

A Draft for Rapid Modeling Using a Non-Iterative Function of the GUE, Riemann, GOE Spectral Distributions and the Wigner Semicircle

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Abstract

In this short paper, we present a draft approach for modeling spectral distributions using a non-iterative function. With a simple change of constants, this function can reproduce multiple distributions with a precision of up to 10^{-17} in the computation of normalized standard deviation, when using 10^7 samples and a computation time under one second, without any optimization or parallelization. The computation was performed on an HP Z workstation. This function successfully models the GUE, GOE distributions, the spacing distribution of the non-trivial zeros of the Riemann zeta function, and the Wigner semicircle. All cases were validated using the Kolmogorov-Smirnov test.

Unlike traditional methods based on random matrices or integrable equations, this approach starts from a highly precise empirical result, without reference to any theoretical model, suggesting the existence of an as-yet unknown underlying structure. This function belongs to a family of non-iterative functions that have already been used for other achievements such as: Synthesizing shapes from the simplest (circle, square, etc.) to much more complex structures. Modeling a long series of known and unknown distributions. Modeling dynamic phenomena (bifurcations, hysteresis, R. May's logistic equation for $r=4$). The point cloud modeling Shaw's dripping faucet experiment. Heavy-tailed distributions and other types.

Introduction

Complex iterative methods have traditionally been used to model the spacing distribution of the zeros of the Riemann zeta function, notably in the work of E. Wigner [1], who introduced an analogy between the zero spacings of the zeta function and energy levels of a complex quantum system. F. Dyson [2] expanded on Wigner's ideas by analyzing random matrices and their link to the zeta zeros. A. Odlyzko [4] conducted computational studies of the zeta zeros and provided numerical evidence. H. Montgomery [3] formalized connections between the statistical distribution of the zeta zeros and the eigenvalues of random matrices.

Other approaches include two-point correlation functions, combinatorial and probabilistic methods. However, despite the importance of these distributions, combined modeling remains difficult due to the computational complexity involved.

Objective

In this paper, we propose a non-iterative function capable of reproducing the spectral distributions of GOE, GUE, and the spacing distribution of the non-trivial zeros of the Riemann zeta function through a simple change of constants. To model the Wigner semicircle, we also draw inspiration from the same function. All distributions are validated via the Kolmogorov-Smirnov test. This approach allows not only fast and flexible reproduction—computation time is less than one second without any optimization or parallelization, with 10^7 samples—but also perfect agreement with the normalized standard deviations reported in the literature. For the GUE case, we reach a precision of 10^{-17} . This function is not unique; other non-recursive functions from the same family can produce similar results. In the Wigner semicircle example, we present two functions that give approximately the same outcomes. In addition to modeling spectral distributions, it is worth noting that all achievements described here stem from the same family of non-iterative functions, without the use of differential systems or traditional iterations. Below, we list conceptually significant applications enabled by this family:

Massive synthesis of thousands of shapes, from the simplest (disks, squares, etc.) to the most complex (biological, geometric, or abstract structures) [5]. In many cases, these shapes can be transformed into a straight line segment by simply changing a constant from an integer to a real number (e.g., 3 to 3.5), or from a disk to a square, or from an empty square to one with a point cloud—just by changing a constant by 10^{-14} [6], [7], [5].

A long series of known and unknown distributions produced by a single family of non-iterative functions. The focus was on the shape of the distributions rather than their statistical properties. The number of these distributions is not limited to those presented; more can be constructed by modifying constants, the number of terms, and

making combinations—independently of traditional statistical methods [8].

Reproducing dynamic systems phenomena, such as R. May's logistic equation for $r=4$ [9], Shaw's dripping faucet point cloud [10], various types of bifurcations (simple, double, triple), simple and multiple hysteresis loops with or without intersections, and sigmoids. For instance, a double bifurcation can transform into a triple one and then into a sigmoid; a simple hysteresis can become multiple with or without intersections, then evolve into a sigmoid—all driven by changes in constants.

Heavy-tailed distributions.

Modeling distributions derived from fundamental differential equations, such as velocity profiles in the Navier-Stokes equations (with specific statistical constraints), or the distribution profiles from the Black-Scholes model. The goal is not to solve differential equations or find universal analytical solutions, but to reproduce a statistical distribution linked to a given real-world scenario, characterized by a mean value and standard deviation [11].

To view the results, only three clicks are required:

Choose a series of constants (n_1 , n_2 , or n_3) along with its reference distribution.

Paste it into a Matlab environment.

