-shaped distributions appear in several contexts in dynamical systems theory, particularly in the study of bifurcations and deterministic chaos, as illustrated by the foundational work of May on logistic maps [1].
Spectral structures exhibiting symmetries and statistical regularities also emerge in random matrix theory, especially within the Gaussian Unitary Ensemble (GUE) [6], studied by Wigner and further developed by Mehta [2,3].
Multimodal distributions and transitions between modal regimes constitute a central phenomenon in nonlinear dynamics and bifurcation analysis [4].
In statistical physics, Maxwell–Boltzmann distributions describe the distribution of velocities in an ideal gas at thermodynamic equilibrium [5].
In this work, we show that a single family of non-iterative functions allows, through simple parametric variation :

- the generation of U-shaped forms,
- progressive transitions in the number of modes,
- the production of bifurcated structures,
- the approximation of Maxwell–Boltzmann-type profiles.

We demonstrate that the Maxwell–Boltzmann-type profile obtained in an extended regime is not introduced ad hoc, but derives from a distribution initially originating from a modified Gaussian Unitary Ensemble (GUE), whose spectral structure, under controlled transformation, converges toward an MB-like form.
Two constants emerge : the coefficient of variation (Ct1), and a second constant (Ct2), defined as the product of the mean gas value and the maximum peak value of the distribution (PIC).

I. MAY–VOICULESCU–GEOMETRIC–INTERMITTENT DISTRIBUTIONS

```
x=1:1:10^7;

b=1.136;    a=0;      % INTERMITENT-CASE (d)
b=1;        a=0;      % GEOMETRIC-CASE(c)
b=1;        a=0.5;    % VOICULESCU-CASE (b)
b=0.49992;  a=0.5;    % MAY-CASE (a)
```

```
y=a+b*sin(x.^33) ;

mean_y=mean(y);std_y=std(y);cv=std_y/mean_y ;
fprintf('mean=%.4f\n',mean_y);
fprintf('standard deviation=%.4f\=n',std_y);hist(y,400)
xlabel('frequency');  ylabel('Value');gridon;
text(0.5,0.96,sprintf('Std:%
.4f',std_y),'Units','normalized','FontSize',12,'Color','r');
text(0.03,0.97,sprintf('mean:%.4f',mean_y),'Units','normalized','FontSize',12,'Color','r');
```

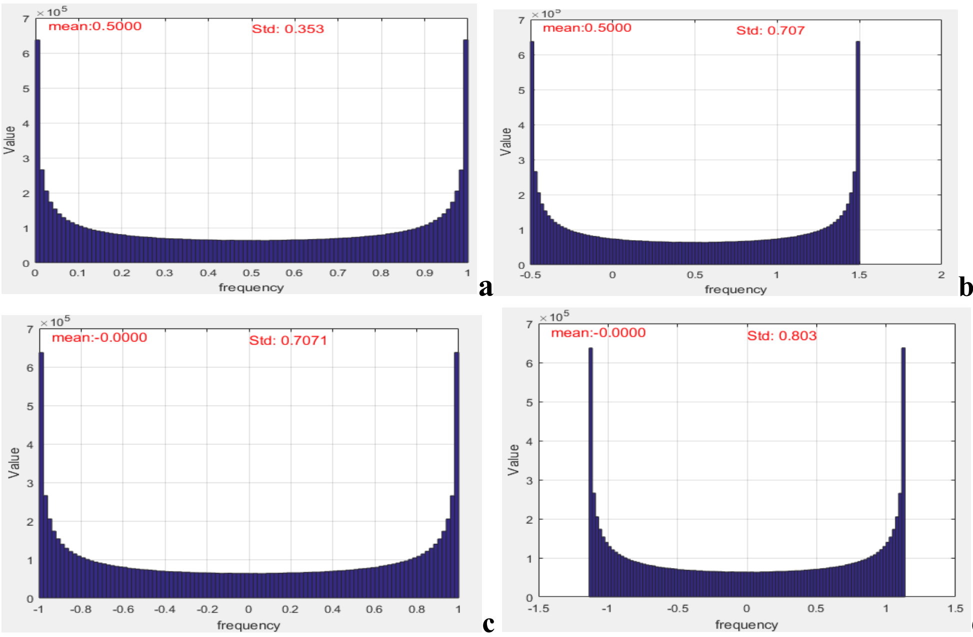


Figure 1. Four U-Shaped Distributions

Four U-shaped distributions obtained from the same family of NIFs, for different sets of constants:

The distributions have bounded supports and variable global symmetry.

