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Unified Modeling of Dynamics, Geometry and Distributions via a Non-Iterative Function Family

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Abstract

We introduce a unified modeling approach based on a single family of non-iterative functions capable of reproducing a wide spectrum of complex behaviors with exceptional precision and speed. Unlike traditional iterative or differential methods, this framework uses closed-form expressions whose qualitative behavior is governed solely by a small number of tunable constants.
Using this family, we successfully model :

- the logistic map of R. May for the fully chaotic regime [5],
- the droplet pattern of a dripping faucet as originally captured by R. Shaw, through point cloud simulations[6],
- a long series of statistical distributions[1], ranging from simple to heavy-tailed forms with controlled parameters ,
- the spectral distributions associated with random matrix theory, including GUE, GOE, the Wigner semicircle, and the spacing distribution of non-trivial zeros of the Riemann zeta function,[9]
- and the synthesis of thousands of geometric and morphogenetic shapes, from elementary forms (disks, squares, rings) to highly complex biological-like or abstract structures[7],[8].

All results are obtained without iteration, without numerical approximation, and in computation times under one second for up to samples. This reveals an underlying functional structure capable of bridging chaotic systems, statistical models, spectral theory, and morphogenesis—all within a unified analytical framework.

Introduction

This work presents a unified and remarkably simple approach to modeling shapes, dynamic transitions, and complex distributions, based on a family of non-iterative functions. In contrast to traditional methods that rely on differential equations, stochastic simulations, or iterative schemes, our method is built exclusively on closed-form analytical expressions, in which the variation of one or two constants is sufficient to generate a wide range of behaviors. This strategy enables rapid and controlled exploration of a broad spectrum of physical, statistical, and morphogenetic phenomena. We present here six groups of curves, all generated by the same family of functions, with extremely short computation times (often under one second for several million points), demonstrating the expressive power of this functional approach.

◈ Group I: Dynamic Transitions – This group includes sigmoids, bifurcations, and hysteresis patterns, all modeled using a single non-iterative function. Each subfigure is generated by simply adjusting the constants of this equation. No iteration is involved, yet the full spectrum of classical dynamic system behaviors is reproduced.

◈ Group II: Geometric Deformations – A single function generates all the figures, from squares to diamonds, including smooth polygonal transformations. By varying the constants, one can smoothly transition from one shape to another, revealing a continuous geometric evolution.

◈ Group III: Morphogenetic Transitions – Figures (a) to (f) are obtained from one function, while curve (j) is generated using another function from the same family. Each pattern emerges from slight variations of constants, illustrating transitions from ordered forms to apparently chaotic structures.

◈ Group IV: Toroidal Networks and Complex Spatial Structures – Again, a single function is sufficient to reproduce various entangled network types, from dense point clouds to toroidal or diamond-like structures. The spatial arrangement is fully controlled by the constants, without relying on graph generation algorithms or optimization routines.

◈ Group V: Heavy-Tailed Distributions – These rare-event distributions, characterized by long tails and strong asymmetry, were all modeled using a single non-iterative function, by modifying constants to control the mean-to-standard deviation ratio, set to 0.1254, 0.01, and 0.001 respectively. The displayed histograms are zoomed-in views, illustrating precise control over dispersion and tail behavior.

◈ Group VI: Statistical Distributions – We successfully reproduced the characteristic shapes of distributions derived from the Navier-Stokes and Black-Scholes equations.

While the curve shapes may not perfectly overlay the theoretical references, the key statistical values (mean, standard deviation, skewness) are accurately replicated, confirming the quantitative relevance of our method in applied contexts.
At each step, you verify these results by proceeding as follows :
1-Copy the equation with its respectve constants,
2-Past it into a Matlab code environment,
3-Execute it.
To fully appreciate the expressive breadth of this functional family, the reader is invited to explore a dedicated online gallery containing thousands of generated patterns :
https://jelloul-elmesbahi.academy/wp-content/uploads/2025/04/Classeur1.pdf