Execute

REPRODUCING THE DISTRIBUTIONS

```
%% GOE      ETN= 0.53864900000000016
```

```
d2=0;d1=1;
```

```
a1=1.3952445652;a2=5.5;a3=0.08;b1=2;b2=5.5;b3=0.1563771; % (n3)
```

```
c1=1 ;c2=8.607153897300483968998;
```

```
%% RIEMANN  ETN=0.42296000000000011
```

```
d2=0;d1=1;
```

```
a1=1.08154;a2=5.8;a3=0.08;b1=2;b2=5.8;b3=0.08281781; % (n2)
```

```
c1=1;c2=8.64411291230259504223;
```

```
% GUE      ETN=0.42298754596461996
```

```
d2=0;d1=1;
```

```
a1=1.08154000000000005698;a2=5.8;a3=0.08;b1=2;b2=5.8;b3=0.08281781; %
```

```
(n1)
```

```
c1=1;c2=8.644716473516317;
```

```
% FONCTION GENERATRICE
```

```
x=1:10^-4:10^3;
```

```
y1=a1*acot(cot((x).^45)).^0.38+a2*acot(cot((x).^33)).^a3 ;
```

```
y2=b1*acot(cot((x).^3)).^0.2+b2*acot(cot((x).^47.3)).^b3;
```

```
y=c1*d1*abs(y1+y2).^(c2)-d2 ;
```

```

tic;figure(1);mean_y=mean(y);std_y=std(y);y_normalized=y/mean_y;
mean_normalized=mean(y_normalized);std_normalized=std(y_normalized);
normalized_std=std_normalized/mean_normalized;fprintf('Mean of original y:
%.6f\n',mean_y);
fprintf('Mean of normalized y: %.12f\n',mean_normalized);
fprintf('Standard deviation of normalized y: %.17\n',std_normalized);
fprintf('Normalized standard deviation (ETN): %.17f\n',normalized_std);
histogram(y_normalized,300,'Normalization','pdf');
title('Dyson Threefold Way-Normalized Histogram of y');xlabel('Normalized
Spacing');
ylabel('Probability Density');grid on;elapsed_time=toc;
fprintf('The computation time is%.5f seconds.\n', elapsed_time);
text(0.32,0.90,sprintf('Normalized
Std: %.18f',normalized_std),'Units','normalized','FontSize',12,'Color','r');
text(0.50,0.75,sprintf(' Time:
%.4fs',elapsed_time),'Units','normalized','FontSize',12,'Color','r');

```

```

%% KOLMOGOROV-SMIRNOV TEST

```

```

x=0:0.01:4;
y3=@(x)(pi/2*x.*exp(-(pi*x.^2)/4)); % GOE (n3)

x=0:0.01:3;
y3= @(x)(32/pi^2)*x.^2.*exp(-((4/pi)*x.^2)); % GUE, RIEMANN (n1, n2)

```

```

y= y_normalized;data2 =y;nbins = 300;
[counts2,edges2]=histcounts(data2,nbins,'Normalization','pdf');
bin_centers2=(edges2(1:end-1)+edges2(2:end))/2;
theoretical_curve=y3(bin_centers2);
[h,p_value,ks2stat]=kstest2(counts2,theoretical_curve);
figure;histogram(y,'Normalization','pdf');hold on;
plot(bin_centers2,theoretical_curve,'r-','LineWidth',2);
legend('H2','theoretical curve (y3)');xlabel('Normalized Spacing');
title('Comparaison of the histogram H2 with the theoriticalque curve');
ylabel('Probability Density');grid on;elapsed_time=toc;
hold off;figure;y7=counts2-theoretical_curve;
plot(edges2(1:300),y7);
mean_err=mean(abs(counts2-theoretical_curve))
MSE=mean((counts2-theoretical_curve).^2)
RMSE=sqrt(mean((counts2-theoretical_curve).^2))
p_value
h

