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A Fuzzy Linear Programming Model for Octagonal Fuzzy Numbers

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Abstract

In this paper, we explore the fuzzy linear programming problem using octagonal fuzzy numbers. We begin by presenting the fundamental definition of octagonal fuzzy numbers and proceed to formulate an exact multiplication operation based on the α-cut method. Building on this foundation, we develop a solution approach for linear programming problems involving octagonal fuzzy numbers by transforming the fuzzy system into a series of crisp programming problems. To demonstrate the effectiveness of the proposed method, we provide a detailed numerical example and compare the results with those obtained using a classical approach to highlight the efficiency and improved performance of the new method.

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1 Introduction

Fuzzy numbers are a natural progression of the traditional crisp numbers that are usually used to treat real life mathematical problems. Many new engineering and economical applications include some amount of uncertainty which make it impractical to present them using crisp numbers. The fuzzy approach introduced first by Zadeh [17] formed a suitable solution for the vagueness surrounding such applications. Basically, A fuzzy number is the fuzzy subset of a crisp number which describes natural physical problems in a more realistic way. Nowadays, fuzzy mathematics forms powerful solutions for problems in robotics, decision making and artificial intelligence fields [6, 11, 12, 19].

Fuzzy linear programming is an central topic of optimization where fuzzy models are developed to treat problems that include uncertainty. Recently, a rising amount of research work and effort is devoted to study programming problems on both theoretical and computational levels [1, 2, 7, 8]. Authors in this field considered different types of fuzzy linear programming problems and proposed specific solutions for each type. For example in [13], the authors proposed a solution for problems where all decision parameters are fuzzy numbers. In [4], a new method for solving fuzzy linear programming problems in which the elements of the coefficient matrix are represented by real numbers while other parameters are represented using symmetric trapezoidal fuzzy numbers. A method for solving fuzzy linear programming problems involving symmetric octagonal fuzzy numbers is developed in [9]. Some researchers considered problems where all parameters are represented by fuzzy numbers referred to as fully fuzzy linear programming (FFLP). For example, in [14], a method for solving FFLP is proposed to obtain the fuzzy optimal solution with unrestricted variables and parameters. In [10], a method to find the optimal solution of the same type for the linear programming problems is presented.

Most of the previous methods adopted either the α-cut or extension principal approach with approximate fuzzy operations for two fuzzy numbers when building up their linear programming problems. However, the approximate operations of two fuzzy numbers may not preserve the type of the sign of the fuzzy number and in many cases lead to more fuzziness in the solution [5,15,16]. Moreover, many research work Oversees fuzzy numbers with more sophisticated shapes - such as octagonal numbers – despite their indisputable role in solving applied problems. This motivated us to introduce this work where a fuzzy linear programming model is considered and dedicated for problems modeled by octagonal fuzzy numbers. The fuzzy multiplication operation presented in [20] is extended to accommodate octagonal fuzzy numbers. An algorithm to solve the fuzzy linear programming model is proposed. A numerical example is then presented to verify the ideas presented in this work. The numerical results is then compared to those presented in [9] to demonstrate the efficiency and performance of proposed method.

The reminder of this paper is organized as follows. In Section 2, we present basic definition of octagonal fuzzy numbers then we move to and formulate their basic operations. Section 3 introduce the linear programming problem with octagonal fuzzy numbers along with the solution structure. To illustrate the concepts discussed, we solve a numerical example in Section 4, where we also compare the results with existing approaches. Finally, we provide our conclusions in Section 5.

2 Octagonal Fuzzy Numbers

In this section, we present some basic definitions from fuzzy set theory for fuzzy numbers [3]. We extend the generalized approach presented in [20] to define the main operations for octagonal fuzzy numbers.

