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# Independent 2-Domination Number of Some Special Graphs

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**Abstract**

Given a nontrivial graph, a set of vertices of a graph is an independent set if every pair of distinct vertices are not adjacent and it is a 2-dominating set if each vertex in its complement is adjacent to at least two vertices in the set. A set of vertices of a graph is an independent 2-dominating set if it is both an independent set and a 2-dominating set. The independent 2-domination number of a nontrivial graph is the cardinality of a minimum independent 2-dominating set. In this paper, we formulate the independent 2-domination number of some special graphs using some properties of the independent 2-dominating sets and the independent 2-domination number.

**Mathematics Subject Classification:** 05C69

**Keywords:** independent 2-domination, 2-domination, independent domination, domination.

# 1 Introduction

Independent 2-domination in a graph is a combination of the concept of 2-domination and the notion of an independent set of vertices of a graph. Berge [1] initiated the concept of independent domination and Cockayne and Hedetnieme [5] introduced the notation for the independent domination number. Goddard and Henning [6], gave an upper bound for the sum of the independent domination numbers of a graph and its complement. The concept 2-domination was studied by Blidia et. al. [2]. They found properties and bounds on the 2-domination number. In [3], V. Maheswari et.al. derived the 2-dominating sets and 2-domination numbers of some special graphs. Also, Domoloan and Canoy, Jr. [5] obtained the 2-domination numbers of the join and corona of two graphs.

The concept of independent 2-domination was introduced by Leonida and Allosa in [7]. They characterized the independent 2-dominating set of the join of two graphs and obtained its independent 2-domination number. In this paper, we formulate the independent 2-domination number of some special graphs using some properties of the independent 2-domination number.

# 2 Terminology and Notation

Let $G = (V(G), E(G))$ be a nontrivial graph, where $V(G)$ is the *vertex-set* of $G$ and $E(G)$ is the *edge-set* of $G$. An element $v$ of $V(G)$ is called a *vertex* and an element of $uv$ of $E(G)$ is called an *edge*. Two vertices $u$ and $v$ of $G$ are *adjacent* or *neighbors* if $uv$ is an *edge* of $G$. The set of neighbors of a vertex $v$ in $G$, denoted by $N_G(v)$, is called the *open neighborhood* of $v$ in $G$. The set $N_G[v] = \{v\} \cup N_G(v)$ is called the *closed neighborhood* of $v$ in $G$.

The *degree* of a vertex $v$ of $G$, denoted by $\deg_G(v)$, is the number of neighbors of $v$ in $G$, that is, $\deg_G(v) = |N_G(v)|$. Moreover, $v$ is called an *isolated vertex* if $\deg_G(v) = 0$, that is, $v$ is not adjacent to any other vertex of $G$; $v$ is an *end-vertex* if $\deg_G(v) = 1$, that is, $v$ is adjacent to exactly one vertex of $G$; and $v$ is a *support* if $v$ is adjacent to an end-vertex.

A *path* in a graph $G$ is a sequence of distinct vertices $v_0, v_1, ..., v_n$, where there is an edge between each consecutive pair of vertices. A graph $G$ is called a *connected* graph if every two vertices in $G$ are joined by a path; otherwise, it is called a *disconnected* graph. Every disconnected graph can be split up into a number of connected subgraphs called *components*.

The *union* $G_1 \cup G_2$ of graphs $G_1$ and $G_2$ with disjoint vertex-sets $V(G_1)$ and $V(G_2)$, respectively, is the graph $G$ where $V(G) = V(G_1) \cup V(G_2)$ and $E(G) = E(G_1) \cup E(G_2)$.

A subset $S$ of $V(G)$ is a *dominating set* in $G$ if for every $v \in V(G) \backslash S$, there exists $u \in S$ such that $uv \in E(G)$. The *domination number* of $G$, denoted by $\gamma(G)$, is the cardinality of a minimum dominating set in $G$.

A subset $S$ of $V(G)$ is called an *independent set* in $G$ if for evey $u, v \in S$, $uv \notin E(G)$, that is, if every pair of vertices in $S$ are not adjacent. A subset $S$ of $V(G)$ is an *independent dominating set* in $G$ if $S$ is an independent set and a dominating set in $G$. The *independent domination number* of $G$, denoted by $i(G)$, is the cardinality of a minimum independent dominating set in $G$.

