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Loss-Sensitive Rating Plans and Solvency Capital under Solvency II

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Abstract

Loss-Sensitive Rating Plans (LSRPs) introduce explicit risk sharing between insurer and insured by linking premiums to realized losses. Recent actuarial research has shown that retrospective rating plans, large deductible plans and stop-loss arrangements can be represented within a unified framework based on layered loss functions and stop-loss transforms. A key implication of this framework is that pricing rules based solely on expected values fail to capture the variability and tail risk borne by the insurer. This paper embeds the loss-sensitive rating framework within the Solvency II regime, where capital requirements are calibrated to tail risk through a Value-at-Risk measure at the 99.5% confidence level. We show that loss-sensitive contracts primarily affect solvency capital by reshaping the upper tail of the insurer's loss distribution rather than by reducing expected losses. We numerically assess how different contract designs with similar expected outcomes can lead to substantially different capital requirements. The results highlight the need to integrate pricing, tail-risk measures and capital considerations when evaluating loss-sensitive rating plans.

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1 Introduction

Loss-Sensitive Rating Plans (LSRPs) constitute a class of insurance contracts in which the premium is not fully determined at inception but depends, wholly or partially, on the realized loss experience of the insured. Compared with guaranteed-cost policies, such contracts introduce explicit risk sharing between insurer and insured, affecting pricing, incentives for loss control and the allocation of underwriting risk (Fisher et al. (2019), Campana et al. (2022) and references therein). The contribution of Campana et al. (2024) provides a unified mathematical formalization of loss-sensitive contracts, showing that retrospective rating plans, large deductible plans and stop-loss arrangements can be described within a common framework based on layered loss functions and stop-loss transforms. A key insight of their work is that pricing criteria based solely on expected values fail to capture the variability and tail behaviour of the insurer's net result. This observation naturally connects loss-sensitive rating to the prudential framework of Solvency II, where solvency capital requirements are driven by tail-risk measures rather than by average outcomes. The purpose of this paper is to embed the formal results of Campana et al. (2024) within a Solvency II perspective, highlighting the implications for underwriting risk, catastrophe risk and counterparty default risk.

2. Aggregate Loss Decomposition and Layered Representation

Let $L \geq 0$ denote the aggregate annual loss generated by an insured portfolio. Following Campana et al. (2024), the total loss is decomposed as

$$L = L_I + L_P,$$

where L_I represents the insurer's loss burden and L_P the insured's loss burden. The insurer's loss can be expressed in general form as a layer:

$$L_I = (L-d)^+ - (L-u)^+,$$

where d denotes the minimum ratable loss (deductible) and u the maximum covered loss (limit). This formulation provides a unified representation of several contractual structures:

- guaranteed-cost contracts ($d = 0, u = \infty$),
- large deductible plans (high d),
- stop-loss arrangements (high d, finite u).

The expected loss borne by the insurer is $\mathbb{E}[L_I] = \mathbb{E}[(L-d)^+] - \mathbb{E}[(L-u)^+]$. Let $\mu = \mathbb{E}[L]$. The stop-loss transform also known as the Table M charge (Gillam, 1991) is defined as $\gamma(d) = \frac{\mathbb{E}[(L-d)^+]}{\mu}$.

The function $\gamma(\cdot)$ summarizes the proportion of expected loss lying beyond a given attachment point and plays a central role in the pricing of excess-of-loss and stop-loss structures.

3. Retrospective Rating and Expected-Value Balance

In a retrospective rating plan, the premium is typically specified as (Meyers, 2014)

$$\Pi = B + kL_I,$$

where B is the basic premium and k the loss conversion factor. In practice, the premium is constrained by minimum and maximum bounds:

$$\Pi_{\min} \leq \Pi \leq \Pi_{\max}.$$

A standard balance condition requires

$$\mathbb{E}[\Pi] = \mathbb{E}[\Pi_{\mathrm{GC}}],$$

where Π_{GC} denotes the premium of a guaranteed-cost policy. While this condition ensures equivalence in expectation, it does not control the variability of the insurer's net result. Indeed, contracts satisfying $\mathbb{E}[\Pi - L_I] = \text{constant}$ may differ substantially in terms of $\mathrm{Var}(\Pi - L_I)$ and $\mathrm{VaR}_\alpha(\Pi - L_I)$.[2] Expected-value balance constrains $\mathbb{E}[X]$ but leaves $\mathrm{VaR}_\alpha(X)$ unconstrained. Consequently, pricing rules based solely on expected values are unable to control exposure to adverse tail events.