Let us first start be introducing the definition of the general fuzzy set [17]:

Definition 2.1. *The fuzzy set $\hat{A}$ is defined by $\hat{A} = \{(x, \mu_{\hat{A}}(x)) : x \in A$ and $\mu_{\hat{A}}(x) \in [0, 1]\}$ where $\mu_{\hat{A}}(x)$ is called the membership function.*

In contrast to a classical set, where its elements have memberships of either one (fully belong in the set) or zero (do not belong in the set), Fuzzy sets are represented by their membership functions, which determine the degree of membership of each element (e.g., each point in the universe of discourse) in the fuzzy set. Following we define the octagonal fuzzy number using membership function:

Definition 2.2. *An octagonal fuzzy number denoted as $\hat{A} = (a, b, \alpha_{11}, \beta_{11}, \alpha_{12}, \beta_{12}, \alpha_{13}, \beta_{13}; k, 1)$ where $k \in (0.1)$ is a fuzzy number whose membership function is given as:*

$$\mu_{\hat{A}}(x) = \begin{cases} \frac{k}{(\alpha_{11}-\alpha_{12})}(x - a + \alpha_{11}), & a - \alpha_{11} \leq x \leq a - \alpha_{12} \\ k, & a - \alpha_{12} \leq x \leq a - \alpha_{13} \\ 1 + \frac{(1-k)}{\alpha_{13}}(x - a), & a - \alpha_{13} \leq x \leq a \\ 1, & a \leq x \leq b \\ 1 - \frac{(1-k)}{\beta_{13}}(x - b), & b \leq x \leq b + \beta_{13} \\ k, & b + \beta_{13} \leq x \leq b + \beta_{12} \\ \frac{k}{\beta_{12}-\beta_{11}}(x - b - \beta_{11}), & b + \beta_{12} \leq x \leq b + \beta_{11} \\ 0, & \text{Otherwise} \end{cases} \quad (1)$$

Figure 1 depicts the Octagonal fuzzy number defined by the membership function $\mu_{\hat{A}}(x)$ in Definition 2.2.

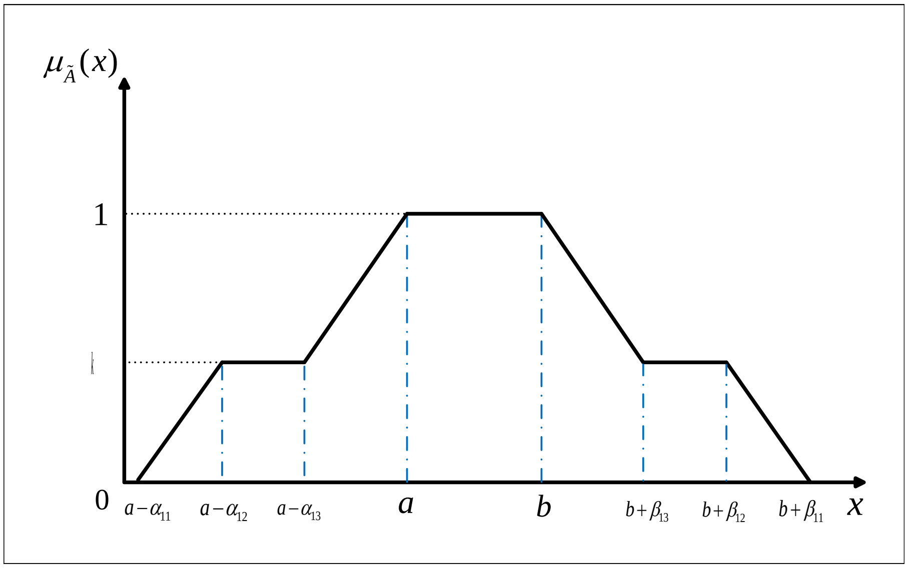

Figure 1: Octagonal fuzzy number $\tilde{A} = (\alpha_{11}\ , \alpha_{12},\ \alpha_{13},\ a, b, \beta_{13}\ , \beta_{12}, \beta_{11})$.

We now move on to defined the $\alpha-cut$ for the octagonal fuzzy numbers as a first step to define the basic operations. We base our work in this paper on the basic idea that we are defining operations such as addition and multiplication without the approximation that is used normally in fuzzy sets. The goal is to use operations that produces the exact results than just an approximate operation [20].