Realizations Generated by a Single Non-Iterative Functional Framework

I. Dynamic transitions and sigmoid bifurcations

```
x=1:1:10^6 ;
        %sigmoid (d)
m=18; n=5;p=10^-5; d=0; e=1;
A1=m;A2=1.95;A3=0.3;A4=0.3;A5=0.3;A6=0.3;A7=e;A8=n;A9=-
16.25;A10=p;A11=-1;
         %degenerate fork (c)
A1=-1.65;A2=2;A3=0.1;A4=0.1;A5=0.1;A6=0.1;A7=0.1;
A8=8;A9=-3.25;A10=1;A11=1;
        %triple and simple fork (a, b)
m=18; n=2; p=1; d=0; e=0.1;
A1=m;A2=1.95;A3=0.3;A4=0.3;A5=0.3;A6=0.3;A7=e;A8=n;A9=-16.25;A10=p;
% (a,b,d)
A11=1;       %% simple fork
A11=-1;      %% triple fork

y2=A1*cos(asin(A2*sin(x.^A3))).*(cos(x.^A4));
y1=A11*cos(acos(2*sin(x.^A5))).*(sin(x.^A6));
y1=A7*y2.^A8+A9*y1 ;
plot(A10*y1,y2,'.')
```

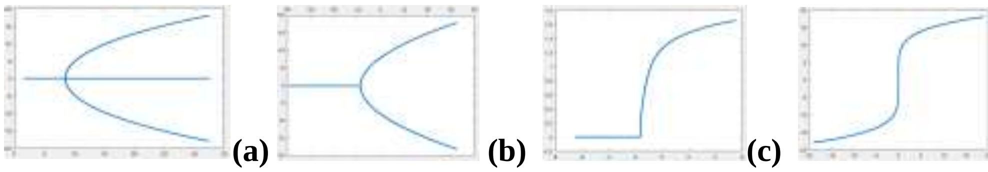


Fig.1. (a) Graph of the triple fork transformation into: (b) a simple fork, (c) degenerate fork, (d) a sigmoid.

3. Simple hysteresis, multiple hysteresis without intersections and with intersections, sigmoid.

```
x=1:1:10^6;
A=0.2885;     B=3;  c=1;       % multiple hysteresis with intersections
A=0.13;       B=3;  c=1;       % multiple hysteresis without intersections
A=0.3;        B=3;  c=1;       % Simple hysteresis
A=0.3;        B=5;  c=10^-3;   % sigmoid

y2=20*cos(asin(0.34595*sin(x.^0.3))).*cos(x.^0.3);
y1=-2.5*(acos(1+0.851*sin(x.^0.3))).*sin(x.^A);
y1=2*10^-3*(y2.^B-655*y1);
plot(y1*c,y2,'.');
```

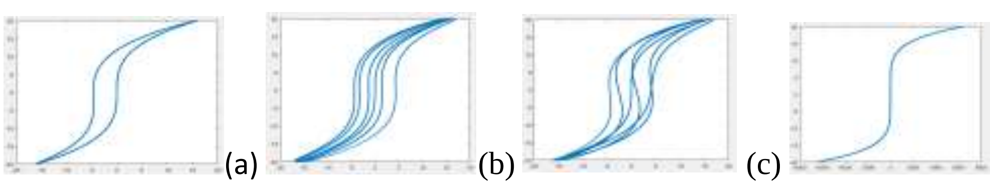


Fig.2. Graph of the hysteresis transformation into: (a) simple hysteresis loop, (b) multiple non- intersecting loops, (c) multiple intersecting loops, (e) sigmoid curve.

II. Polygonal Deformations and Geometric Transitions

```
x=1:1:10^6;                 %6th rounded curves
C1=01.75;C2=01.75;
A1=0;B1=0.85;A2=0;B2=0.85;

x=1:1:10^6;                 % Started figures
C1=1.3;C2=1.3;
```

```
A1=0;     B1=1.4;  A2=0;     B2=1.4;   %5th case
A1=0.01;  B1=1.4;  A2=0.01;  B2=1.4;   %4th case
A1=0.01;  B1=1.4;  A2=0.1;   B2=1.34;  %3rd case
A1=0.015; B1=1.42; A2=0.015; B2=1.42;  %2nd case
A1=0;     B1=1.42; A2=0;     B2=1.42;  %1st case

y1=asin(A1+B1*sin(x.^C1));
y2=acos(A2+B2*cos(x.^C2));

plot(y1.*y2,4*y2,'.');    % geometric  figure 4
plot(y1-y2,y2+y1,'.');    % geometric  figure 3
plot(y1-y2,y2,'.');       % geometric  figure 2
plot(y1,y2,'.');          % geometric  figure 1
```

