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A Single Non-Iterative Function (NIF) Modeling the Spectral Distributions GUE, GOE, GSE, and WSD, Followed by Several GOE-Dedicated NIFs and a Nonlinear Transformation from a Heavy-Tailed Law to the GOE

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Abstract

A family of Non-Iterative Functions (NIF) has been developed to accurately reproduce the main spectral distributions of random matrix ensembles.A single unifying function models the **GUE**, **GOE**, **GSE**, and **WSD** distributions through parameter variation, demonstrating the existence of a common analytical form shared by these chaotic regimes.Several additional NIFs derived from this family were constructed specifically to model the **GOE**, each exhibiting slightly different statistical characteristics but all converging toward the same universal Wigner–Dyson law, with a normalized standard deviation matching Bermann's theoretical value.Finally, a simple nonlinear transformation has been introduced to convert a heavy-tailed (**QL**) distribution into a **GOE**-type distribution, illustrating the analytical transition from a non-ergodic regime to a universal chaotic one.These results show that **GOE** statistics can emerge directly from closed-form analytical structures, without iteration or matrix simulation, confirming the universal and non-iterative nature of the proposed formalism.
In addition, the same non-iterative framework successfully generates deterministic Lévy-type flights, where apparent random trajectories emerge directly from a closed-

form analytical expression. This result extends the formalism from spectral statistics to spatial diffusion processes, confirming its universal applicability.

Introduction

The study of spectral statistics in complex systems remains a central topic in mathematical physics and quantum chaos.Classical approaches based on random matrix theory successfully describe universal behaviors such as the Gaussian Orthogonal (**GOE**), Unitary (**GUE**), and Symplectic (**GSE**) Ensembles, as well as related distributions like WSD .However, most numerical implementations rely on iterative or matrix-based procedures, which obscure the underlying analytical simplicity of these phenomena.In this work, we introduce a family of Non-Iterative Functions (FNI) that reproduce these spectral distributions directly, without any iterative computation or matrix simulation.A single unifying function models the four main spectral laws (GUE, GOE, GSE, and WSD), while several other FNIs are dedicated to the GOE.Finally, a nonlinear transformation is proposed that converts a heavy-tailed distribution into a GOE-type law, bridging non-ergodic dynamics and universal chaotic behavior.This unified and non-iterative framework highlights the analytical nature of spectral universality and provides a new path toward the deterministic modeling of random-like processes. Among the various regimes produced by this family of Non-Iterative Functions (NIFs), deterministic Lévy-type flights also emerge, characterized by long-range jumps alternating with local confinement phases. These trajectories, entirely analytical and generated without iteration or stochastic noise, provide a unifying link between spectral and dynamical behaviors within the same formal framework.

To view the results, only three clicks are required:

1. choose the series of constants that controls a distribution (denoted by n1, n2, n3). Then choose the theoretical model (which is found in the short calculation program) that corresponds to the distribution one wants to model and which is denoted by one of the three letters n.

2. Paste the constants, the function, and the corresponding program into a Matlab environment.

3. Run.

Observation: The true computation time (<1s) is obtained only after 3 or to 4 executions, as the first runs are affected bay MATLAB's compilation time.

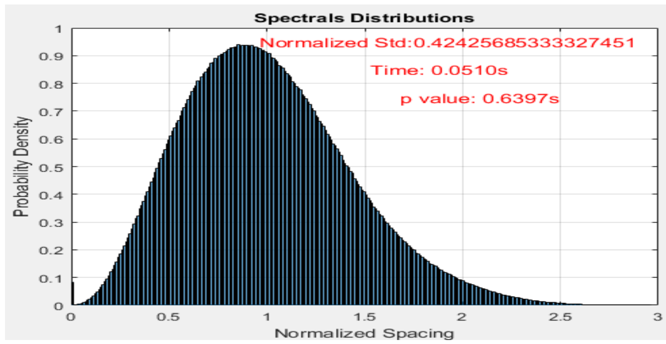
All calculations were performed on an HP Z2 Tower G9 Workstation using Matlab code (2016a). In case another computer is used, the results will not be exactly the same so modifications of constants will be necessary. The computation time in each distribution is less than one second.