Definition 2.3. *Let $\hat{A} = (a, b, \alpha_{11}, \beta_{11}.\alpha_{12}, \beta_{12}, \alpha_{13}, \beta_{13}; k, 1)$ be an octagonal fuzzy number, the $\alpha - cut$ of $\hat{A}$ denoted by A_α is defined as:*

1. *If $0 \leq \alpha < k$, then $A_\alpha = [a - \alpha_{11} + \frac{\alpha}{k}(\alpha_{11} - \alpha_{12}), b + \beta_{11} + \frac{\alpha}{K}(\beta_{12} - \beta_{11})]$.*
2. *If $\alpha = k$, then the outer interval is $A_\alpha^{outer} = [a - \alpha_{12}, b + \beta_{12}]$ and the inner interval is $A_\alpha^{inner} = [a - \alpha_{13}, b + \beta_{13}]$.*
3. *If $k < \alpha \leq 1$, then $A_\alpha = [a + \frac{(\alpha-1)}{(1-k)}\alpha_{13}, b + \frac{(\alpha-1)}{(1-k)}\beta_{13}]$.*

In order to define the basic operations, we find the sum of lower and upper limits for fuzzy intervals presented in the $\alpha-cut$ Definition 2.3. So we introduce the following definition:

Definition 2.4. *Let $\hat{A} = (a, b,\ \alpha_{11},\ \beta_{11}.\alpha_{12},\ \beta_{12}, \alpha_{13}, \beta_{13}; k, 1)$ and $\hat{B} = (c, d,$ $\alpha_{21},\ \beta_{21}.\alpha_{22}, \beta_{22}, \alpha_{23}, \beta_{23}; k, 1)$ be two octagonal fuzzy numbers and $A_\alpha + B_\alpha =$ $[L_\alpha, U_\alpha]$, where L_α and U_α are the lower and upper limits of the sum of lower and upper limits of the intervals A_α and B_α respectively. Regarding the cases above, the sum is defined as follows:*

1. *If* $0 \leq \alpha < k$, *then*

$$\begin{aligned} L_\alpha &= a - \alpha_{11} + \frac{\alpha}{k}(\alpha_{11} - \alpha_{12}) + c - \alpha_{21} + \frac{\alpha}{k}(\alpha_{21} - \alpha_{22}) \\ &= a + c - (\alpha_{11} + \alpha_{21}) + \frac{\alpha}{k}(\alpha_{11} - \alpha_{12} + \alpha_{21} - \alpha_{22}) \\ U_\alpha &= b + d + (\beta_{11} + \beta_{21}) + \frac{\alpha}{k}(\beta_{12} - \beta_{11} + \beta_{22} - \beta_{21}) \end{aligned}$$

2. *If* $\alpha = k$, *then*

$$\begin{aligned} A_\alpha^{outer} + B_\alpha^{outer} &= [a + c - (\alpha_{12} + \alpha_{22}), b + d + (\beta_{12} + \beta_{22})] \\ A_\alpha^{inner} + B_\alpha^{inner} &= [a + c - (\alpha_{13} + \alpha_{23}), b + d + (\beta_{13} + \beta_{23})] \end{aligned}$$

3. *If* $k < \alpha \leq 1$, *then*

$$\begin{aligned} L_\alpha &= a + c + \frac{(\alpha - 1)}{(1 - k)}(\alpha_{13} + \alpha_{23}) \\ U_\alpha &= b + d + \frac{(\alpha - 1)}{(1 - k)}(\beta_{13} + \beta_{23}) \end{aligned}$$

Using Definition 2.4 and the operations presented in [20], we have the following:

