Inventory Decisions with Scarcity and Spillover

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Abstract

We study how fast-fashion retailers should manage their inventories when customer demand is impacted by inventory scarcity. We characterize the optimal stocking levels for a single fast-fashion retailer under an economic order quantity (EOQ) model. We show that under this EOQ model, the fast fashion retailer benefits from deliberate understocking. We conclude that deliberate understocking may stimulate demand for a fast-fashion retailer by creating a sense of scarcity.

Keywords: Fast Fashion, Inventory, Economic Order Quantity

1 Introduction

Fast fashion offers many innovative lessons for retailers on how they can manage their supply chains to quickly respond to rapidly changing customer needs. It brings high clock-speed and results in rapid and regular new product introductions at affordable prices [3]. Fast-fashion retailers such as H&M and Zara can generate on average 120 new product introductions per week while a traditional retailer Gap introduces on average 40 new products per week [3]. While products have average shelf lives of six months in traditional retailers, fast fashion retailers replace products with new designs in just a few weeks [6]. They use their high clock-speed in bringing new products as a mechanism to attract customers into stores. Fast fashion also converts customers to buy quickly. Customers know that fast-fashion products are reasonably priced and are not replenished. Thus, they tend to buy them on the spot rather than postponing their purchases for the sake of any possible discounts. Fast-fashion customers no longer have the luxury of telling “I will wait
until it goes on sale.” They know that products are sold out very quickly. Fast-fashion retailers aim at creating a sense of scarcity and opportunity to customers in the form of “Buy now before it is sold out” [4]. As a result, on average 15% of products at fast-fashion retailers eventually experience discount prices compared to 40% industry average [6]. They emphasize the feeling of scarcity through deliberate undersupply and short shelf lives at stores [4]. This paper sheds some light on how fast-fashion retailers benefit from deliberate undersupply. We analyze the accumulated impact of inventory scarcity at a fast-fashion retailer on customer behavior under an economic order quantity (EOQ) model. Under this model, we characterize how much inventory for a particular product a fast-fashion retailer should stock to maximize its profits. Using this model we show that a fast-fashion retailer benefits from deliberate understocking through its demand energizing impact. The remainder of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes our EOQ model where we consider the accumulated impact of stock-outs. We conclude the paper by summarizing our main findings and pointing out interesting future extensions.

2 Literature Review

The marketing literature has recognized the impact of retail inventories on customer demand. Wolfe [8] provides empirical evidence from retail sales in apparel industry that display inventory is highly correlated to the sales of each style within a selling season. Koschat [5] presents empirical evidence from magazine retailing that customer demand can be affected by the on-shelf inventory. Urban [7] and Balakrishnan et al. [2] provide a detailed review on the inventory management models that reflect the demand stimulation effect of higher inventories. Many studies in the operations management literature solve extensions of the classical EOQ model assuming demand rate is increasing in on-hand inventory level. Baker and Urban [1] are the first to analyze an EOQ model with an inventory-dependent demand rate for managing the inventory of a single product with a deterministic demand in continuous time. They propose a demand rate function that is polynomial and increasing in inventory level. Balakrishnan et al. [2] indicate that high inventories may stimulate demand through promoting product visibility, signaling product popularity or providing high service levels. In contrast to these studies in the literature, our model focuses on the fast-fashion retailers’ intentional undersupply practice to induce customers to buy earlier. Thus, our model view shortages rather than inventory as a tool to stimulate demand. Yin et al. [10] propose a game-theoretical model to analyze strategies that intentionally limit the stock display to create a scarcity effect. Yang and Zhang [9] provide a detailed review on the scarcity effect of inventory.

3 Economic Order Quantity Model

We consider an inventory stocking problem for a single product at a single fast-fashion retailer based on a classical EOQ model. The retailer stocks $Q_0$ units of a
product at the beginning of the selling horizon. The product is sold over a selling horizon of length T and is then replaced by a new product afterwards. Customers arrive at the retailer for the product according to a homogenous Poisson process with rate $\lambda$ units per unit-time. Each arriving customer observes the current inventory level of the product and makes a purchasing decision. We assume that the customer utility has both a static component $U_S$ and a dynamic component $U_D(Q_t)$ that depends on the inventory level $Q_t$ at time $t$. We assume that customers perceive less inventory as a sign of product attractiveness [9]; therefore $U_D(Q_t)$ increases as the inventory gets depleted. We call this the *exclusivity effect* of inventory scarcity on customer purchasing decision. We define the total utility $U(Q_t)$ for a customer as follows,