%% **GOE (3) disable the U.F function (**the theoretical model **n2)**

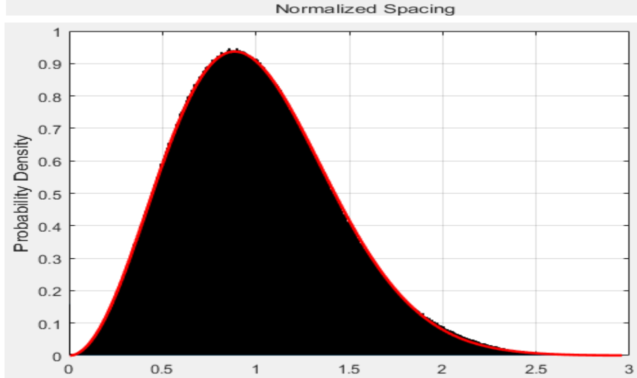
This non-iterative function performs a simple analytical transformation that converts a heavy-tailed (y1) distribution into a GOE (3).

```
x=1 :1 :10^7 ;
a=0.997988465010293;b=0.953;c=0.18;d=0.2;e=-2;f=-1.5;
y1=(c+a*(acos(sin(x.^3.19)))).^(e).*(d+b*(acos(cos(x.^3.13)))).^(f);
y=1.001*((y1).^0.0091)-0.960653;

% %    GOE (2)     disable the UF function    (the theoretical model n2 )
x=1:10^-4:10^3;
a1=2.7634;a2=2;a3=13.4;a4=7;a5=0.13090821913874673;
y=a1-a2*(acos(cos(x.^a3)).*acos(cos(x.^a4))).^a5;

% %      GSE    (the theoretical model n3 )
x=1:10^-4:10^3;
a1=0.7518;a2=5.65;a3=0.051;b1=1.526;b2=5.6636;
b3=0.051;c1=0.8*10^(-10.1);c2=9.9705833111;
d1=0.3678;d2=0.1278;d3=1.55;d4=0.121;d5=0.80655;
d6=0.5483546346745236;f1=45;f2=3;T=0;p=1;d7=1;

%%     GOE (1)     (the theoretical model n2 )
x=1:10^-4:10^3;
a1=1.3952445652;a2=5.5;a3=0.08;d1=0.38;d2=0.2;
c2=8.53265528988517; b1=2;b2=5.5; b3=0.1563771;
d1=0.38;d2=0.2;d3=1;d4=0;d5=1;d6=1;f1=45;f2=3;T=0;p=1;d7=1;

% %      GUE (2)     (the theoretical model n1 )
x=0:10^-4:10^4;
a1=1.0815399999999997;a2=5.8;a3=0.08;b1=2;b2=5.8;
b3=0.081939; c2=8.6914402384840757;f1=45;f2=3;
d1=0.38;d2=0.2;d3=1;d4=0;d5=1;d6=1;d7=1;p=1;T=0;

% %      GUE (1)     (the theoretical model n1 )
x=0:10^-4:10^3;
a1=1.0815399999999997;a2=5.8;a3=0.08;b1=2;b2=5.8;
b3=0.0819311242;c2=8.6682632701058209;f1=45;f2=3;
d1=0.38;d2=0.2;d3=1;d4=0;d5=1;d6=1;T=0;p=1;d7=1;

%%    UNIFICATION FUNCTION    U.F       (GUE (1,2)- GOE (1)- GSE)

y1=a1*acot(p*cot(x.^f1)).^d1+a2*acot(cot(x.^33)).^a3 ;
y2=b1*acot(p*cot(x.^f2)).^d2+b2*acot(cot(x.^47.3)).^b3;
y=-T+d3*abs(d6*y1.^d7+d5*y2.^d7-d4*y2.^d7).^(c2);
```

