

Uncertain Demand and the Gathering of Information: A Game-Based Perspective

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Abstract

I study a dynamic problem with uncertainty about demand when there are two monopolists selling in independent markets with unknown demand. Each monopolist's objective is to find a production plan that maximizes the expected profit by balancing the incentive to increase revenues and the cost of information gathering. Meanwhile, it is possible to freely obtain information from the other firm. Information reduces uncertainty for everyone and, therefore, is a public good. Hence, firms have an incentive to free ride on others' information acquisition. This incentive leads to underinvestment in information acquisition on a two-period horizon and to delayed information acquisition over an infinite horizon.

Keywords: coordination game, backward induction, war of attrition

1 Introduction

This work represents the second part of an investigation into information gathering by assuming a strategic environment. This analysis completes the dynamic problem under monopoly proposed in [1]. The topic of information learning is of great interest in the economic literature (see [2], [3], [4], [5]). In this framework, the study developed in [1] shows that the monopolist adopts an adaptive learning scheme that takes into account the benefit that the information gathered in one period produces for future profits. In this paper, I consider two independent markets. In each market, there is a monopolist. Demand is uncertain in both markets, but some information can be inferred from observing the monopolist in the other market because there is perfect correlation between the demand levels in the two markets, for example, due to the presence of common shocks. Information

reduces uncertainty for everyone and, therefore, is a public good. This implies that players have an incentive to free ride on others' information gathering, which would lead to underinvestment in information acquisition. As assumed in [1], the monopoly power of the firm derives from some external reason, such as a patent. Demand is unknown and described by a probability distribution function. Differently from [1], it is possible to update information through two different channels. First, by producing larger and larger quantities of goods and then attempting to sell them. Second, the demand functions of the two markets are correlated. This means that information updates can come from the other monopolist. Hence, there is a trade-off between the benefit of gathering information and the cost of information revelation, given its public good nature. This cost can be avoided if a monopolist can get an update from the other monopolist's action. The strategic interaction between the two monopolists gives rise to a coordination game with multiple equilibria in a two-period framework. The analysis also shows that the monopolists are involved in a war of attrition in the symmetric equilibrium of an infinite-horizon game. This result is linked to a large strand of literature regarding the war of attrition [6]. It is a framework widely used to study many forms of conflicts, including oligopolistic competition with the option to exit ([3], [5]), patent races [4], and public good provision [2]. In this paper, players slowly acquire information and essentially play a war of attrition regarding the information acquisition. This creates inefficiency in terms of a delay in information acquisition.

2 A two-period game of information acquisition

As a first step in the analysis, I assume a two-period model in which two monopolists, say A and B, face two different demand functions that are perfectly correlated with each other. This implies that the information disclosed is a public good. Since the information is costly to acquire, there is an incentive to free-ride. I use backward induction to look for equilibrium in the game. If monopolist A produces more than B in the second period, then A will compute the first-period production to maximize the intertemporal profit. Otherwise, firm A will maximize the period 1 profit. The same reasoning holds for monopolist B, by symmetry. I assume that each monopolist $i = A, B$ sells the quantity y_i if $y_i < x_i$, where the unknown demand level is denoted by x_i , at price $p_i = 1$, and the quantity $x_i - dx_i$ if $y_i \geq x_i$ at price $p_i = 1$. The production function is $y_i = k_i$. The probability distribution function of the demand is identical for the two markets. Then the random variable x_i is defined on the interval $[L, H]$, where $H > L > 0$. The distribution function is assumed to be uniform, e.g., $pr(x_i = z) = f(z) = 1/(H - L)$ and the cumulative probability that x_i is less than z is $pr(x_i \leq z) = F(z) = (z - L)/(H - L)$. Production requires an operating cost c per unit of good actually sold, e.g., a distribution cost, and a unit cost g that is incurred before demand is known. Therefore, the cost function is $C(y) = (g + c)y$ if $y < x_i$ and $C(y) = gy + cx_i$ if $y \geq x_i$. Production is a way of obtaining useful information to investigate demand. However, this can prove very costly if production is excessive.