```

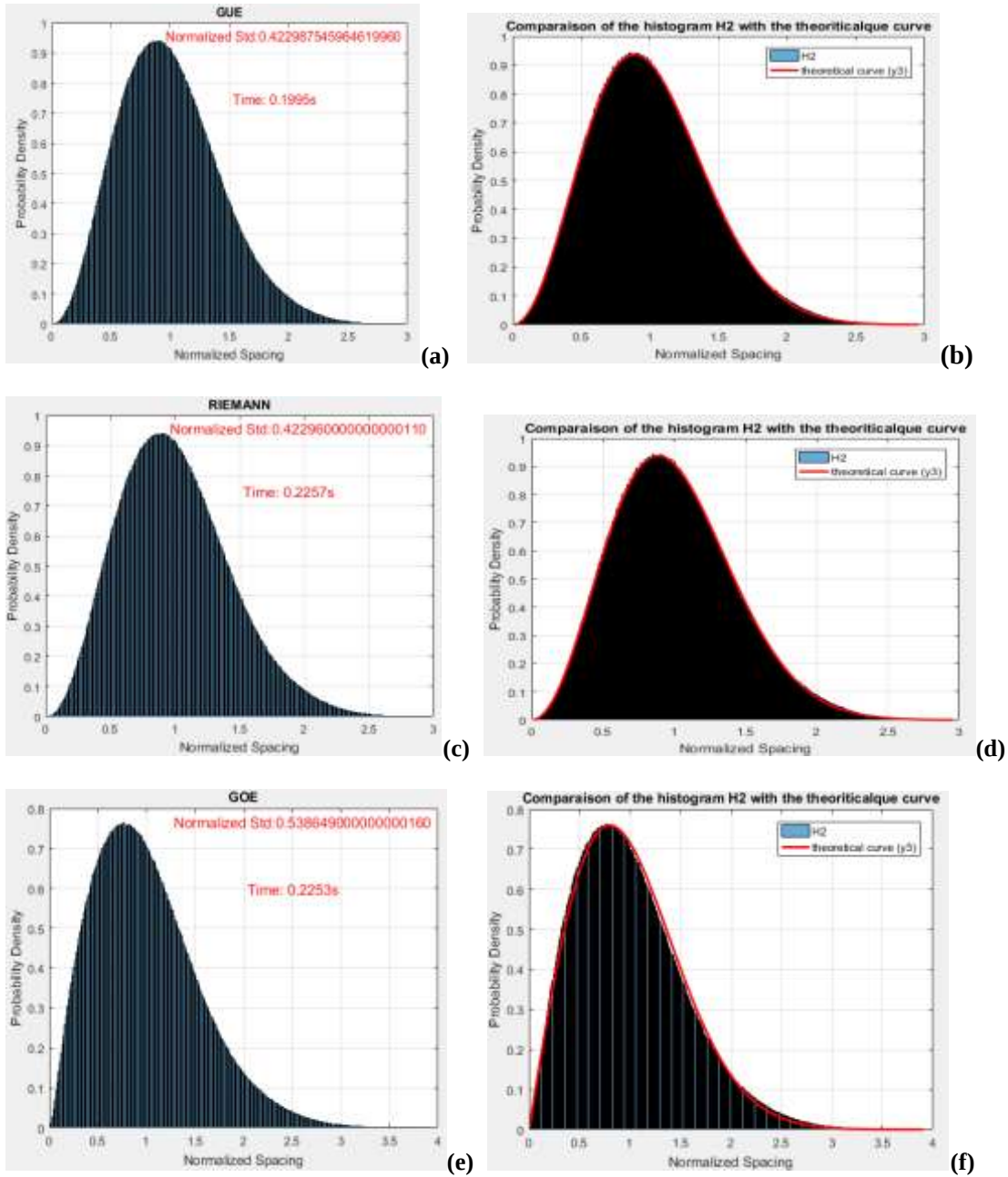


Fig.1 Comparative graphical representations of spectral distributions.(a,b) GUE distribution-Simulated histogram (black) superimposed (red), showing a normalized standard deviation = 0.42298754596461996 and computation time of 0.1995s. (c,d): Distribution of the spacing between non trivial zeros of the Riemann zeta function-Simulated histogram with Std=0.42296000000000110, time 0.2257s. (e,f):GOE distribution, Std=0.53589800000000159 and coputation time of 0.2255s.

```
%% WIGNER DEMICIRCLE (1) Std= 0.9891; Mean=2.1*10^-16
```

```
x=1:10^-4:10^3;
nbins = 300;
p=0.98888589; k =2.44123198721; q =0.545; T=2.000000000000294;
y1=acot(p*cot(x.^37));
y2=acot(p*cot(x.^35));
y=-T+k*abs(y1.^2-y2.^2).^q;
```

```
%% WIGNER DEMICIRCLE (2) Std= 0.9998; Mean= 1.8*10^-16
```

```
x=1:10^-4:10^3;
nbins = 300;
C1=5.50 ; C2= 7.795601; D= 1.070659; T= 2.0488261313523485;
y1=C1*asin(sin(x.^11)).*cos(x.^33);
y2=C2*asin(sin(x.^11)).*sin(x.^33);
y=-T+abs(y1+D*y2).^0.51;
```

```
tic;figure(1);histogram(y,300,'Normalization','pdf');mean_y=mean(y);std_y=std(
y);
fprintf('Mean: %.17f\n',mean_y);fprintf('standard deviation= %.3f\n',std_y);
text(0.01,0.95,sprintf('Mean: %.17f',mean_y),'Units','normalized','FontSize',12,'
Color','r');
title('SEMICIRCLE OF WIGNER');xlabel('X');
ylabel('Probability Density');grid on;elapsed_time=toc;
fprintf('time is %.6f seconds.\n', elapsed_time);
text(0.01,0.85,sprintf('Time: % .5f
s',elapsed_time),'Units','normalized','FontSize',12,'Color','r');
text(0.035,0.90,sprintf('Std: %
.5f',std_y),'Units','normalized','FontSize',12,'Color','r');
```

```
%% KOLMOGOROV-SMIRNOV TEST
```

```
data1=y;[f,xi]=ksdensity(data1);cdf_wigner=cumtrapz(xi,f);
xy1=cdf_wigner;figure;histogram(data1,nbins,'Normalization','pdf');
hold on;grid on;R=2;N=10^7;nbins=300;x=linspace(-R,R,N);
pdf_wigner=(2/(pi*R^2))*sqrt(R^2-x.^2);
cdf_wigner=cumtrapz(x,pdf_wigner);xy2=cdf_wigner;u=rand(N, 1);
data2=interp1(cdf_wigner,x,u,'linear');
plot(x,pdf_wigner,'r-','LineWidth',2);xlabel('X');ylabel('Density');
title('SEMICIRCLE OF WIGNER');
grid on;[h,p]=kstest2(xy1,xy2)
```