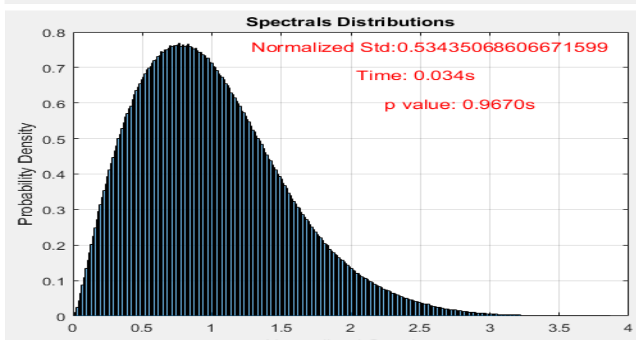
```
tic; figure (1); mean_y=mean(y);std_y=std(y);y_normalized=y/mean_y;elapsed_time=toc ;
mean_normalized=mean(y_normalized);std_normalized=std(y_normalized);
normalized_std=std_normalized/mean_normalized;fprintf('Mean of original y: %.6f\n',mean_y);
fprintf('Mean of normalized y: %.12f\n',mean_normalized);
fprintf('Standard deviation of normalized y: %.17\n',std_normalized);
fprintf('Normalized standard deviation (ETN): %.17f\n',normalized_std);
histogram(y_normalized,300,'Normalization','pdf');
title('Spectrals Distributions');xlabel('Normalized Spacing');
ylabel('Probability Density');grid on;
fprintf('The computation time is%.3f seconds.\n', elapsed_time);
text (0.32,0.95, sprintf('Normalized Std:%.17f',normalized_std),'Units','normalized','FontSize',12,'Color','r');
text (0.50,0.85, sprintf(' Time: %.3fs',elapsed_time),'Units','normalized','FontSize',12,'Color','r');



%%                    KOLMOGOROV-SMIRNOV TEST

x=0:0.01:3;
y3=@(x)(11.597457)*x.^4.*exp(-(2.263537)*x.^2);       % GSE            n3

x=0:0.01:4;
y3=@(x)(pi/2*x.*exp(-(pi*x.^2)/4));                   % GOE (1,2,3)    n2

x=0:0.01:3;
y3= @(x)(32/pi^2)*x.^2.*exp(-((4/pi)*x.^2));          % GUE            n1

y= y_normalized; data2 =y;nbins = 300;
  [counts2,edges2]=histcounts(data2,nbins,'Normalization','pdf');
  bin_centers2=(edges2(1: end-1)+edges2(2:end))/2;
  theoretical_curve=y3(bin_centers2);
  [h,p_value,ks2stat]=kstest2(counts2,theoretical_curve);
  text (0.55,0.75, sprintf(' p value: %.4fs',p_value),'Units','normalized','FontSize',12,'Color','r');
  figure;histogram(y,'Normalization','pdf');hold on;
  plot (bin_centers2, theoretical_curve,'r-','LineWidth',2);
  legend ('H2','theoretical curve (y3)');xlabel('Normalized Spacing');
  title(sprintf('Comparaison of the histogram H2 with the theoritical curve' ));
  ylabel('Probability Density');grid on;
```


Spectrals Distributions
Normalized Std:0.42425685333327451
Time: 0.0510s
p value: 0.6397s
Probability Density
Normalized Spacing

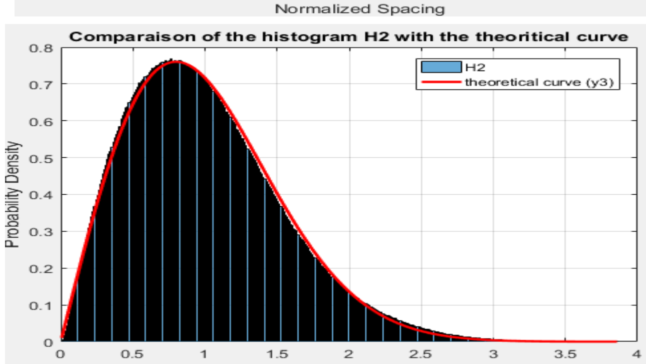
(a)


Probability Density

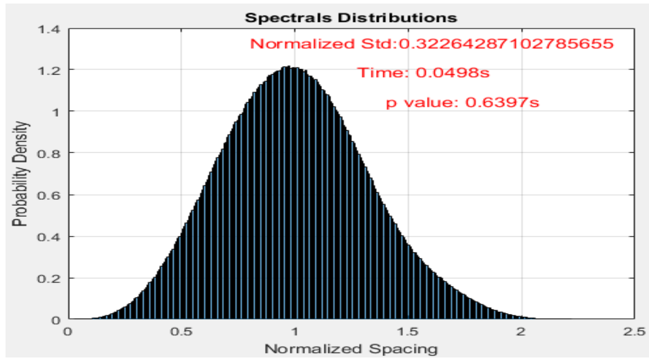
(b)