4. Solvency II and Tail-Based Capital Requirements

Within the Solvency II regulatory framework[3], insurers' solvency is assessed through a market-consistent and risk-based capital requirement. The central quantitative measure is the Solvency Capital Requirement (SCR), defined as the amount of capital required to ensure that the probability of insolvency over a one-year horizon does not exceed 0.5%. Formally, the SCR is defined as

$$\mathrm{SCR} = \mathrm{VaR}_{99.5\%}\left(\Delta\mathrm{BOF}_{1y}\right),$$

where $\Delta\mathrm{BOF}_{1y}$ denotes the one-year change in Basic Own Funds, i.e. the variation in the economic value of own funds over the following year. From an actuarial standpoint, $\Delta\mathrm{BOF}_{1y}$ represents the insurer's stochastic annual result and aggregates all relevant sources of uncertainty affecting solvency. Abstracting from investment and valuation effects, it may be expressed in simplified form as

$$\Delta\mathrm{BOF}_{1y} = \text{Premiums} - \text{Claims} - \text{Expenses}.$$

This representation emphasizes that solvency assessment is ultimately driven by the distributional properties of underwriting results.

A fundamental implication of the Solvency II definition is that capital requirements are determined by extreme but plausible adverse outcomes, corresponding to a "1-in-200 year" event. Consequently, the SCR depends on the upper tail of the annual loss distribution, rather than on its expected value. Portfolios or contract structures

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[2] For a loss random variable X and a confidence level $\alpha \in (0,1)$, the Value-at-Risk is defined as

$$\mathrm{VaR}_\alpha(X) = \inf\{x \in \mathbb{R}: \mathbb{P}(X \leq x) \geq \alpha\}.$$

[3] The Solvency II framework is formally defined in Directive 2009/138/EC (European Parliament and Council, 2009); for an actuarial treatment, see Sandström (2011).

that are equivalent in expectation may therefore give rise to substantially different capital requirements if they differ in terms of volatility, skewness or tail thickness. This tail-based perspective establishes a natural conceptual link with the loss-sensitive rating framework developed by Campana et al. (2024). As shown in their analysis, pricing rules based on expected-value balance conditions constrain only the first moment of the insurer's net result and leave higher-order characteristics of the loss distribution unconstrained. In contrast, Solvency II explicitly targets tail risk through a Value-at-Risk measure, making it sensitive to the distributional effects induced by contract design. Loss-Sensitive Rating Plans interact with this framework by reshaping the distribution of underwriting results. Retrospective rating mechanisms affect the dispersion of outcomes through premium responsiveness, large deductible structures reallocate a portion of losses to the insured, and stop-loss arrangements truncate the insurer's exposure beyond predefined thresholds. While these mechanisms may preserve expected-value balance, they generally induce markedly different values of $\mathrm{VaR}_{99.5\%}(\Delta\mathrm{BOF}_{1y})$ and, hence, different solvency capital requirements. The Solvency II framework implements this tail-based assessment through a modular structure, in which the SCR is decomposed into distinct sources of risk. The implications of loss-sensitive rating plans for solvency capital therefore emerge more clearly when analysing their impact on the individual risk modules of the Standard Formula, which is the focus of the following section.

5. Impact on the Standard Formula Risk Modules

5.1. Premium and Reserve Risk

Within the Non-Life Underwriting Risk Module, the Premium and Reserve Risk sub-module captures deviations of claims and reserves from their expected levels. In simplified form, the Standard Formula specifies

$$\mathrm{SCR}_{\mathrm{PremRes}} = 3 \cdot \sigma_{\mathrm{nl}} \cdot V_{\mathrm{nl}},$$

where V_{nl} is the volume of net premiums and reserves, and σ_{nl} the standard deviation of net losses. Retrospective rating plans reduce σ_{nl} through loss-dependent premium adjustments, while large deductible plans reduce V_{nl} by eliminating high-frequency losses. Non-proportional reinsurance further mitigates volatility through tail transfer.