In this sense, collecting information is costly. It is also possible to exploit the information revealed by the other monopolist under the assumption of perfectly correlated demand between the two markets so that the unknown level of demand is identical, e.g., $x_A = x_B = x$.

With the objective of finding the monopolists' payoffs and the Nash equilibrium of the game, I first identify the players' strategies in terms of period 1 and period 2 production levels (y_i^1, y_i^2) , $i = A, B$, as displayed in Proposition 1.

Proposition 1: Player A has the following strategies:

– If $y_A^1 > y_B^1$, monopolist A produces

$$y_A^1 = \frac{a^2 R}{1+R(1+a^2)} \left[\left(1 - \frac{1+a}{aR}\right) H + \frac{1+a}{aR} L \right] \quad (1)$$

in the first period and

$$\begin{cases} y_A^2 = (1-a)H + ay_A^1 & \text{if } y_A^1 < x \\ y_A^2 = x - dx & \text{if } y_A^1 \geq x \end{cases} \quad (2)$$

in the second period, where $a \equiv g/(1-c)$ and $(1-a)$ is a profitability index of the investment (net profit over operating margin); $R \equiv \frac{1}{1+r}$ is the discount factor of future profits.

– If $y_A^1 < y_B^1$, monopolist A produces

$$y_A^1 = (1-a)H + aL \quad (3)$$

in the first period and

$$\begin{cases} y_A^2 = (1-a)H + ay_B^1 & \text{if } y_B^1 < x \\ y_A^2 = x - dx & \text{if } y_B^1 \geq x \end{cases} \quad (4)$$

in the second period. Player B's strategies are symmetrically identical.

Proof: The strategies are identified by backward induction. Let's start with monopolist A's optimal production level at period 2, after observing the production levels of both firms at period 1. If the demand level is known at period 2, because one of the two monopolists has discovered the exact value of demand at period 1, each firm i produces $y_i^2 = x - dx$. If the demand level is still unknown at period 2, the probability distribution function is updated according to period 1 actions. Then, the probability of $x_i = z$ at period 2 is $f(z) = 1/[H - \max(y_A^1, y_B^1)]$ to calculate the optimal production level at period 2. In this case, the problem is studied in Proposition 1 in [1]. Therefore, I get $y_i^2 = (1-a)H + a \max(y_A^1, y_B^1)$. When analyzing period 1, I distinguish two cases: $x > y_A^1 > y_B^1$ and $y_A^1 < y_B^1 < x$. *Case 1:* If monopolist A produces more than B in period 1, $x > y_A^1 > y_B^1$, production in period 1, y_A^1 , is obtained by maximizing the intertemporal profit

$$E\Pi_A = E\Pi_A^1 + R E\Pi_A^2 \quad (5)$$

where

$$E\Pi_A^1 = (1-c) \int_L^{y_A^1} z f(z) dz + (1-c) y_A^1 \int_{y_A^1}^H f(z) dz - g y_A^1$$

or, given the uniform probability distribution,

$$E\Pi_A^1 = \frac{1-c}{H-L} \left\{ -\frac{1}{2}(y_A^1)^2 - \frac{1}{2}L^2 + [H - a(H-L)]y_A^1 \right\}$$

Period 2 expected profit is

$$E\Pi_A^2 = (1-c) \int_{y_A^1}^{y_A^2} zf(z)dz + (1-c)y_A^2 \int_{y_A^2}^H f(z)dz - gy_A^2$$

or, given the uniform probability distribution and period 2 production plan (2),

$$E\Pi_A^2 = S + \frac{1-c}{H-L} y_A^1 \left\{ a^2 L - \frac{1}{2}(1+a^2)y_A^1 \right\}$$

where S is a constant obtained as a combination of structural parameters. The first-order condition of (5) gives the optimal period 1 production reported in (1).