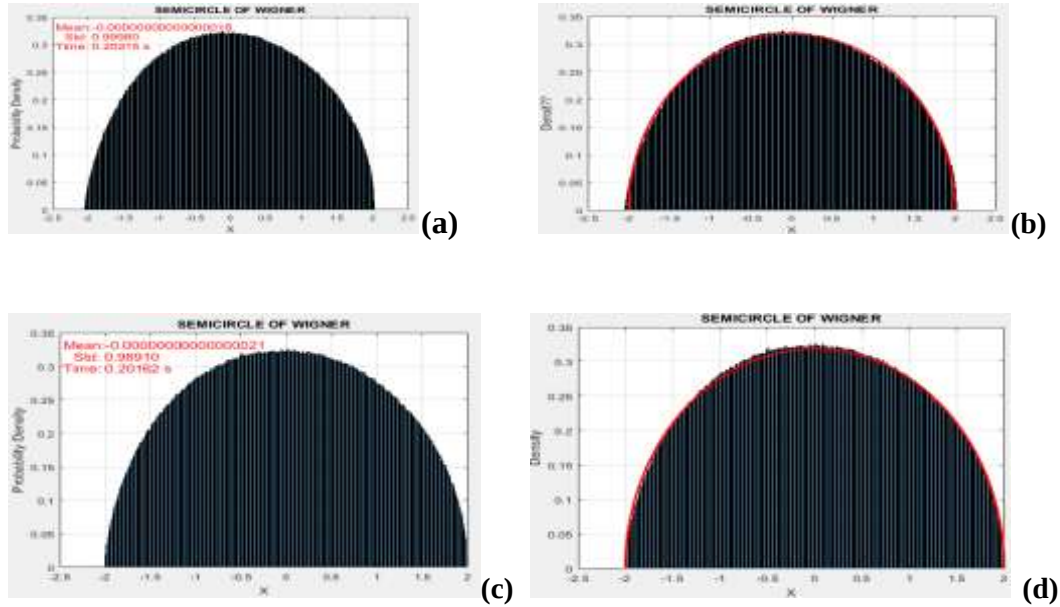


Fig.2. Modeling of the Wigner semicircle distribution—two variants.

(a,b) First variant—Simulated histogram (black) with mean= 1.8×10^{-17} and Std=0.9998 and computation time of 0.2016s, theoretical Wigner (red).

(c,d) Second variant— Simulated histogram (black) with mean= 2.1×10^{-17} and Std=0.98919 and computation time of 0.2016s, theoretical Wigner (red).

Conclusion

This preliminary study highlights progress in modeling spectral distributions using a non-iterative function. With simple changes in constants, this function accurately reproduces the normalized standard deviations of fundamental distributions such as GUE, GOE, the spacing distribution of the Riemann zeta zeros, and the Wigner semicircle—all validated by the Kolmogorov-Smirnov test. Belonging to a family of non-iterative functions previously used to model complex dynamic phenomena (bifurcations, hysteresis, deterministic chaos), this function opens up a new field of exploration: that of a unified, underlying mathematical structure capable of linking statistical distributions, nonlinear dynamics, and morphogenesis. One promising extension to other spectral distributions involves the use of artificial intelligence to select constants. This entire method is based on a family of non-iterative functions—not on random matrices or integrable equations. Its origin lies in highly

precise empirical results, unrelated to any theoretical model, suggesting an as-yet unknown underlying structure.

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