A subset $D$ of $V(G)$ is a 2-*dominating set* in $G$ if every $v \in V(G)\backslash D$, $|D \cap N_G(v)| \geq 2$, that is, $v$ is adjacent to at least two vertices in $D$. The 2-*domination number* of $G$, denoted by $\gamma_2(G)$, is the cardinality of a minimum 2-dominating set in $G$.

**Definition 2.1** Let $G$ be a nontrivial graph. A subset $S$ of $V(G)$ is an *independent* 2-*dominating set* in $G$ if $S$ is an independent set in $G$ and a 2-dominating set in $G$.

**Example 2.2** Consider the graph $G$ in Figure 1. The set $S_0 = \{u, w\}$ is an independent 2-dominating set in $G$.

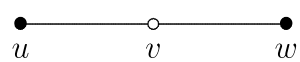


Figure 1: A nontrivial graph $G$ with an independent 2-dominating set.

**Example 2.3** Consider the graph $G$ in Figure 2.

(1) The set $S_1 = \{a\}$ is not an independent 2-dominating set in $G$ since it is not a 2-dominating set in $G$. Similarly, $S_2 = \{b\}$ is not an independent 2-dominating set in $G$.

(2) The set $S_3 = \{a, b\}$ is not an independent 2-dominating set in $G$ since it is not an independent set.

Therefore, $G$ is a nontrivial graph without an independent 2-dominating set.

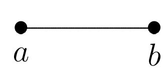


Figure 2: A nontrivial graph $G$ with no independent 2-dominating set.

In case a nontrivial graph $G$ has an independent 2-dominating set, we shall say "$G$ **admits an independent 2-dominating set**."

Let us now define the independent 2-domination number of a nontrivial graph.

**Definition 2.4** Let $G$ be a nontrivial graph. Suppose $G$ admits an independent 2-dominating set. Then the *independent* 2*-domination number* of $G$, denoted by $i_2(G)$, is the cardinality of a minimum independent 2-dominating set in $G$.

The following Corollary follows from Definition 2.4.

**Corollary 2.5** *Let $G$ be a nontrivial graph.*
*($i$) If $S_0$ is a minimum independent 2-dominating set in $G$, then $i_2(G) = |S_0|$.*
*($ii$) If $S$ is an independent 2-dominating set in $G$, then $i_2(G) \leq |S|$.*

**Theorem 2.6** *Let $G$ be a nontrivial graph. If $S_0$ is a minimum independent 2-dominating set in $G$, then*
*($i$) $S_0$ contains every end-vertex of $G$.*
*($ii$) $S_0$ does not contain a support of $G$.*

*Proof*: ($i$) Let $v$ be an end-vertex of $G$. Suppose that $v \notin S_0$. Then $v \in V(G)\backslash S_0$. Since $\deg_G(v) = 1$, it follows that $v$ is adjacent to exactly one vertex in $S_0$. This contradicts the fact that $S_0$ is a 2-dominating set in $G$. Therefore, $S_0$ contains every end-vertex of $G$.

($ii$) By part ($i$), $S_0$ contains every end-vertex of $G$. Since a support is adjacent to an end-vertex and $S_0$ is an independent set, it follows that $S_0$ does not contain a support of $G$.□

**Theorem 2.7** *Let $G$ be a nontrivial graph. Suppose $G$ admits an independent 2-dominating set. Then*
*($i$) $i_2(G) \geq |\Omega(G)|$, where $\Omega(G)$ is the set of all end-vertices of $G$;*
*($ii$) $i_2(G) \leq |V(G)| - |S(G)|$, where $S(G)$ is the set of all supports of $G$.*

*Proof*: ($i$) Let $\Omega(G)$ be the set of all end-vertices of $G$ and $S_0$ a minimum independent 2-dominating set in $G$. Then, by Theorem 2.6($i$), $S_0 \supseteq \Omega(G)$, which implies that $|S_0| \geq |\Omega(G)|$. By Corollary 2.5($i$), $i_2(G) \geq |\Omega(G)|$.