Definition 2.5. *Let* $\hat{A} = (a, b, \alpha_{11}, \beta_{11}.\alpha_{12}, \beta_{12}, \alpha_{13}, \beta_{13}; k, 1)$ *and* $\hat{B} = (c, d, \alpha_{21}, \beta_{21}.\alpha_{22}, \beta_{22}, \alpha_{23}, \beta_{23}; k, 1)$ *be two octagonal fuzzy numbers. The sum of* $\hat{A}$ *and* $\hat{B}$ *denoted by* $\hat{A} \oplus \hat{B}$ *is given by:*

$$\hat{A} \oplus \hat{B} = (a+c, b+d, \alpha_{11}+\alpha_{21}, \beta_{11}+\beta_{21}, \alpha_{12}+\alpha_{22}, \beta_{12}+\beta_{22}, \alpha_{13}+\alpha_{23}, \beta_{13}+\beta_{23}) \tag{2}$$

Definition 2.6. *Let* $\hat{A} = (a, b, \alpha_{11}, \beta_{11}.\alpha_{12}, \beta_{12}, \alpha_{13}, \beta_{13}; k, 1)$ *and* $\hat{B} = (c, d, \alpha_{21}, \beta_{21}.\alpha_{22}, \beta_{22}, \alpha_{23}, \beta_{23}; k, 1)$ *be two octagonal fuzzy numbers. We can deduce the product of* $\hat{A}$ *and* $\hat{B}$ *denoted by* $\hat{A} \otimes \hat{B}$ *as following:*

$$\begin{aligned} \hat{A} \otimes \hat{B} = \; & (ac, bd, a\alpha_{21} + c\alpha_{11} - \alpha_{11}\alpha_{21}, b\beta_{21} + d\beta_{11} + \beta_{11}\beta_{21}, a\alpha_{22} + c\alpha_{12} - \alpha_{22}, \\ & b\beta_{22} + d\beta_{12} + \beta_{12}\beta_{22}, a\alpha_{23} + c\alpha_{13} - \alpha_{13}\alpha_{23}, b\beta_{23} + d\beta_{13} + \beta_{13}\beta_{23}; k, 1). \end{aligned}$$

Definition 2.7. *Let* $\hat{A} = (a, b, \alpha_{11}, \beta_{11}.\alpha_{12}, \beta_{12}, \alpha_{13}, \beta_{13}; k, 1)$ *and* λ *be any real number, then*

$$\lambda\hat{A} = \begin{cases} (\lambda a, \lambda b, \lambda\alpha_{11}, \lambda\beta_{11}.\lambda\alpha_{12}, \lambda\beta_{12}, \lambda\alpha_{13}, \lambda\beta_{13}; k, 1) & \lambda \geq 0 \\ (\lambda b, \lambda a, |\lambda|\beta_{11}, |\lambda|\alpha_{11}.|\lambda|\beta_{12}, |\lambda|\alpha_{12}, |\lambda|\beta_{13}, |\lambda|\alpha_{13}; k, 1) & \lambda < 0. \end{cases}$$

So, from Definitions 2.7 and 2.5 we can define the subtraction operation as follows:

Definition 2.8. *Let $\hat{A} = (a, b, \alpha_{11}, \beta_{11}.\alpha_{12}, \beta_{12}, \alpha_{13}, \beta_{13}; k, 1)$ and $\hat{B} = (c, d, \alpha_{21}, \beta_{21}.\alpha_{22}, \beta_{22}, \alpha_{23}, \beta_{23}; k, 1)$ be two octagonal fuzzy numbers. The subtraction of $\hat{A}$ and $\hat{B}$ denoted as $\hat{A} \ominus \hat{B}$ is given as:*

$$\hat{A} \ominus \hat{B} = (a - d, b - c, \alpha_{11} + \beta_{21}, \beta_{11} + \alpha_{21}, \alpha_{12} + \beta_{22}, \beta_{12} + \alpha_{22}, \alpha_{13} + \beta_{23}, \beta_{13} + \alpha_{23}) \quad (3)$$

Remark 2.1. *A real number $n \in \mathbb{R}$ can be expressed by the octagonal number definition as $\hat{N} = (n, n, 0, 0, 0, 0, 0, 0; k, 1)$ so for two real numbers n, m the basic operations can be applied directly as presented in Definitions 2.5, 2.6 and 2.8.*