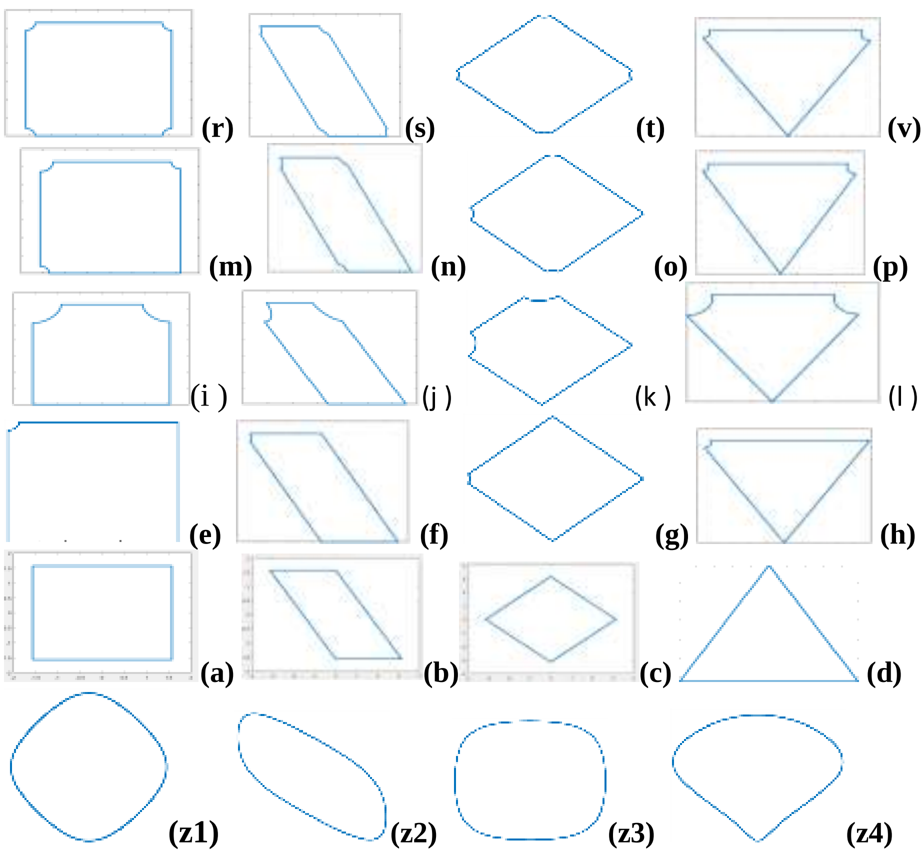


Fig.3. (a-o) Progressive geometric transformations within a unified functional framework.

III. Structured Filling and Internal Wave Patterns

```
k=1; A1=1.3; A2=1.31; x=1:10^-4:21; % network (f)
 % waves (a,b,c,d,e)
    k=1.51;
A2=1.359;A1=1.35; % wave(e)
A2=1.357;A1=1.35; % wave(d)
A2=1.355;A1=1.35; % wave(c)
A2=1.353;A1=1.35; % wave(b)
A2=1.350;A1=1.35; % wave(a)
y1=asin(k*sin(x.^A1)); y2=asin(k*cos(x.^A2)); plot(y1,y2,'.')
```

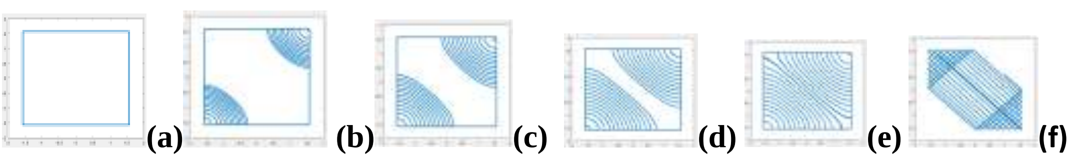


Fig.4. Progressive evolutionof a curved pattern within a square,generated by the same non-iterative function (a, b,c, d, e, f).

Chaotic arrangement of curves

```
x=1:10^-4:1*10^2;
 y1=(asin(1.51*sin(x.^1.135)))-(acos(1.51*cos(x.^1.35)));
 y2=(asin(1.51*sin((x).^1.355)));
 plot (y1,y2);
```

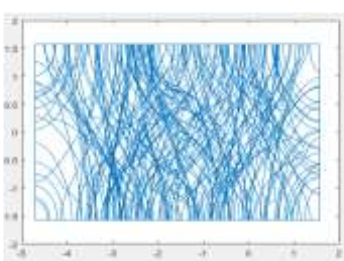


Fig.5. Autonomous chaotic field enclosed within self-generated polygonal limits.