($ii$) Let $S(G)$ be the set of all supports of $G$ and $S_0$ a minimum independent 2-dominating set in $G$. Then, by Theorem 2.6($ii$), $S_0$ does not contain a support of $G$. This implies that $S_0 \subseteq V(G)\backslash S(G)$, which implies that $|S_0| \leq |V(G)| - |S(G)|$. By Corollary 2.5($i$), $i_2(G) \leq |V(G)| - |S(G)|$.□

# 3 Independent 2-Domination Number of Some Special Graphs

In this section we derive and prove formulas for the independent 2-domination numbers of some special graphs.

The complete graph $K_n$ does not admit an independent 2-dominating set since every pair of distinct vertices are adjacent. This implies that any subset $S$ of $V(K_n)$ containing at least two vertices is not an independent set and hence not an independent 2-dominating set.

The empty graph $\overline{K_n}$ admits an independent 2-dominating set. Since every vertex of $\overline{K_n}$ is an isolated vertex, $V(\overline{K_n})$ is clearly a minimum independent 2-dominating set in $\overline{K_n}$. Hence, by Corollary 2.5$(ii)$, $i_2(\overline{K_n}) = |V(\overline{K_n})| = n$.

**Theorem 3.1** *Let $P_n$ be a path graph of order $n \geq 2$. Then $i_2(P_n) = \frac{n+1}{2}$ if $n$ is odd.*

*Proof*: Let $P_n = [v_1, v_2, ..., v_n]$, where $n$ is odd. Let $S = \{v_{2k-1} : 1 \leq k \leq \frac{n+1}{2}\}$. Then $S$ is an independent set in $P_n$. We have $V(P_n)\backslash S = \{v_{2k} : 1 \leq \frac{n-1}{2}\}$. Then every $v_{2k} \in V(P_n)\backslash S$ is adjacent to two vertices $v_{2k-1}$ and $v_{2k+1}$ in $S$. Thus, $S$ is a 2-dominating set in $P_n$. By Definition 2.1, $S$ is an independent 2-dominating set in $P_n$. Hence, by Corollary 2.5$(ii)$,

$$i_2(P_n) \leq |S| = |\{v_{2k-1} : 1 \leq k \leq \tfrac{n+1}{2}\}| = \tfrac{n+1}{2}.$$

Therefore, $i_2(P_n) \leq \frac{n+1}{2}$.

On the other hand, suppose that $S_0$ is a minimum independent 2-dominating set in $P_n$. Let $v_iv_{i+1} \in E(P_n)$ for $1 \leq i \leq n-1$. If $v_i \in S_0$, then $v_{i+1} \notin S_0$ since $S_0$ is an independent set in $P_n$. Since $v_1$ and $v_n$ are end-vertices, by Theorem 2.6$(i)$, $v_1, v_n \in S_0$. Also, $v_2, v_{n-1} \notin S_0$ by Theorem 2.6$(ii)$ since they are supports of $P_n$. Thus, $v_i \in S_0$ and $v_{i+1} \notin S_0$ if $i$ is odd. This implies that $S_0 \supseteq \{v_1, v_2, ..., v_{n-2}, v_n\}$, that is, $|S_0| \geq |\{v_1, v_3, ..., v_{n-2}, v_n\}| = \frac{n+1}{2}$. By Corollary 2.5$(i)$, $i_2(P_n) = |S_0| \geq \frac{n+1}{2}$.

Combining the two inequalities, we obtain $i_2(P_n) = \frac{n+1}{2}$.□

If $n$ is even, the path $P_n$, for $n \geq 2$, does not admit an independent 2-dominating set.