Main statistics (mean, standard deviation, coefficient of variation) are indicated on each subfigure.

The U-shape is preserved, while the opening, edge concentration, and dispersion evolveaccording to the parameters.

II. GENERATION OF TWELVEDIFFERENT DISTRIBUTIONS WITH A SINGLE PARAMETER

k=3.9;k=3;k=1.42;k=1.1;k=0.92;k=1;

```
k=0.8;k=0.6;k=0.5;k=0.3;k=0.41;k=0.001;

x=1:1:10^7 ;

y=17+acot(0.75*cot(x.^(33)).^k);

y=abs(y);        hist (y,400)
```

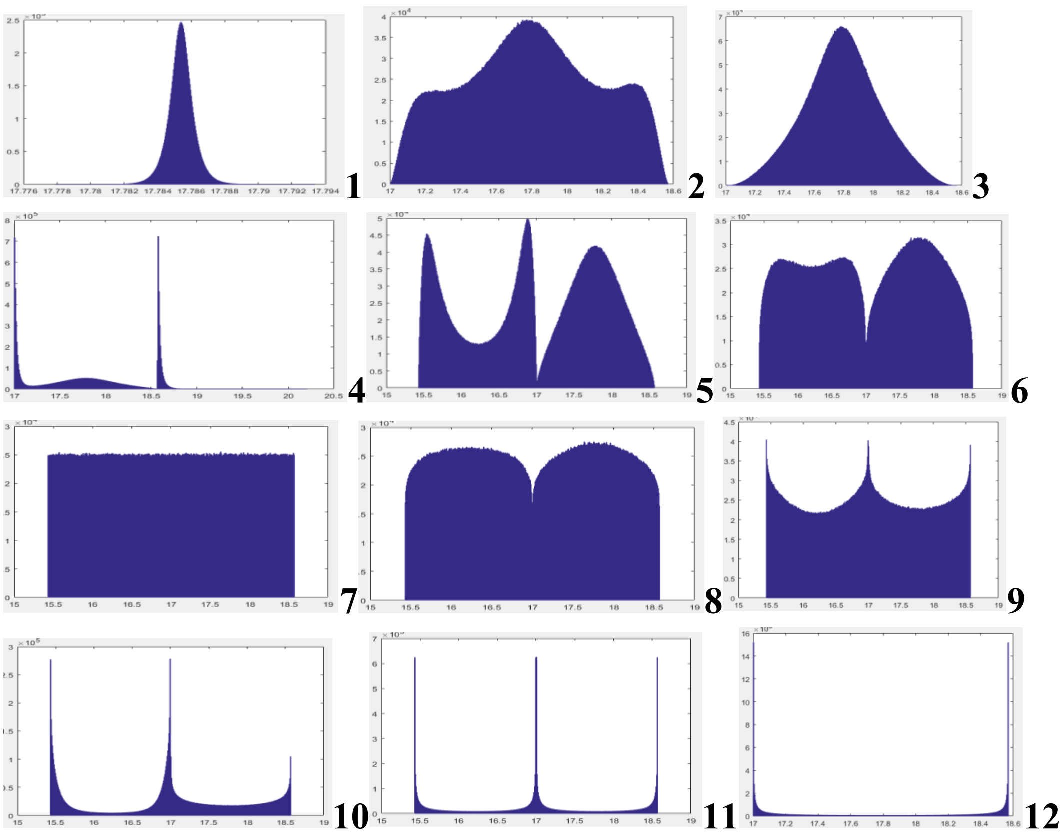


Figure 2. Twelve Distributions Generated from a Single NIF (Single-Parameter Variation)

Twelve different distributions generated by a single NIF through continuous variation of one constant, with all other parameters fixed.
The morphological transition is progressive:
From unimodal or bimodal shapes
Toward multimodal structures

The number of modes evolves, revealing a strong structural dependence on this single parameter.