IV. Extreme regimes

These two networks illistrate two extreme regimes :one adopts an ordered toroidal tructure (a),while the is a fluid and chaotic entanglement(b).
This section party relies on the analytical expression of the staircase function y_1 :

$$y_1 = A \cdot \left(x - \frac{B}{\pi} \cdot \operatorname{acot}\left(\cot\left(\frac{x \cdot \pi}{B} \right) \right) \right)$$

```
x=0:0.001:100;k=1; s=0.3;g=15;u=5; f=1; a=0.0132+f*10^(-8);
b=150+f*10^(-8);c=25+f*10^(-8); q=5;
X1=16*sin(a*(x-b/pi*acot(cot(pi*x./b))).^q);
X2=0.52*cos(x.^2)+8*sin(a*(x-c/pi*acot(cot(pi*x./c))).^q);
X3=2*asin(2*sin(0.1*x.^1.25)); Y1=5*cos(a*(x-b/pi*acot(cot(pi*x./b))).^q);
Y2=0.52*sin(x.^2)+8*cos(a*(x-c/pi*acot(cot(pi*x./c))).^q);
Y3=0.75*acos(s*cos(k*x.^1.25));
X=X1.*cos(0.1*x)+X2.*sin(0.3*x)+X3.*sin(u*x);
Y=Y1.*cos(0.05*pi*x)+Y2.*cos(3*pi*x)+Y3.*cos(g*x); %% toroidal tructure
 plot(X,Y);

x=0:0.1:30000; f=998;v=0.12; w=0.12;
b=150+f*10^(-8); c=25+f*10^(-8); q=2.5;a=0.0132+f*10^(-8);
X1=3*sin(a*(x-b/pi*acot(cot(pi*x./b))).^q);
X2=v*cos(x.^32)-4*sin(a*(x-c/pi*acot(cot(pi*x./c))).^q);
Y1=3*cos(a*(x-b/pi*acot(cot(pi*x./b))).^q);
Y2=w*sin(x.^32)+4*cos(a*(x-c/pi*acot(cot(pi*x./c))).^q);
X=X1.*X2+X1.*Y2+Y1;Y=Y1.*Y2-Y1.*X2;     % double rhombus
X=X1+0.4*X2; Y=Y1-0.4*Y2;                % ring
plot (X,Y);
```

These two networks illistrate two extreme regimes :one adopts an ordered toroidal tructure (a),while the is a fluid and chaotic entanglement (b).

(a) **(b)** **(c)**

Fig.6. (a) , (b) and (c) illustrate two complex structures generated by sam non-iterative function.

V. Rares event distributions

```
x=1:1:10^6;
A=0.001; B=-3.9;    C=-3.8;D=0.000981;    % Mv/Std=0.001    (c)
A=0.001; B=-3.9;    C=-3.8;D=0.87;        % Mv/Std=0.01     (b)


A=0.1;   B=-2;      C=-1.5;D=0.1;         % Mv/Std=0.1254   (a)

y= (A+(acos(sin(x.^3.17)))).^B.*(D+(acos(cos(x.^3.13)))).^C;

figure(1);histogram(y,200,'Normalization','pdf');mean_y=mean(y);std_y=std(y);
fprintf('Mean:%.2f\n',mean_y);fprintf('standard deviation=%.3f\n',std_y);
text(0.05,0.9,sprintf('Mean:
%.2f',mean_y),'Units','normalized','FontSize',12,'Color','r');
text(0.05,0.83,sprintf('Std:
%.2f',std_y),'Units','normalized','FontSize',12,'Color','r');
title(' Rare Event DISTRIBUTION');xlabel('Observed Value');
ylabel('Probability Density');grid on;
```

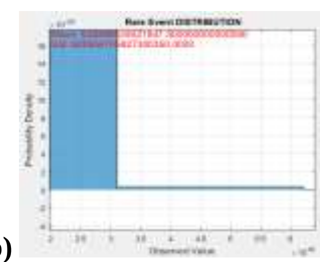
 **(a)**  **(b)**  **(c)**

Fig.7. Heavy-tailed distributions generated by a single non-iterative function belonging to the same family as those used in the previous figures. Each subfigure (a), (b), and (c) corresponds to a case where the mean-to-standard deviation ratio is precisely controlled and set to 0.1254, 0.01, and 0.001, respectively. These distributions illustrate rare events with a concentration near low values and a significant long tail. The histograms shown are zoomed-in views of the high-density regions.