**Theorem 3.2** *Let $C_n$ be a cycle graph of order $n \geq 3$. Then $i_2(C_n) = \frac{n}{2}$ if $n$ is even.*

*Proof*: Let $C_n = [v_1, v_2, ..., v_n, v_1]$. Let $S = \{v_{2k-1} : 1 \leq k \leq \frac{n}{2}\}$. From the definition of $C_n$, $S$ is an independent set in $C_n$. We have $V(C_n)\backslash S = \{v_{2k} : 1 \leq \frac{n}{2}\}$. Then every $v_{2k} \in V(C_n)\backslash S$ is adjacent to two vertices $v_{2k-1}$ and $v_{2k+1}$ in $S$. Thus, $S$ is a 2-dominating set in $C_n$. By Definition 2.1, $S$ is an independent 2-dominating set in $C_n$. Hence, by Corollary 2.5$(ii)$, $i_2(C_n) \leq |S| = |\{v_{2k-1} : 1 \leq k \leq \frac{n}{2}\}| = \frac{n}{2}$. Therefore, $i_2(C_n) \leq \frac{n}{2}$.

On the other hand, suppose that $S_0$ is a minimum independent 2-dominating set in $C_n$. Let $v_iv_{i+1} \in E(C_n)$ for $1 \leq i \leq n-1$. If $v_i \in S_0$, then

$v_{i+1} \notin S_0$ since $S_0$ is an independent set in $C_n$. Since $v_1, v_{n-1} \in S_0$, it follows that $v_i \in S_0$ if $i$ is odd. Thus, $S_0 \supseteq \{v_1, v_2, ..., v_{n-1}\}$, which implies that $|S_0| \geq |\{v_1, v_2, ..., v_{n-1}\}| = \frac{n}{2}$. By Corollary 2.5($i$), $i_2(C_n) = |S_0| \geq \frac{n}{2}$.

Combining the two inequalities, we obtain $i_2(C_n) = \frac{n}{2}$.□

If $n$ is odd, then $C_n$ does not admit an independent 2-dominating set.

**Theorem 3.3** *Let $c_{n,2}$ be a centipede graph of order $3n$, where $n \geq 1$. Then the independent 2-domination number of $c_{n,2}$ is given by $i_2(c_{n,2}) = 2n$.*

*Proof*: A centipede graph $c_{n,2}$ is a connected graph obtained from a path $P_n$ and attaching exactly two pendant vertices to each vertex of the path. Let $S = \Omega(c_{n,2}) = \{x_i : 1 \leq i \leq n\} \cup \{y_i : 1 \leq i \leq n\}$. Then $S$ is an independent set of $c_{n,2}$. We have $V(c_{n,2})\backslash S = V(P_n) = v_i : 1 \leq i \leq n\}$. By the definition of $c_{n,2}$, every $v_i \in V(c_{n,2})\backslash S$ is adjacent to two vertices $x_i, y_i \in S$ for all $i$. Thus, $S$ is a 2-dominating set in $c_{n,2}$. Hence, by Definition 2.1, $S$ is an independent 2-dominating set in $c_{n,2}$. By Corollary 2.5($ii$), $i_2(c_{n,2}) \leq |S| = 2n$.

On the other hand, let $\Omega(c_{n,2}) = \{x_i : 1 \leq i \leq n\} \cup \{y_i : 1 \leq i \leq n\}$. By Theorem 2.6($i$), $i_2(c_{n,2}) \geq |\Omega(c_{n,2})| = |\{x_i : 1 \leq i \leq n\}| + |\{y_i : 1 \leq i \leq n\}| = 2n$. Hence, $i_2(c_{n,2}) \geq 2n$.

Combining the two inequalities, we obtain $i_2(c_{n,2}) = 2n$.□

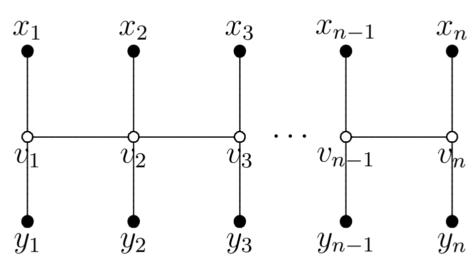


Figure 3: A centipede graph $c_{n,2}$ with $i_2(c_{n,2}) = 2n$.