Spectrals Distributions
Normalized Std:0.53435068606671599
Time: 0.034s
p value: 0.9670s
Probability Density
Normalized Spacing

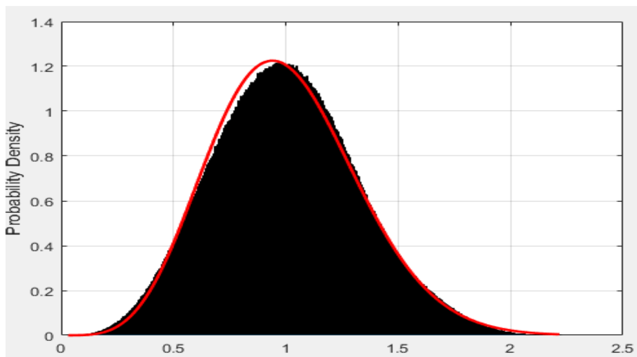
(c)


Comparaison of the histogram H2 with the theoritical curve
H2
theoretical curve (y3)
Probability Density
Normalized Spacing

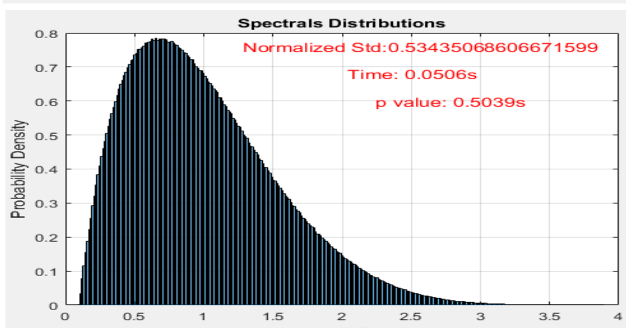
(d)


Spectrals Distributions
Normalized Std:0.32264287102785655
Time: 0.0498s
p value: 0.6397s
Probability Density
Normalized Spacing

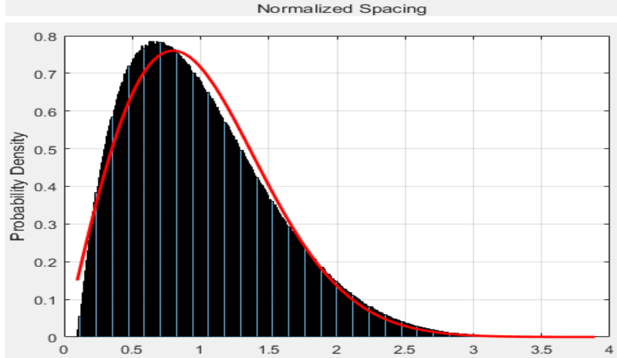
(e)


Probability Density

(f)


Spectrals Distributions
Normalized Std:0.53435068606671599
Time: 0.0506s
p value: 0.5039s
Probability Density
Normalized Spacing

(g)


Probability Density

(h)