Case 2: If in period 1, $y_A^1 < y_B^1 < x$, monopolist A has no incentive to gather information since firm B is doing it. In period 2, firm A can freely take advantage of it and produces $x - dx$ if $y_B^1 \geq x$; otherwise, it is sufficient to take account of the updated distribution function $f(z) = 1/(H - y_B^1)$ to obtain $y_A^2 = (1-a)H + ay_B^1$, as shown in Proposition 1 in [1].

So far, we have established that of all possible levels of production, each monopolist can use two distinct profit-maximising strategies, given the choice of the other monopolist. If the quantity produced by firm A is greater than that produced by firm B, A maximises intertemporal profit and gathers information. If the quantity produced by firm A is smaller, A maximises uniperiodic profit and observes the decisions of the other firm to gain information. B firm's strategies are symmetric. It is easy to show that period 1 output is larger when gathering information, provided that the monopolist is sufficiently patient, i.e., $R > \frac{a^2}{(1+a^2)}$. The normal form representation of the game includes all possible strategies for A and B (G, NG), and their corresponding payoffs expressed in terms of intertemporal profits (Figure 1).

Corollary 1: Firm A and B payoffs satisfy $Z > T > W$.

Proof: Let indicate information gathering with subscript G and no information gathering with NG . The optimal production plan in period 1 (2) - when strategy G is adopted - is given by equation (1) (equation (2)). Otherwise, when strategy NG is adopted, the optimal production plan is given by (3) and (4) in periods 1 and 2, respectively. The payoffs corresponding to all possible strategy combinations for each monopolist $i = A, B$ are

$$\begin{aligned} Z &= E\Pi_i^1(s_{NG}) + RE\Pi_i^2(s_G) \\ T &= E\Pi_i^1(s_G) + RE\Pi_i^2(s_G) \\ W &= E\Pi_i^1(s_{NG}) + RE\Pi_i^2(s_{NG}) \end{aligned}$$

where

$$E\Pi_i^1(s_k) = \frac{1-c}{H-L} \left\{ -\frac{1}{2}(s_k)^2 - \frac{1}{2}L^2 + [(1-a)H + aL]s_k \right\}$$

$$E\Pi_i^2(s_k) = S + \frac{1-c}{H-L} s_k \left\{ a^2 L - \frac{1}{2} (1+a^2) s_k \right\}$$

with $k = G, NG$

$$s_G = \frac{a^2 R}{1+R(1+a^2)} \left[\left(1 - \frac{1+a}{aR} \right) H + \frac{1+a}{aR} L \right]$$

$$s_{NG} = (1-a)H + aL$$

and $s_G > s_{NG}$ if $R > \frac{a^2}{(1+a^2)}$. Remember that s_{NG} maximises period 1 profit, then $E\Pi_i^1(s_{NG}) > E\Pi_i^1(s_G)$ and $Z > T$. Second, s_G maximizes the intertemporal profit, therefore $T > W$. Finally, since $Z > T$ and $T > W$, the inequality $Z > W$ holds too.

Figure 1. Information game in normal form

		Monopolist B	
		NG	G
Monopolist A	NG	W, W	Z, T
	G	T, Z	T, T

Note: Z, T and W indicate intertemporal profits of monopolist A (left) and monopolist B (right) for all possible combinations of players' strategies. In Corollary 1, it is shown that $Z > T > W$.

Proposition 2: The equilibria of the information game in Figure 1 are: i) two pure-strategy Nash equilibria: A gathers information (G) and B does not (NG), B gathers information (G) and A does not (NG). ii) In the unique mixed strategy equilibrium, player i uses the strategy

$$q = \frac{T - W}{Z - W}$$

which is the probability to gather information at time 2, if the other player has not gathered information at time 1.

