Faithful Reproduction of the Statistical Properties of the Robert May (r=4) Logistic Equation via a Simple Non-Recursive Formula

Jelloul Elmesbahi

Abstract

In this brief article, we present a simple non-recursive equation sensitive to initial conditions, which faithfully reproduces the histogram, mean value, and standard deviation generated by Robert May's logistic equation for the growth parameter r=4.

Keywords: logistic equation, dynamical system, chaos, non-recursive modelling.

Introduction

Robert May's [4] equation is a cornerstone in the study of chaotic dynamic systems used to model phenomena ranging from the evolution of a biological population to the dynamics of epidemics. While the recursive forms of this equation are powerful, they present specific analytical challenges due to its chaotic nature. Proposed formula

\[ y = 0.5(\sin(x^{49.6}) + 1); \]

This equation was inspired by a family of equations presented in [3].

Results

The mean value, standard deviation, and histogram are accurately produced by the proposed equation despite its great simplicity. The following table shows the results obtained with the two methods, namely the mean value and standard deviation. Histograms of both methods are depicted in Figure 1. They were conducted with \(10^6\) samples.
These results are obtained in the case where the exponent is 49.6.

<table>
<thead>
<tr>
<th></th>
<th>Mean Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logistic Equation</td>
<td>0.50001</td>
<td>0.35355</td>
</tr>
<tr>
<td>Non-Recursive Equation</td>
<td>0.50008</td>
<td>0.3535</td>
</tr>
</tbody>
</table>

**Figure 1**. Histogram from by the non-recursive equation (a), logistic equation (b).

In Figure 2, for a small number of samples, two graphs of the function Y are presented for two exponent values, 49.6 and 49.6001, illustrating the sensitivity to initial conditions. The small number of samples allows for a better visualization of the two graphs of the function Y with exponent differing by $10^{-4}$. 
The exponent was varied over a wide range from 49.6 to 3.6. We find exactly the same histogram, the same mean value, while the standard deviation differs by $10^{-4}$.

![Figure 2. Overlay of two plots of the function Y, with one exponent at 49.6 and the other at 49.6001.](image)

We have not studied the minimum or maximum values of the exponent to determine if the non-recursive equation produces data compatible with those produced by the logistic equation in this case.

**Conclusion**

We have produced the same statistical values as those generated by Robert May's logistic equation, using a very simple non-recursive function. Dozens of tests were conducted by varying the exponent, and each time we found stable statistical values (mean value, standard deviation, and histogram). There is a strong dependency on initial conditions, demonstrated by the dynamics of the values produced by the non-recursive function, which change even with a variation of $10^{-4}$ in exponent (Fig. 2). The formula used is not unique. We chose the simplest one.

Matlab program for calculating: Non recursive equation case and Logistic equation case

```matlab
x=0:1:10^6;
y=0.5*(sin(x.^49.6)+1);
mean_y=mean(y);std_y=std(y);
fprintf('mean=%.5f
',mean_y);
fprintf('standard deviation=%.5f
',std_y);hist(y,100);
```
x0 = 0.1; r = 4; n_samples = 1e6;
samples = zeros(n_samples, 1);
samples(1) = x0;
for i = 2:n_samples
    samples(i) = r * samples(i-1) * (1 - samples(i-1));
end
mean_val = mean(samples);
std_dev = std(samples);
figure;
histogram(samples, 100);
title('Histogram of the logistic equation for r = 4');
xlabel('Values');
ylabel('Frequency');
grid on;
disp(['mean value: ', num2str(mean_val)]);
disp(['standard deviation: ', num2str(std_dev)]);

References


Received: May 5, 2024; Published: May 26, 2024