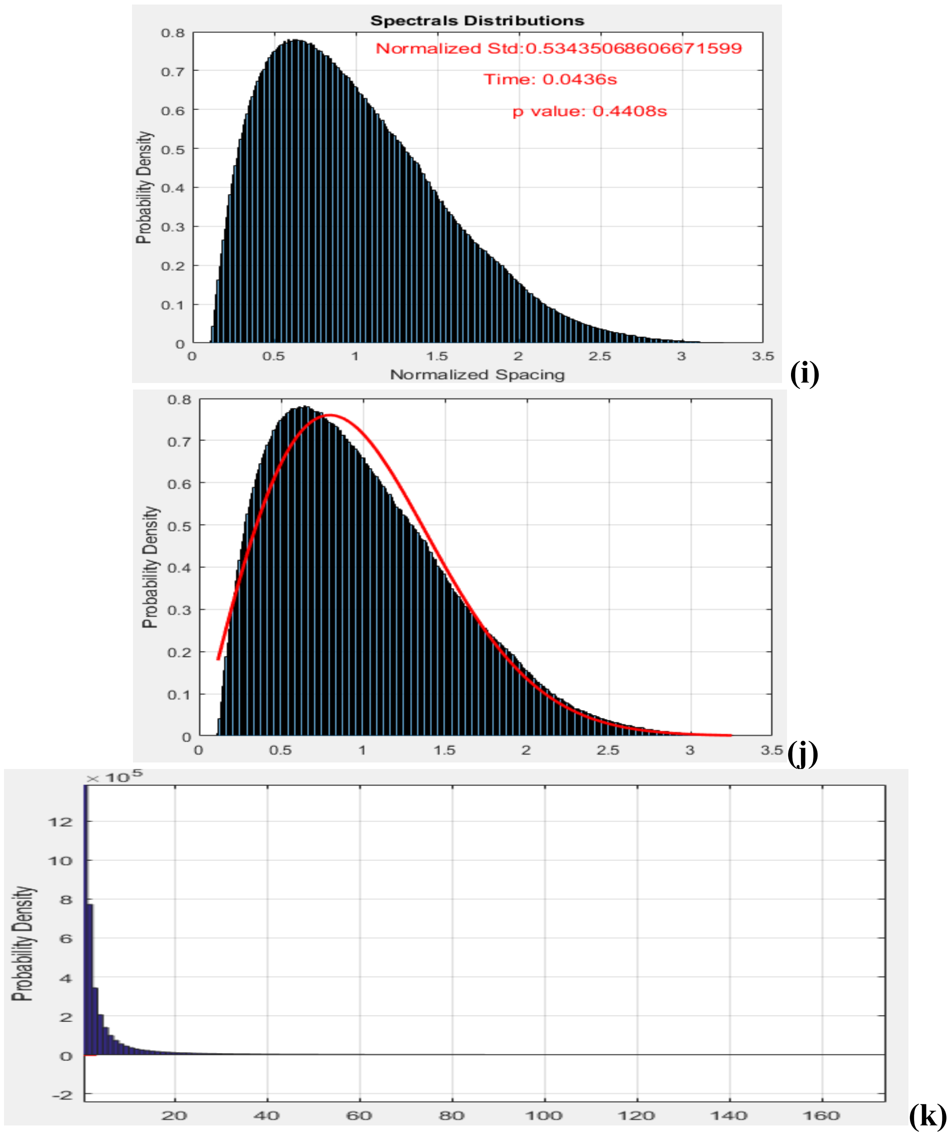
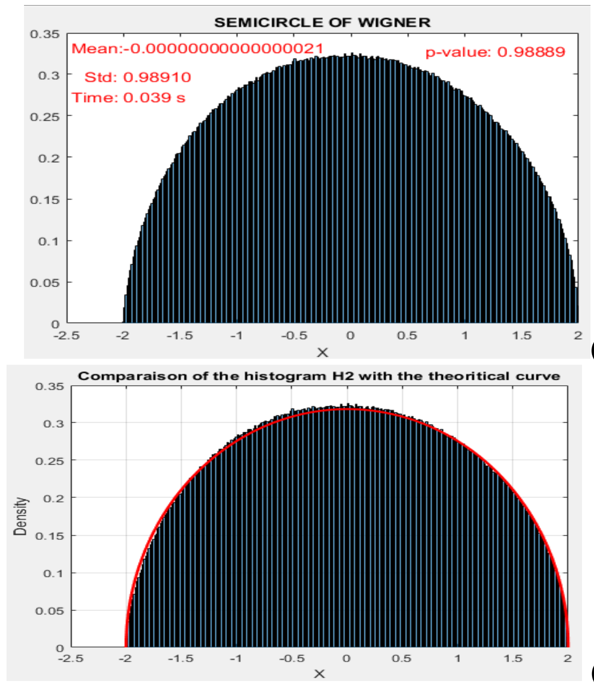