III. GENERATION OF MULTIMODAL DISTRIBUTIONS WITH A SINGLE PARAMETER

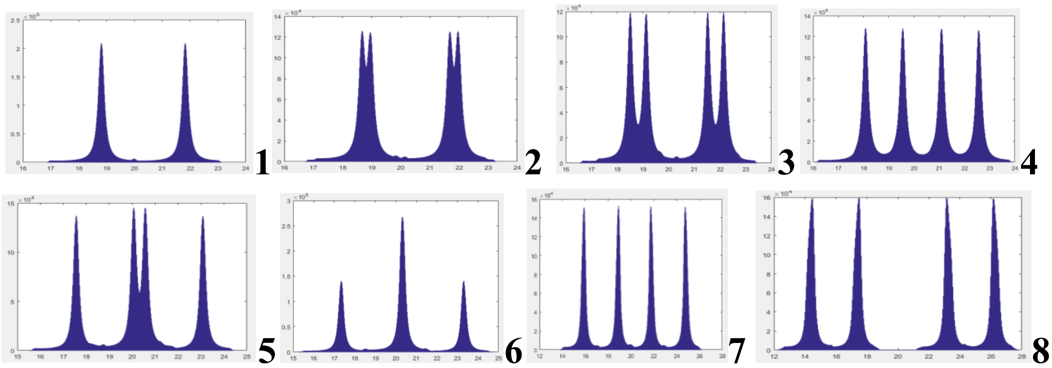
```
a=0;b=1 ;% 3 MOD
(1)  c=0;(2) c=0.1; (3) c=0.2; (4) c=0.5;(5) c=0.85 (6) c=1 ; (7) c=2 ; (8) c=3 ;

a=2 ;b=1 ;;     % 7 MOD
 (1) c=0; (2) c=1;  (3) c=2;(4) c=3; (5) c=4;(6) c=5 ; (7) c=6 ;C(8)=5.5 ;


x=1:1:10^7;
y1=1.25*acot(0.43*sin(x.^5))+acot(0.15*cos(x.^15));
y2=acot(2.652+cot(x.^25))+acot(0.1*cos(x.^35));
y3=acot(0.15*cos(x.^37));
y=20+a*y1+b*y2+c*y3;

y=abs(y);      hist(y,400)
```

SERIES.I



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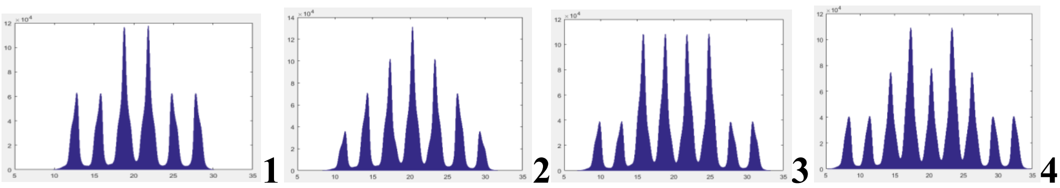


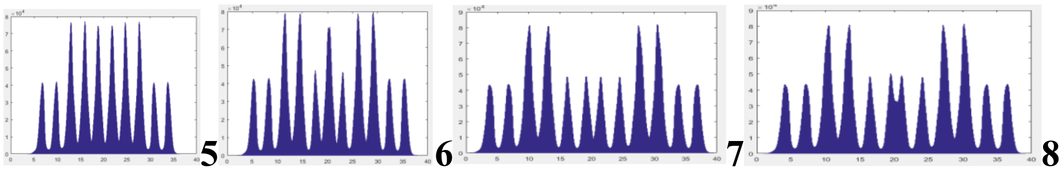


Figure 3. Two Series of Multimodal Distributions

Two series of multimodal distributions, each produced by continuous variation of a single constant, for two distinct parameter configurations.

Series I:

Progressive evolution in the number of modes (2 → 3 → 4).

Series II:

Progressive evolution in the number of modes (6 → 7 → … → 12).

A denser structure appears, with an increasing number of peaks (up to 12 modes).

In each series, the transformation is continuous and reversible when the parameter varies in the opposite direction.

IV. SIMULATION OF A MAXWELL–BOLTZMANN-LIKE DISTRIBUTION

```
x=1:10^-4:10^3; % Cts for GUE modified

a1=1.08154;a2=5.8;a3=0.08;b1=2;b2=5.8;d7=1;
d1=0.38;d2=0.2;d3=1;d4=0;d5=1;d6=1;T=0;p=1;
b3=0.081158734;c2=8.6682632701;f1=45;f2=3;

%%   Y   FUNCTION

y1=a1*acot(p*cot(x.^f1)).^d1+a2*acot(cot(x.^33)).^a3;
y2=b1*acot(p*cot(x.^f2)).^d2+b2*acot(cot(x.^47.3)).^b3;
y=-T+d3*abs(d6*y1.^d7+d5*y2.^d7-d4*y2.^d7).^(c2);

y=411.551506*10^-10*y;   %Xe(MS=283.956207-Std=119.833976) (T=500K)
y=1543.201713*10^-10*y;                          %N2(MS=1064.755441-
Std=449.343505)(T=1500K)
y=155.3487820*10^-10*y;  %He(MS=107.185250-Std=45.233857)  (T=2.17K)
y=216.5661270*10^-10*y;  %He(MS=149.4230086-Std=63.058887) (T=4.22K)
y=664.5843100*10^-10*y;  %H2(MS=458.540030-Std=193.511089) (T=20K)
```

```
figure(1);histogram(y,200,'Normalization','pdf');mean_y=mean(y);std_y=std(
y);
fprintf('Mean:%.6f\n',mean_y);fprintf('standard deviation=%.6f\n',std_y);
cv=std_y/mean_y ;
text(0.5,0.65,sprintf('cv:
%.9f',cv),'Units','normalized','FontSize',12,'Color','r');
text(0.55,0.9,sprintf('Mean:
%.6f',mean_y),'Units','normalized','FontSize',12,'Color','r');
text(0.55,0.85,sprintf('Std:
%.6f',std_y),'Units','normalized','FontSize',12,'Color','r');
title(' DISRIBUTION');xlabel('Frequency of occurences');
ylabel('Probability Density');gridon;
```