5.2. Catastrophe Risk

The Catastrophe Risk sub-module captures rare but severe losses and can be interpreted as $\mathrm{SCR}_{\mathrm{Cat}} \approx \mathrm{VaR}_{99.5\%}(L_I)$.

Large deductible plans reduce expected losses but do not necessarily mitigate tail exposure, whereas stop-loss reinsurance directly truncates the tail of the loss distribution, leading to substantial reductions in catastrophe capital.

5.3. Counterparty Default Risk

Risk transfer mechanisms introduce counterparty default risk. Credit exposure may be summarized as $\mathrm{EL}_{\mathrm{credit}} = PD \cdot LGD \cdot EAD,$ where PD is the probability of default, LGD the loss given default and EAD the exposure at default. Although expected losses may be limited, the associated capital requirement can partially offset reductions in underwriting risk.

6. Numerical Assessment

The aim of this numerical results section is to compare the tail behaviour of the insurer's underwriting result under different loss-sensitive contract designs that can be aligned in expectation but differ substantially in their exposure to extreme outcomes. Since solvency capital under Solvency II is calibrated to a 99.5% Value-at-Risk over a one-year horizon, the analysis focuses on tail-risk measures of the underwriting loss distribution.

In addition to the Value-at-Risk, we report the Expected Shortfall (ES) as a complementary tail-risk indicator. Although the Solvency Capital Requirement is formally based on VaR, the Expected Shortfall is included to assess the severity of losses beyond the regulatory threshold, thereby providing additional information on the shape and thickness of the tail.

6.1. Expected Shortfall

Let X be a real-valued loss random variable and let $\alpha \in (0,1)$. The Expected Shortfall at level α is defined as

$$\mathrm{ES}_{\alpha}(X) = \mathbb{E}\,[X \mid X > \mathrm{VaR}_{\alpha}(X)].$$

While the Value-at-Risk identifies the entry point of the loss tail, the Expected Shortfall measures the average loss conditional on exceeding that threshold. In the present context, ES is used to distinguish between contract designs that may exhibit similar $\mathrm{VaR}_{99.5\%}$ values but differ in the magnitude of losses occurring in the most adverse scenarios.

6.2. Aggregate loss model

Let $L \geq 0$ denote the aggregate annual loss of an insurance portfolio. We assume that L follows a lognormal distribution, reflecting the positive skewness and heavy-tailed nature typically observed in aggregate insurance losses. The distribution is calibrated to $\mathbb{E}[L] = 1000,\ \mathrm{SD}(L) = 800,$ corresponding to a coefficient of variation of 0.8.

Monte Carlo simulations with $N = 2 \times 10^5$ independent scenarios are performed to obtain stable estimates of tail-risk measures, in particular at the 99.5% confidence level.

6.3. Contractual structures and underwriting loss

Four stylised contractual structures are considered. In each case, L_I denotes the portion of the aggregate loss borne by the insurer, and the underwriting loss is defined as

$$U = L_I + E - \Pi,$$

where $E = 100$ represents fixed underwriting expenses.

- Guaranteed-cost (GC): $L_I^{\mathrm{GC}} = L$.
- Large deductible (LD): $L_I^{\mathrm{LD}} = (L - 500)^+$.
- Stop-loss layer (SL): $L_I^{\mathrm{SL}} = (L - 500)^+ - (L - 1500)^+$.
- Retrospective rating with corridor (RR):

$$\Pi = \min\{\max(B + 1.1L_I^{SL}, 800), 1600\}.$$

Premiums are determined so that the expected underwriting result is identical across the four contract types, with $\mathbb{E}[U] = -400$ in each case. This construction ensures that any differences observed in the subsequent analysis are attributable to the distributional properties of the underwriting loss, and in particular to its tail behaviour, rather than to differences in mean profitability. For the RR contract, the intercept parameter B in the premium corridor is obtained by numerical calibration, yielding $B = 106.21$.