**Theorem 3.4** *Let $B(r, s)$ be a bi-star graph of order $r + s + 1$, where $r, s \geq 2$. Then the independent 2-domination number of $B(r, s)$ is given by $i_2(B(r, s)) = r + s$.*

*Proof*: A bi-star graph $B(r, s)$ is a connected graph obtained by joining the centers of two stars $K_{1,r}$ and $K_{1,s}$. Let $v_1$ and $v_2$ be the centers of the two stars $K_{1,r}$ and $K_{1,s}$, respectively. Also, let $\{x_i : 1 \leq i \leq r\}$ and $\{y_j : 1 \leq j \leq s\}$ be the set of end-vertices of the stars $K_{1,r}$ and $K_{1,s}$, respectively (see Figure 4).

Let $S = \{x_i : 1 \leq i \leq r\} \cup \{y_j : 1 \leq j \leq s\}$. Since $x_i$ and $y_j$ are end-vertices, it follows that $S$ is an independent set. Next, $V(B(r, s))\backslash S = \{v_1, v_2\}$. Thus, $v_1$ is adjacent to at least two vertices $x_i$ in $S$ and $v_2$ is adjacent to at least two vertices $y_i$ in $S$. Hence, $S$ is a 2-dominating set in $B(r,s)$. By Definition 2.1, $S$ is an independent 2-dominating set in $B(r,s)$. Therefore, by Corollary 2.5($ii$), $i_2(B(r,s)) \leq |S| = |\{x_i : 1 \leq i \leq r\}| + |\{y_j : 1 \leq j \leq s\}| = r + s$, that is, $i_2(B(r,s)) \leq r + s$.

On the other hand, let $\Omega(B(r,s)) = \{x_i : 1 \leq i \leq r\} \cup \{y_j : 1 \leq j \leq s\}$. Then by Theorem 2.6($i$), $i_2(B(r,s)) \geq |\Omega(B(r,s))| = |\{x_i : 1 \leq i \leq r\}| + |\{y_j : 1 \leq j \leq s\}| = r + s$. Hence, $i_2(B(r,s)) \geq r + s$.

Combining the two inequalities, we obtain $i_2(B(r,s)) = r + s$.□


Figure 4: A bi-star graph $B(r,s)$ with $i_2(B(r,s)) = r + s$.

**Theorem 3.5** *Let $H_n$ be a helm graph of order $2n+1$, where $n \geq 3$. Then the independent 2-domination number of $H_n$ is given by $i_2(H_n) = n + 1$.*

*Proof*: A helm graph $H_n$ is a connected graph obtained from a wheel by attaching a pendant vertex at each of the $n$ vertices of the cycle. Denote the central vertex of the wheel by $v$, the vertices of the cycle by $v_i$, and the end-vertices by $u_i$ for $1 \leq i \leq n$ (see Figure 7).

Let $V(H_n) = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$, where $S(H_n) = \{v_i : 1 \leq i \leq n\}$. By Theorem 2.9($ii$),

$$i_2(H_n) \leq |V(H_n)| - |S(H_n)| = 2n + 1 - n = n + 1.$$

Hence, $i_2(H_n) \leq n + 1$.

On the other hand, let $S_0$ be a minimum independent 2-dominating set in $H_n$. By Theorem 2.6($i$), $S_0 \supseteq \Omega(H_n) = \{u_i : 1 \leq i \leq n\}$. Now, by Theorem 2.6($ii$), $S_0$ does not contain $S(H_n) = \{v_i : 1 \leq i \leq n\}$. But $vv_i \in E(H_n)$ for all $i$. Consequently, $v \in S_0$. Thus, $S_0 \supseteq \Omega(H_n) \cup \{v\}$, which implies that $|S_0| \geq |\Omega(H_n)| + |\{v\}| = n + 1$. By Corollary 2.5($i$), $i_2(H_n) = |S_0|$. Therefore, $i_2(H_n) \geq n + 1$.

Combining the two inequalities, we obtain $i_2(H_n) = n + 1$.□

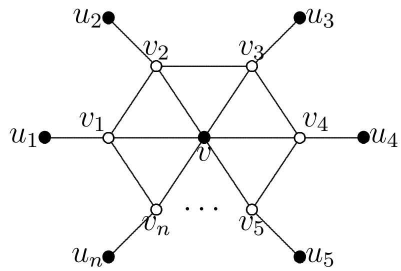


Figure 5: A helm graph $H_n$ with $i_2(H_n) = n + 1$.

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