In the following section, we use the defined operations presented in this section in order to build up a mathematical model of fuzzy linear programming along with all sub linear systems with octagonal fuzzy numbers. $\hat{c_j}, \hat{a_j}$ and $\hat{q_j}$

3 Fuzzy Linear Programming

We start by defining $\hat{c_j} = (c_{1j}, c_{2j}, ..., c_{8j}), \hat{a_j} = (a_{1j}, ..., a_{8j}), \hat{q_j} = (q_{1j}, ..., q_{8j})$ and $\hat{x_j} = (x_{1j}, ..., x_{8j})$ to be octagonal fuzzy numbers. The mathematical model of fuzzy linear programming can be given as:

$$Min\ \hat{Z} \ = \ \sum_{j=1}^{n} \hat{c_j} \otimes \hat{x_j} \quad (4)$$

subject to the constraints

$$\begin{aligned} &\sum_{j=1}^{n} \hat{a_j}^{(i)} \otimes \hat{x_j} \geq \hat{q_i},\ i = 1, 2, ..., n \\ &\hat{x_j} \geq \hat{0},\ j = 1, 2, ..., n \end{aligned} \quad (5)$$

Now, Using Definitions 2.5 and 2.6, we can expand the fuzzy linear programming problem so that we have:

$$\begin{aligned} Min\ \hat{Z} &= \sum_{j=1}^{n} \hat{c}_j \otimes \hat{x}_j \qquad (6) \\ &= (\sum_{j=1}^{n} c_{1j}x_{1j}, \sum_{j=1}^{n} c_{2j}x_{2j}, \sum_{j=1}^{n}(c_{1j}x_{3j} + c_{3j}x_{1j} - c_{3j}x_{3j}), \sum_{j=1}^{n}(c_{2j}x_{4j} + \qquad (7) \\ &\quad c_{4j}x_{2j} + c_{4j}x_{4j}), \sum_{j=1}^{n}(c_{1j}x_{5j} + c_{5j}x_{1j} - c_{5j}x_{5j}), \sum_{j=1}^{n}(c_{2j}x_{6j} + c_{6j}x_{2j} + c_{6j}x_{6j}), \\ &\quad \sum_{j=1}^{n}(c_{1j}x_{7j} + c_{7j}x_{1j} - c_{7j}x_{7j}), \sum_{j=1}^{n}(c_{2j}x_{8j} + c_{8j}x_{2j} + c_{8j}x_{8j})) \end{aligned}$$

where

$$\begin{aligned} \sum_{j=1}^{n} \hat{a_j}^{(i)} \otimes \hat{x}_j &= (\sum_{j=1}^{n} a_{1j}^{(i)}x_{1j}, \sum_{j=1}^{n} a_{2j}^{(i)}x_{2j}, \sum_{j=1}^{n}(a_{1j}^{(i)}x_{3j} + a_{3j}^{(i)}x_{1j} - a_{3j}^{(i)}x_{3j}), \qquad (8) \\ &\quad \sum_{j=1}^{n}(a_{2j}^{(i)}x_{4j} + a_{4j}^{(i)}x_{2j} + a_{4j}^{(i)}x_{4j}), \sum_{j=1}^{n}(a_{1j}^{(i)}x_{5j} + a_{5j}^{(i)}x_{1j} - a_{5j}^{(i)}x_{5j}), \\ &\quad \sum_{j=1}^{n}(a_{2j}^{(i)}x_{6j} + a_{6j}^{(i)}x_{2j} + a_{6j}^{(i)}x_{6j}), \sum_{j=1}^{n}(a_{1j}^{(i)}x_{7j} + a_{7j}^{(i)}x_{1j} - a_{7j}^{(i)}x_{7j}), \\ &\quad \sum_{j=1}^{n}(a_{2j}^{(i)}x_{8j} + a_{8j}^{(i)}x_{2j} + a_{8j}^{(i)}x_{8j})). \qquad (9) \end{aligned}$$