6.4. Tail-risk comparison

Table 1 reports the main distributional characteristics of the underwriting loss U under each contractual structure, with particular emphasis on the tail-risk measures $\mathrm{VaR}_{99.5\%}$ and $\mathrm{ES}_{99.5\%}$.

Table 1. Underwriting loss distribution and tail risk (E[U]=–400)

Contract	$\mathbb{E}[\Pi]$	$\mathrm{VaR}_{99.5\%}(U)$	$\mathrm{ES}_{99.5\%}(U)$
Guaranteed cost (GC)	1502	3443	4866
Large deductible (LD)	1043	3402	4825
Stop-loss layer (SL)	894	206	206
Retrospective rating (RR)	894	-70	-70

These findings provide a numerical illustration of how loss-sensitive contract features interact with tail-based solvency assessment under Solvency II.
By construction, all contracts share the same expected underwriting result. Differences therefore arise exclusively from the distributional characteristics of U. The GC contract concentrates the entire tail of the aggregate loss distribution on the insurer, resulting in the largest values of $\mathrm{VaR}_{99.5\%}$ and $\mathrm{ES}_{99.5\%}$. The LG reduces the contribution of moderate losses but leaves extreme outcomes largely unaffected. The SL layer substantially truncates the insurer's exposure beyond the upper attachment point, leading to a marked reduction in tail risk. The RR contract, calibrated to the same expected result, further modifies the distribution through premium responsiveness within the corridor. Although its expected outcome coincides with that of the stop-loss layer, its tail behaviour differs due to the interaction between premium bounds and the loss layer. These results confirm that contracts equivalent in expectation may generate materially different tail-risk pro-

files. The Expected Shortfall provides additional insight into the severity of losses beyond the regulatory VaR threshold, reinforcing the view that solvency capital under Solvency II is primarily driven by tail exposure rather than by differences in mean performance.

7. Concluding Remarks

Loss-Sensitive Rating Plans transform the distribution of aggregate losses through layered contract structures and premium adjustment mechanisms, rather than eliminating risk altogether. The unified framework developed by Campana et al. (2024) provides a rigorous basis for analysing these effects and highlights the limitations of pricing criteria based solely on expected values. Within the Solvency II regime, where solvency capital requirements are calibrated to extreme but plausible adverse outcomes, such distributional effects are of primary importance. As shown in the analytical discussion and confirmed by the numerical illustration in Section 6, contract designs that can be aligned in expectation may exhibit markedly different tail-risk profiles. In particular, the comparison of $\mathrm{VaR}_{99.5\%}$ and $\mathrm{ES}_{99.5\%}$ across guaranteed-cost, large deductible, stop-loss and retrospective rating structures illustrates how tail reshaping (rather than mean reduction) drives capital efficiency under Solvency II. The numerical results emphasize that large deductible arrangements reduce loss frequency but leave extreme outcomes largely unaffected, while stop-loss structures achieve substantial reductions in tail risk by truncating the insurer's loss exposure. Retrospective rating mechanisms further compress the distribution of underwriting results through premium responsiveness, although their effectiveness in reducing extreme losses ultimately depends on the position of contractual premium corridors. Overall, the findings support the view that loss-sensitive contract features should be evaluated not only from a pricing perspective but also in terms of their impact on tail-risk measures relevant for solvency. A coherent assessment therefore requires integrating expected-value balance, tail-risk analysis and capital considerations. From this perspective, a risk-adjusted pricing rule can be written as

$$\text{Risk-adjusted price} = \mathbb{E}[\Pi] + \mathrm{CoC} \times \mathrm{SCR},$$

where CoC denotes a cost-of-capital rate associated with holding solvency capital. This representation accounts for tail risk in the pricing of loss-sensitive contracts. As future extension, we plan to consider a more sophisticated risk-adjusted price formulation by modelling it as a non-convex optimization problem to be efficiently solved, using advanced non-convex optimization techniques, such as those proposed in G. Scutari et al. (2016).

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