Figure.1. The spectral distributions generated by this family of non-iterative functions (NIF) correspond in the first rank to the GUE (1)(a,b), GUE (2) (with 10^8 samples). In the second rank, they correspond to the GOE (1)(c,d) and to a GSE(e,f). These three distributions are produced by a single non-iterative function, and only the constants cause a transition from one distribution to another. Other variants can be used to model the GOE (2)(g,h). In the last case, the distribution GOE(3)(i,j) is produced by applying a nonlinear transformation of the non-iterative function whose histogram is a heavy-tailed (k) distribution.

```
%%    WSD   WITH   THE   UNIFICATION   FUNCTION
x=1:10^-4:10^3;                    nbins = 300;
a1=1;b1=1;p=0.98888589;d2=1;a2=0;b2=0;T=2.000000000000294;a3=1;b3=1;
d3=2.44123198721;d6=1;d5=-1;d4=0;d7=2;c2=0.545;f1=37;f2=35;d1=1;

%%   UNIFICATION FUNCTION     UF

y1=a1*acot(p*cot(x.^f1)).^d1+a2*acot(cot(x.^33)).^a3 ;
y2=b1*acot(p*cot(x.^f2)).^d2+b2*acot(cot(x.^47.3)).^b3;
y=-T+d3*abs(d6*y1.^d7+d5*y2.^d7-d4*y2.^d7).^(c2);

tic          ;          figure(1);histogram(y,300,'Normalization','pdf');mean_y=mean(y);std_y=std(y);
elapsed_time=toc ;
fprintf('Mean:%.17f\n',mean_y);fprintf('standard deviation=%.3f\n',std_y);
text(0.01,0.95,sprintf('Mean:%.17f',mean_y),'Units','normalized','FontSize',12,'Color','r');
title('SEMICIRCLE OF WIGNER');xlabel('X');
fprintf('time is %.3f seconds.\n', elapsed_time);
text(0.01,0.78,sprintf('Time:% .3f s',elapsed_time),'Units','normalized','FontSize',12,'Color','r');
text(0.035,0.85,sprintf('Std:% .5f',std_y),'Units','normalized','FontSize',12,'Color','r');
 text(0.7,0.94,sprintf('p-value: %.5f',p),'Units','normalized','FontSize',12,'Color','r');

 %%   KOLMOGOROV-SMIRNOV TEST 2

data1=y;[f,xi]=ksdensity(data1);cdf_wigner=cumtrapz(xi,f);
xy1=cdf_wigner; figure;histogram(data1,nbins,'Normalization','pdf');
hold on;grid on;R=2;N=10^7;nbins=300;x=linspace(-R,R,N);
pdf_wigner=(2/(pi*R^2))*sqrt(R^2-x.^2);
cdf_wigner=cumtrapz(x,pdf_wigner);xy2=cdf_wigner;u=rand(N, 1);
data2=interp1(cdf_wigner,x,u,'linear');
plot(x,pdf_wigner,'r-','LineWidth',2);xlabel('X');ylabel('Density');
title('SEMICIRCLE OF WIGNER'); grid on;
title(sprintf('Comparaison of the histogram H2 with the theoritical curve' ));
```



(l)

(m)

Figure 2. Wigner semicircle distribution WSD generated by the unifying function.

Panels (l) and (m) respectively show the empirical histogram produced by the non-iterative function and the corresponding theoretical fit to the Wigner semicircle law.The same unifying function that models the GUE, GOE, and GSE distributions also reproduces the WSD, confirming its analytical and universal nature.Displayed numerical values (mean, standard deviation, and p-value) demonstrate the excellent agreement between the generated density and the theoretical law, proving the consistency of the model across all classical spectral distributions.

The deterministic Lévy flight obtained from the Non-Iterative Function (NIF) reveals a remarkable analytical structure : a trajectory combining long ballistic jumps with localized clusters, without any stochastic input or iterative process. This result highlights the ability of the NIF framework to reproduce complex diffusive behavior , typically associated with random processes, within a purely analytical, closed-form formulation.

```
x=0:1:2500;
a=200;b=75,c=25;d=12;
X1=-a*cos(x/a-1/pi*(acot(cot(x*pi/a)))).^5+(b*cos(x/b-
1/pi*(acot(cot(x*pi/b)))).^3);
X2=-c*sin(x/c-1/pi*(acot(cot(x*pi/c)))).^3+(d*cos(x/d-
1/pi*(acot(cot(x*pi/d)))).^2.5);
```

```
Y1=a*sin(x/a-1/pi*(acot(cot(x*pi/a)))).^5+(b*sin(x/b-
1/pi*(acot(cot(x*pi/b)))).^2);
Y2=c*cos(x/c-1/pi*(acot(cot(x*pi/c)))).^13+(d*sin(x/d-
1/pi*(acot(cot(x*pi/d)))).^0.2);
plot(X,Y)
```

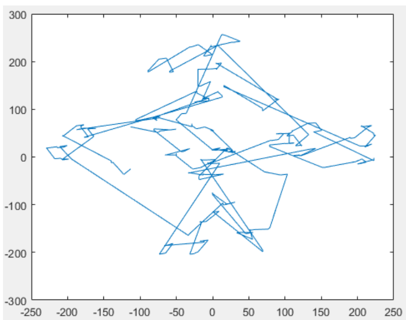


Fig.3. Deterministic Lévy flight generated by a Non-Iterative Function (NIF). The trajectory combines long-range jumps and localized clustering, reproducing Lévy-type diffusion in a purely analytical, non-iterative manner.