$$U(Q_t) = U_S + U_D(Q_t) - p,$$

where $p$ is the unit selling price of the product. To capture customer heterogeneity in the market, we multiply $U_S + U_D(Q_t)$ by a random variable $\Phi$ that is uniformly distributed on $[0,1]$:

$$U(Q_t) = \Phi(U_S + U_D(Q_t)) - p.$$  

Using the above utility function, we can determine the probability of a purchase for a customer arriving at time $t$ as follows,

$$Pr(U(Q_t) > 0) = \left(1 - \frac{p}{u_s + u_d(Q_t)}\right)^+,$$

where $[x]^+ = \max(x, 0)$. For algebraic tractability we assume the following forms for static component $U_S$ and dynamic component $U_D(Q_t)$,

$$U_S = p, \quad U_D(Q_t) = \frac{V}{Q_t},$$

where $V > 0$ is the additional utility of purchasing the last remaining product. The form of $U_D(Q_t)$ fits well to cases where customers’ utilities and purchasing decisions are highly impacted by each other through inventory levels. We note that $U(Q_t) = \frac{V}{Q_t}$ corresponds to a customer’s highest utility; therefore, a small percent of the customers whose initial utility $U(Q_0) = \frac{V}{Q_0} > 0$ will ignite the system by making an immediate purchase. We can then represent the change in inventory level through the following differential equation,

$$\frac{\partial Q_t}{\partial t} = -\lambda Pr(U(Q_t) > 0) = -\frac{\lambda V}{pQ_t + V} \quad (1)$$

The solution to the above differential equation with the boundary condition that the firm stocks $Q_0$ units at the beginning of the selling season yields the following expression,
\[(pQ_t + V)^2 = (pQ_0 + V)^2 - 2\lambda Vpt\] (2)

The above expression for \(Q_t\) shows how the inventory level for the single product evolves over time during the selling horizon. Using this expression for the inventory dynamics, we can determine stock-out time \(\tau\) which is the duration until the whole initial inventory \(Q_0\) is depleted. Let \(Q_t(t = \tau) = 0\), therefore,

\[(pQ_0 + V)^2 = 2\lambda Vp\tau + V^2,\] (3)

which results in the following expression for the initial inventory to be stocked as a function of stock-out time \(\tau\),

\[Q_0 = \frac{V}{p}\left(\sqrt{1 + \frac{2\lambda p\tau}{V}} - 1\right)\]

We can easily show that stock-out time \(\tau\) is decreasing in additional utility of purchasing the last remaining unit, \(V\), indicating that the product is sold quicker as the exclusivity effect of scarcity increases. Also, the lower selling price \(p\), the lower initial inventory level \(Q_0\), and the higher the unit demand rate \(\lambda\) shortens the duration of sales as we expect. Clearly, \(Q_t(t > \tau) = 0\), as there cannot be negative inventory after the inventory is depleted. During the stock-out interval \([\tau, T]\), customer demand is not satisfied, which motivates them to make a purchase next period. This creates a demand spillover effect in the next period. The backlogged demand during the stock-out interval \([\tau, T]\) may be expressed as \(Q_T\), which evolves according to the following,

\[(pQ_T + V)^2 = 2\lambda VpT - (pQ_0 + V)^2 + 2V^2, T \geq \tau.\]

Eliminating \(Q_0\), using equation (3), we obtain the following expression for the backlogged demand \(Q_T\)

\[(pQ_T + V)^2 = 2\lambda Vp(T - \tau) + V^2, T \geq \tau,\] (4)

which can be equivalently expressed as

\[Q_T = \frac{\sqrt{2\lambda Vp(T - \tau) + V^2} - V}{p}\]