CV=0.422015693 **Ct_2=Mean*PIC**

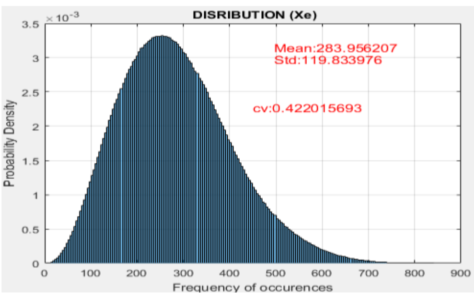
Xe (Mean=283.956207) PIC=$3.324*10^{-3}$ Ct2(Xe)=0.9438704321

N2 (Mean=1064.755441) PIC=$8.856*10^{-4}$ Ct2(N2) =0.9429474185

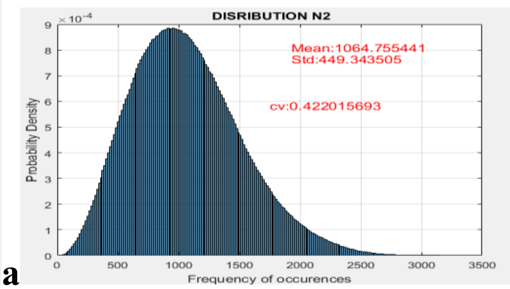
He (Mean=107.185250) PIC=$8.801*10^{-3}$ Ct2(He)=0.9433373853

H2 (Mean=458.540030) PIC=$2.057*10^{-3}$ Ct2(H2) =0.9432168417

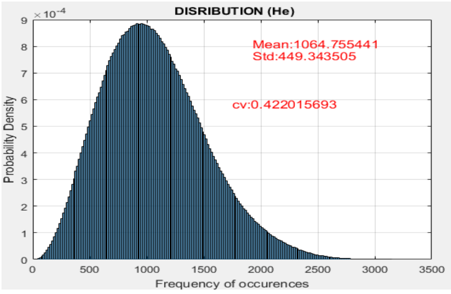
Mean (Ct2) =0.9433430194



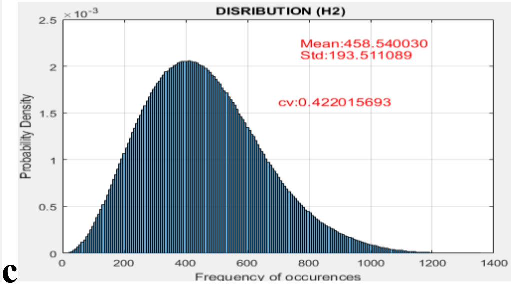
a



b



c



d

Figure 4. Maxwell–Boltzmann-type distributions obtained by modifying one constant of a NIF initially used to model a GUE-type distribution.The histograms correspond to profiles close to molecular velocity distributions for:The coefficient of variation obtained is indicated on each figure.It results from adjusting a single constant while keeping the other parameters fixed**.**

Conclusion

The presented results highlight the ability of a simple functional architecture to generate a significant diversity of statistical forms.
The observations include:

- Four distinct U-shaped distributions,
- Twelve distributions obtained by varying a single parameter,
- Progressive and reversible multimodal transitions controlled by a single parameter,
- Profiles close to known physical distributions.

All transformations appear to be governed by a limited number of constants.
This work constitutes a descriptive and exploratory study. The MB-like approximation does not represent an independent fitting procedure but results from a deterministic transformation applied to a modified GUE distribution, suggesting a deep structural link between random matrix theory and statistical physics.
It opens perspectives toward:

- The theoretical analysis of modal bifurcation conditions,
- The study of limiting regimes.

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