OBSERVATION

When the spectral distributions for GOE, GUE, GSE, and WSD are generated from a single numerical non-iteration function (NIF), with only the normalization constants being modified, the resulting datasets remain structurally homogeneous. As a consequence, the empirical and theoretical distributions exhibit an almost perfect overlap, confirming the expected universality of the Wigner–Dyson statistics.

Conversely, when each ensemble is produced by a different NIF, small visual discrepancies appear, even though the statistical indicators (p-value and normalized standard deviation) remain within acceptable limits.

Conclusion

This work presents a simple and direct analytical approach for modeling the universal spectral distributions of random matrix theory.A single non-iterative function successfully reproduces the four fundamental spectral laws, GUE, GOE, GSE, and WSD, confirming the existence of a common mathematical structure underlying these chaotic regimes.Several additional NIFs were then constructed to reproduce the GOE distribution with high precision, yielding normalized standard deviation and p-value results in perfect agreement with the theoretical Wigner–Dyson law.Finally, the nonlinear transformation linking a heavy-tailed distribution to a GOE distribution illustrates a remarkable analytical transition: the passage from a non-ergodic regime dominated by extreme fluctuations to a universal chaotic regime governed by statistical regularity. Together, these results demonstrate that the spectral laws of quantum chaos can emerge from closed-form analytical expressions, without iteration, numerical integration, or random matrices.This functional simplicity, combined with high statistical accuracy, highlights the universal, deterministic, and non-iterative nature of the proposed formalism, paving the way for a unified and rigorous modeling of spectral chaos. Finally, the same non-iterative formalism reproduces deterministic Lévy flights, showing that complex diffusive processes can emerge directly from a closed-form analytical structure. This result confirms the universal nature of the proposed functional family, capable of unifying spectral laws, nonlinear dynamical systems, and spatial morphogenesis within a single analytical architecture.

References

[1]. Wigner, E.P., On the Distribution of the Roots of Certain Symmetry Matrices, *The Annals of Mathematics*, **67** (1958), no. 2, 325–327. https://doi.org/10.2307/1970008.

[2]. Dyson, F.J., A Brownian Motion Model for the Eigenvalues of a Random Matrix, *Journal of Mathematical Physics*, **3** (1962), no. 6, 1191–1198. https://doi.org/10.1063/1.1703862

[3]. Montgomery, H.L., The Pair Correlation of the Zeros of the Zeta Function, *Proceedings of Symposia in Pure Mathematics*, **24** (1973), 181–193. https://doi.org/10.1090/pspum/024/9944.

[4]. Odlyzko, A.M., On the Distribution of Spacing Between Zeros of the Zeta Function, *Mathematics of Computation*, **48** (1987), no. 177, 273–308. https://doi.org/10.1090/s0025-5718-1987-0866115-0

[5]. Elmesbahi, J., Synthesis of Large Diversity of Forms by Non-Recursive Equations, *Applied Mathematical Sciences*, **15** (2021), no.16, 797-811. https://doi.org/10.12988/ams.2021.916578

[6]. Elmesbahi, J., Attractors in Non-Recursive Systems, *Applied Mathematical Sciences*, **18** (2024), no. 6, 253-268. https://doi.org/10.12988/ams.2024.919145.

[7] Elmesbahi, J., Modelling a wide range of Signals of Different Distributions by Non-Iterative Functions, *Applied Mathematical Sciences*, **14** (2020), no. 20, 935-951. https://doi.org/10.12988/ams.2020.914341

[8]. Elmesbahi, J., Faithful Reproduction of the Statistical Properties of the Robert May (r=4) Logistic Equation via a Simple Non-Recursive Formula, *Applied Mathematical Sciences*, **18** (2024), no. 5, 223-226. https://doi.org/10.12988/ams.2024.919125

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