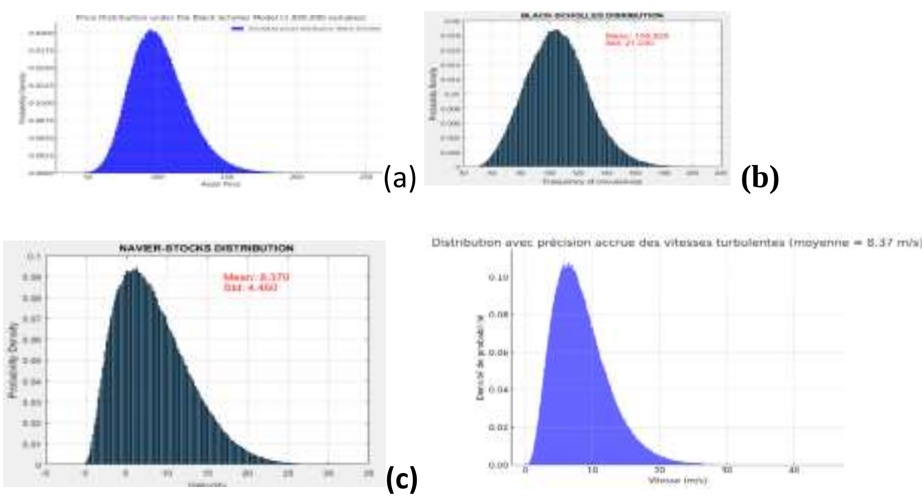
VI. Black-Scholles distribution, Navier-Stocks distribution

```
x=1:1:10^6;
y=1.2792+(36.734*(5.96-
abs(2*(acot(cot(x.^35)).^.117785+abs(.199999*acot(cot(x.^3.3))).^1.029-
0.061*acot(cot(x.^23.327)))).^1.452));
```

```
x=1:1:10^6;
y=1.21*(2.76-2*(acos(cos(x.^13)).*acos(cos(x.^7))).^0.136266)-
0.43*(acos(cos(x.^3)).*acos(cos(x.^7.3))).^0.134; y=10*(y+0.5122);


figure(1);histogram(y,200,'Normalization','pdf');mean_y=mean(y);std_y=std(y);
fprintf('Mean:%.3f\n',mean_y);fprintf('standard deviation=%.3f\n',std_y);
text(0.55,0.9,sprintf('Mean:
%.3f',mean_y),'Units','normalized','FontSize',12,'Color','r');
text(0.55,0.85,sprintf('Std:
%.3f',std_y),'Units','normalized','FontSize',12,'Color','r');
title('BLACK-SCHOLLES DISRIBUTION');xlabel('Frequency of occurences');
ylabel('Probability Density');grid on;
```

MV target=105.82 Std target=21.23; MV calculated=8.37 Std calculated=4.46



(a) (b) (c) (d)

Fig. 8. (a) Distribution calculated using the classical Black-Scholes method. (b) Distribution produced by the non-iterative function y.
(c) Distribution produced by the non-iterative function y. (d)Velocity distribution obtained from the Navier-Stokes equation (mean velocity of 8.37) calculated classically.

A preliminary framework has been developed to model these two distributions. Further work will focus on enhancing the structural accuracy of these distributions.
Observation : It is possible to identify other non-iterative function within this same family that can simultaneously reproduce all these curves

Conclusion

This series of figures demonstrates the expressive power of a single family of non-iterative functions. by simply modifying a few constants within closed-form analytical expressions, we are able to reproduce — with precision, richness, and diversity — classical dynamic transitions (sigmoids, bifurcations, hysteresis), geometric deformations, morphogenetic networks, complex spatial structures, as well as realistic statistical distributions (Navier-Stokes, Black-Scholes) and heavy-tailed behaviors. All of this is achieved without iterative schemes, without numerical approximations, and with computation times under one second. This approach opens a new pathway toward a unified, fast, and controlled modeling of natural and mathematical complexity.

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