We assume that only a portion \(\alpha \in (0,1)\) of the backlogged demand \(Q_T\) is realized the next period through the demand spillover effect. We assume that the higher the frequency of introduction of new designs, the higher the impact of demand spillover effect through \(\alpha\). Thus, we assume that \(\alpha\) can be used as a proxy for the ability of a retailer to introduce new designs quicker. We next consider the profit per period
with an inventory replenishment cycle time of $T$. We assume that the retailer incurs a fixed cost $K$ per cycle for placing an order, transportation, and warehousing. Assuming unit margin for the product $m = p - c$, where $p$ and $c$ are unit price and cost respectively, we can express per-period profit as

$$\pi(\tau, T) = (mQ_0 + amQ_T - K)/T \quad (5)$$

Substituting the closed-form expressions developed earlier for $Q_0$ and $Q_T$ through equations (3) and (4) results in the following equivalent expression for the per-period profit,

$$\pi(\tau, T) = \left( \frac{\nu}{p} \left( \sqrt{1 + \frac{2\lambda \nu p (T - \tau)}{\nu}} - 1 \right) + \alpha \left( \frac{\sqrt{2\lambda \nu p (T - \tau) + \nu^2}}{p} - K \right) \right)/T.$$  

We next analyze $\pi(\tau, T)$ in more detail to determine the optimal stock-out time $\tau$ and the optimal selling horizon $T$ maximizing the retailer’s per-period profits in the long run. Furthermore, solving for the optimal stock-out time $\tau$ will provide us with the optimal stocking quantity $Q_0$ through the expression in (3).

### 3.1 Optimal Stock-out Time

Let us consider the first derivative of $\pi(\tau, T)$ in $\tau$ for a given value of the selling horizon $T$,

$$\frac{\partial \pi(\tau, T)}{\partial \tau} = \frac{m}{T} \left( \frac{\partial Q_0}{\partial \tau} + \alpha \frac{\partial Q_T}{\partial \tau} \right)$$

We can show using equations (3) and (4) that $\frac{\partial Q_0}{\partial \tau} = \frac{\lambda V}{pQ_0 + V}$ and $\frac{\partial Q_T}{\partial \tau} = -\frac{\lambda V}{p Q_T + V}$. Therefore, we obtain the following first-order condition, $\frac{\partial \pi(\tau, T)}{\partial \tau} = \frac{m}{T} \left( \frac{\lambda V}{pQ_0 + V} - \frac{\alpha \lambda V}{p Q_T + V} \right)$.

Since $\frac{\partial^2 \pi(\tau, T)}{\partial \tau^2} = -\frac{m}{T} \left( \frac{p(\lambda V)^2}{(pQ_0 + V)^3} + \frac{\alpha p(\lambda V)^2}{(p Q_T + V)^3} \right) < 0$, we conclude that $\pi(\tau, T)$ is concave in $\tau$ for a given value of $T$. Hence, from the first-order condition, we have

$$pQ_T + V = \alpha (pQ_0 + V) \quad (6)$$

From equations (3), (4), and (6), it follows that

$$\alpha^2 (2\lambda V p \tau + V^2) = 2\lambda V p (T - \tau) + V^2,$$

which simplifies to the following expression for the optimal stock-out time for a given selling horizon $T$
\[
\tau = \frac{T}{(1+\alpha^2)} + \frac{(1-\alpha^2)V}{2(1+\alpha^2)\lambda p}
\]  

(7)

Based on the above expression we observe that the optimal stock-out time \( \tau \) for a given selling horizon \( T \) is increasing in additional utility of purchasing the last remaining product \( V \), decreasing in unit demand rate \( \lambda \), and decreasing in demand spillover effect \( \alpha \). We observe that the firm has no incentive to create intentional stock-out period, i.e., \( \tau = T \) when there is no benefit from demand spillover effect, i.e., \( \alpha = 0 \). Also, when it fully benefits from demand spillover effect (i.e., \( \alpha = 1 \)), the optimal stock-out time simply becomes \( \tau = \frac{T}{2} \). We also observe that the optimal stock-out time \( \tau \) decreases in \( p \) unless \( \alpha = 1 \). Since the probability of purchase for a customer seeing \( Q(t) \) units of inventory upon arrival is \( V/(pQ(t) + V) \), which is decreasing in unit selling price \( p \), there will be more customers with demand spillover effect with higher selling price, which results in earlier stock-out time for a given selling horizon \( T \). We also note that the following relationship holds true \( T \geq \frac{1-\alpha^2}{2\alpha^2} \frac{V}{\lambda p} \) since \( \tau \leq T \).

In this section we have developed the expression for the optimal \( \tau \) for a given selling horizon \( T \). We showed that it is optimal for the retailer to stock-out earlier before the selling season is over, \( \tau < T \) when a portion of the unfilled demand in the current season energizes the demand for the product in the next selling season: the retailer benefits from intentional stock-outs through demand spillover. This important result is aligned with the practice of fast fashion firms: create scarcity to energize the demand. Next, we use this result to derive the optimal selling horizon \( T \) maximizing the firm’s profits per period.