In order to obtain the optimal fuzzy solution for the problem, we transform the fuzzy system into a system of crisp linear programming problems where each is solved to get the optimal solution. The final optimal solution for the fuzzy problem $\hat{Z}$ and $\hat{x}_j$ is then formed from the sub solutions for the crisp problems. For the octagonal fuzzy number based linear programming problem, The corresponding crisp linear models can be expressed in three general systems as follows:

System I (basic problems):

$$\begin{aligned} &\min Z_r = \sum_{j=1}^{n} c_{rj}x_{rj}, \\ &\text{s.t. } \sum_{j=1}^{n} a_{rj}^{(i)}x_{rj} \geq q_{ir}, \quad x_{rj} \geq 0,\ j = 1, \ldots, n, \end{aligned} \qquad r = 1, 2. \qquad (10)$$

System II (odd-derived problems):

$$
\begin{aligned}
&\min Z_r = \sum_{j=1}^{n} \left[(c_{1j} - c_{rj})x_{rj} + c_{rj}x_{1j}\right],\\
&\text{s.t.} \ \sum_{j=1}^{n} \left[(a_{1j}^{(i)} - a_{rj}^{(i)})x_{rj} + a_{rj}^{(i)}x_{1j}\right] \geq q_{ir},\\
&x_{rj} \geq 0, \ j = 1, \ldots, n, \qquad r = 3, 5, 7.
\end{aligned} \tag{11}
$$

System III (even-derived problems):

$$
\begin{aligned}
&\min Z_r = \sum_{j=1}^{n} \left[(c_{2j} + c_{rj})x_{rj} + c_{rj}x_{2j}\right],\\
&\text{s.t.} \ \sum_{j=1}^{n} \left[(a_{2j}^{(i)} + a_{rj}^{(i)})x_{rj} + a_{rj}^{(i)}x_{2j}\right] \geq q_{ir},\\
&x_{rj} \geq 0, \ j = 1, \ldots, n, \qquad r = 4, 6, 8.
\end{aligned} \tag{12}
$$

To solve the resulting crisp linear programming problems, one can use any of the well known simplex methods. In this paper, we use the interior point method [18] to produce the optimal solution for each crisp programming problem. The final fuzzy optimal solution is composed of all the optimal solutions of the crisp problems.

Theorem 3.1. *The solution of problem 4 is the same as the solution of the crisp linear programming problems described in Equations 10-12.*

The fuzzy solution for the problem is said to be optimal if all the crisp problems have optimal solutions. The reader may turn to [9] for more theoretical details on the feasibility of the fuzzy solution with octagonal fuzzy numbers and the relation between the fuzzy and crisp programming problems. The method reduces the fuzzy LP into *eight* crisp LPs, each solvable using classical algorithms (e.g., simplex, interior-point), resulting in polynomial-time solvability. The increase in complexity is linear with respect to the number of fuzzy components. The algorithm we used can be summarized as following:

Algorithm 1 Fuzzy Linear Programming with Octagonal Fuzzy Numbers

1: **Input:** Fuzzy parameters for objective and constraints
2: **Output:** Fuzzy optimal solution $\hat{Z}$ and $\hat{x}_j$
3: Transform all fuzzy input parameters into their octagonal 8-tuple representations.
4: Define fuzzy decision variables as $\hat{x}_j = (x_{1j}, x_{2j}, \ldots, x_{8j})$.
5: Formulate the corresponding crisp subproblems Z_1–Z_8 using the operations defined in Definitions 5–8.
6: Solve each crisp subproblem individually using a linear programming solver (e.g., interior-point method).
7: Aggregate the crisp solutions to construct fuzzy outcome vectors $\hat{Z}$ and $\hat{x}_j$.
8: Return the final fuzzy optimal solution in the octagonal fuzzy domain.

In the following section, we use the proposed algorithm to solve a numerical example and compare the results to those presented in [9].

4 Numerical illustration

In this section, we implement the procedure and algorithm presented in Section 3 using a numerical example. The example deals with a linear programming problem with octagonal fuzzy numbers and constraints with crisp coefficients. The results is compared to those presented in [9].