Proof: Given the normal-form representation of the game (Figure 1) and given that $Z > T > W$, it is easy to show that there are no dominant strategies for both players. A set of strategies $s^* = (s_A^*, s_B^*)$ is a Nash equilibrium if $E\Pi_i(s_i^*, s_{-i}^*) \geq E\Pi_i(s_i, s_{-i}^*)$ for all $s_i \in S_i$, $i = A, B$, and given that s_{-i}^* denotes the strategies of a player other than i . In addition, an optimal strategy can be specified as a probability q^* to gather information. In a mixed equilibrium, any player must be indifferent between G and NG strategies. This is obtained if it meets the following condition:

$$T = qZ + (1 - q)W$$

The left-hand side is the expected profit if a player chooses G, while the right-hand side expression is the expected profit if a player chooses NG. The other player's optimal strategy is probability $q = \frac{T-W}{Z-W}$. Since $Z > T > W$, the condition $0 < q < 1$ is satisfied.

3 An infinite horizon game of information gathering

I consider a framework with an infinite horizon in which players can gather information about uncertain demand at any time during the game. The strategic interaction of the monopolists can be described as a war of attrition. In the standard war of attrition, the game ends only when at least one of the players reveals the true value of demand. Formally, I indicate any player with i and the opponent with $-i$. Each player chooses the time to gather information, t_i , $i = A, B$. If player $-i$ gathers information first ($t_{-i} < t_i$), then player i receives a higher payoff than $-i$. I fully characterize the class of equilibria in which a player collects information with a positive probability. As in the standard war of attrition, at each time t , players must be indifferent between acquiring information (G) and waiting one more period (NG), as shown in the next proposition.

Proposition 3: In equilibrium, player i uses the strategy q^w , which is the probability to gather information at time $t + 1$, if the other player has not gathered information at time t .

Proof: An optimal strategy can be specified as a probability q^w to gather information. Any player i must be indifferent between gathering information at time t and waiting one more period. This is obtained if it meets the condition:

$$\begin{aligned} \text{EP}_t^i(s_G, y_{t-1}^i) + \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \text{EP}_{t+k}^i(s_G, y_{t+k-1}^i) \\ = q^w \left[\text{EP}_t^i(s_{NG}, y_{t-1}^i) + \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \text{EP}_{t+k+1}^i(s_G; y_{t+k-1}^i) \right] + \\ + (1 - q^w) \left[\text{EP}_t^i(s_{NG}, y_{t-1}^i) + \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \text{EP}_{t+k+1}^i(s_{NG}, y_{t+k-1}^i) \right] \end{aligned}$$

The left-hand side is the expected profit if player i chooses G from time t onwards, while the right-hand side is the expected profit if player i chooses to wait for one more period. In the latter case, the other player can either gather information from time t onwards with probability $(1 - q^w)$ or wait one more period with probability q^w . It is easy to obtain q^w as

$$q^w = 1 - \frac{\text{EP}_t^i(s_{NG}, y_{t-1}^i) - \text{EP}_t^i(s_G, y_{t-1}^i)}{\sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \text{EP}_{t+k+1}^i(s_G; y_{t+k-1}^i) - \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \text{EP}_{t+k+1}^i(s_{NG}, y_{t+k-1}^i)}$$

Remember that s_{NG} maximises the uniperiodic profit, then $\text{EP}_t^i(s_{NG}, y_{t-1}^i) > \text{EP}_t^i(s_G, y_{t-1}^i)$; s_G maximizes the multi-period profit, therefore the denominator is positive and greater than the numerator. Thus, the condition $0 < q^w < 1$ is satisfied.

4 Concluding remarks

In this paper, I investigated the optimal production plan of two monopolists who are faced with the trade-off between producing to maximize revenues and demand learning using game theory. When the demand curves are unknown but there is a perfect correlation between them, a coordination game describes the interaction between the two players in a two-period framework. Differently, a war of attrition depicts their decisions. Players have an incentive to free ride on the opponent's information acquisition and, thus, there is excessive delay.

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