If we have two types of food, the first contain 20 units of proteins per gram and 40 unites of minerals per gram. The second type contains 30 units of proteins and the same of minerals per gram. Knowing that the human body requires 900 protein units and 1200 mineral units in minimum daily and these amounts varies depending on the particular individual. Also, the cost of food varies depending on the place of purchase. The goal is finding the approximate amount of those two types of food should be consumed in order to have the needed levels with minimum cost.

The problem is clearly better modeled by fuzzy numbers. Thus, it has been chosen that this linear programming problem is modeled using symmetrical fuzzy octagonal numbers with crisp coefficients for the constraints as they represent the presence of proteins and minerals on those specific types of food. The mathematical problem can be given as the following:

$$Min\ \hat{Z} = \hat{c_1} \otimes \hat{x_1} + \hat{c_2} \otimes \hat{x_2},$$

subject to

$$20\hat{x_1} + 30\hat{x_2} \geq \hat{q_1}$$
$$40\hat{x_1} + 30\hat{x_2} \geq \hat{q_2}$$
$$\hat{x_1} \geq \hat{0}, \hat{x_2} \geq \hat{0}$$

Where $\hat{c_1} = (5, 7, 3, 3, 2, 2, 1, 1)$ and $\hat{c_2} = (7, 9, 4, 4, 3, 3, 2, 2)$ are the cost of the two types of food. $\hat{q_1} = (890, 910, 5, 5, 4, 4, 2, 2)$ and $\hat{q_2} = (1195, 1205, 5, 5, 4, 4, 2, 2)$ are the expected minimum requirement of proteins and minerals for humans.

The crisp linear programming problems which are equivalent to the fuzzy linear programming are

$$\begin{aligned} Min\ Z_1 &= 5x_{11} + 7x_{12}, \quad \text{subject to} \\ & 20x_{11} + 30x_{12} \geq 890 \\ & 40x_{11} + 30x_{12} \geq 1195 \end{aligned}$$

The optimal solution $Z_1 = 212.75$ is occurred when $x_{11} = 15.25$ and $x_{12} = 19.5$

$$\begin{aligned} Min\ Z_2 &= 7x_{21} + 9x_{22}, \quad \text{subject to} \\ & 20x_{21} + 30x_{22} \geq 910 \\ & 40x_{21} + 30x_{22} \geq 1205 \end{aligned}$$

Concerning this part we have to take in account that $x_{21} \geq x_{11}$ and $x_{22} \geq x_{12}$. So, by solving this problem we get $x_{21} = 15.25,\ x_{22} = 20.1667$ and the optimal solution is $Z_2 = 288.25$.

For the remaining parts, we fix the values for x_{11}, x_{12}, x_{21} and x_{22} which are calculated in the first two parts above.

$$\begin{aligned} Min\ Z_3 &= 3x_{11} + 4x_{12} + 2x_{31} + 3x_{32}, \quad \text{subject to} \\ & 20x_{31} + 30x_{32} \geq 5 \\ & 40x_{31} + 30x_{32} \geq 5 \end{aligned}$$

The values of $x_{11} = 15.25, x_{12} = 19.5, x_{31} = 0, x_{32} = 0.1667$ and the optimal solution $Z_3 = 124.25$.

$$\begin{aligned} Min\ Z_4 &= 3x_{21} + 4x_{22} + 10x_{41} + 13x_{42}, \quad \text{subject to} \\ &20x_{41} + 30x_{42} \geq 5 \\ &40x_{41} + 30x_{42} \geq 5 \end{aligned}$$

The values of $x_{41} = 15.25, x_{22} = 20.1667, x_{41} = 0, x_{42} = 0.1667$ and the optimal solution $Z_4 = 128.5835$.

$$\begin{aligned} Min\ Z_5 &= 2x_{11} + 3x_{12} + 3x_{51} + 4x_{52}, \quad \text{subject to} \\ &20x_{31} + 30x_{32} \geq 4 \\ &40x_{31} + 30x_{32} \geq 4 \end{aligned}$$

The values of $x_{11} = 15.25, x_{12} = 19.5, x_{51} = 0, x_{52} = 0.1333$ and the optimal solution $Z_5 = 89.7433$.

$$\begin{aligned} Min\ Z_6 &= 2x_{21} + 3x_{22} + 9x_{61} + 12x_{62}, \quad \text{subject to} \\ &20x_{61} + 30x_{62} \geq 4 \\ &40x_{61} + 30x_{62} \geq 4 \end{aligned}$$

The values of $x_{21} = 15.25, x_{22} = 20.1667, x_{61} = 0, x_{62} = 0.1333$ and the optimal solution $Z_6 = 92.6001$.

$$\begin{aligned} Min\ Z_7 &= x_{11} + 2x_{12} + 4x_{71} + 5x_{72}, \quad \text{subject to} \\ &20x_{31} + 30x_{32} \geq 2 \\ &40x_{31} + 30x_{32} \geq 2 \end{aligned}$$

The values of $x_{11} = 15.25, x_{12} = 19.5, x_{71} = 0, x_{72} = 0.0667$ and the optimal solution $Z_7 = 54.7233$.

$$\begin{aligned} Min\ Z_8 &= x_{21} + 2x_{22} + 8x_{81} + 11x_{82}, \quad \text{subject to} \\ &20x_{41} + 30x_{42} \geq 2 \\ &40x_{41} + 30x_{42} \geq 2 \end{aligned}$$

The values of $x_{21} = 15.25, x_{22} = 20.1667, x_{81} = 0, x_{82} = 0.0667$ and the optimal solution $Z_8 = 56.3167$.

Therefore, the optimal solution $\hat{Z} = (212.75, 288.25,\ 124.25,\ 128.5835,$ $89.7433,\ 92.6001,\ 54.7233,\ 56.3167)$ achieved at the octagonal fuzzy numbers $\hat{x_1} = (15.25,15.25,\ 0, 0, 0, 0, 0, 0)$ and $\hat{x_2} = (19.5,\ 20.1667,\ 0.1667, 0.1667,$ $0.1333, 0.1333,\ 0.0667,\ 0.0667)$.

The results of the proposed method is compared to results presented by [9] in Table 1.

Table 1: Comparison of the proposed method with method presented in [9]

Metric	**Proposed Method**	**Method [9]**
Fuzzy Objective $\hat{Z}$ (center)	[212.75, 288.25]	[203.83, 296.17]
Central Interval Width	75.5	92.34
Maximum Interval Width	231.53	249.83
Solution Time (sec)	∼0.89	∼0.92
$\hat{x}_1$ Central Value	15.25	15.00
$\hat{x}_2$ Central Value	19.83	20.00
Computational Strategy	Exact Operations	Approximate Multiplication

In [9], the method produced an optimal solution $\hat{Z} = (203.833\ , 296.1667, 138.583, 138.584, 100.397, 100.4, 60.61,\ 60.61)$ with $\hat{x_1} = (14.25,\ 15.75,\ 0.5,\ 0.5, 0.4, 0.4, 0.2, 0.2)$ and $\hat{x_2} = (19.1667, 20.833, 0.4997, 0.497, 0.4, 0.4, 0.2, 0.2)$. Looking at the results, the method proposed int this paper have smaller uncertainty intervals. For example, the central interval in our approach is $(212.75, 288.25)$ compared to $(203.833, 296.1667)$.

5 Conclusions

We consider the fuzzy linear programming problem in context of octagonal fuzzy numbers. We have used the multiplication operation for fuzzy numbers without any approximation or any added special conditions to eliminate some terms in the multiplication operation itself. We proposed a new algorithm for the solving the fuzzy linear programming problem where the fuzzy system is converted into a number of crisp problems. The fuzzy optimal solution is then obtained by collecting the crisp solutions solved using classical simplex methods. We apply the proposed method using a numerical example and compare our results with a classical method presented int the field. The numerical results reflect the efficiency and performance of the new approach.

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