### 3.2 Optimal Selling Horizon

We can obtain the following expressions by substituting for the optimal \( \tau \) in equations (3) and (4),

\[
(pQ_0 + V)^2 = \frac{2V}{1+\alpha^2} (\lambda pT + V)
\]

(8)

\[
(pQ_T + V)^2 = \frac{2\alpha^2V}{1+\alpha^2} (\lambda pT + V)
\]

(9)

Next, we consider the variation of the per-period profit in \( T \). From equation (5), we obtain the following

\[
\frac{\partial \pi(\tau, T)}{\partial T} = \frac{m}{T} \left( \frac{\partial Q_0}{\partial T} + \alpha \frac{\partial Q_T}{\partial T} \right) + \left( \frac{mQ_0 + \alpha mQ_T - K}{T^2} \right)
\]

Substituting \( \frac{\partial Q_0}{\partial T} = \frac{\lambda V}{(1+\alpha^2)(pQ_0+V)} \) and \( \frac{\partial Q_T}{\partial T} = \frac{\alpha \lambda V}{(1+\alpha^2)(pQ_0+V)} \), we have the following derivatives
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\[
\frac{\partial \pi(\tau, T)}{\partial \tau} = \frac{m}{T} \frac{\lambda V}{pQ_0 + V} - \left( \frac{m(Q_0 + \alpha Q_T) - K}{T^2} \right)
\]

\[
\frac{\partial^2 \pi(\tau, T)}{\partial T^2} = -\frac{mp(\lambda V)^2}{T(1 + \alpha^2)(pQ_0 + V)^3} - \frac{2m\lambda V}{T^2(pQ_0 + V)} + \frac{2m(Q_0 + \alpha Q_T - K)}{T^3}
\]

We observe that the sign of \( \frac{\partial^2 \pi(\tau, T)}{\partial T^2} \) can be positive or negative, depending on the values of parameters in the model. However, corresponding to the selling horizon \( T \) satisfying the first-order condition in equation (10), and after substituting \( \frac{m(Q_0 + \alpha Q_T) - K}{T^2} = \frac{m}{T} \frac{\lambda V}{pQ_0 + V} \), we would have \( \frac{\partial^2 \pi(\tau, T)}{\partial T^2} = -\frac{mp(\lambda V)^2}{T(1 + \alpha^2)(pQ_0 + V)^3} < 0 \). That is \( \pi(\tau, T) \) is concave in \( T \), when \( T \) satisfies the first-order condition in equation (10). Thus, we obtain the following expression for the optimal selling horizon \( T \),

\[
T = \frac{(pQ_0 + V)(m(Q_0 + \alpha Q_T) - K)}{m\lambda V}
\]

Based on the above expression we observe that the optimal selling season \( T \) is increasing in unit profit margin \( m \); decreasing in fixed cost \( K \) per inventory cycle; decreasing in unit demand rate \( \lambda \); and decreasing in additional utility of purchasing the last remaining product \( V \). We observe that unit cost \( c \) (embedded in \( m \)) impacts \( T \), but not \( \tau \). Clearly, equations (8), (9), and (11) can be solved simultaneously for \( T, Q_0, \) and \( Q_T \) to determine the optimal stock-out time, the optimal duration of the selling season, and the optimal amount of inventory to stock at the beginning of a selling season.

We derive the corresponding optimal results as follows,

\[
\tau = \frac{\left( \frac{2K^2p^2\lambda + 4KmpV(1 + \alpha)\lambda - V(m^2V(1 + \alpha(-4 + 2\alpha + \alpha^3))\lambda + \sqrt{\left(Kp + mV(1 + \alpha)\right)^2(2K^2p^2 - m^2V^2(1 - \alpha)^2 + 2KmpV(1 + \alpha)\lambda^2)} p^2V^2(1 + \alpha^2)^2\lambda^2} \right)}{2m^2pV(1 + \alpha^2)^2\lambda^2}
\]

\[
T = \frac{\left( \frac{K^2p^2\lambda + 2KmpV(1 + \alpha)\lambda - V(m^2V(1 - \alpha)^2\lambda + \sqrt{\left(Kp + mV(1 + \alpha)\right)^2(2K^2p^2 - m^2V^2(1 - \alpha)^2 + 2KmpV(1 + \alpha)\lambda^2)} p^2V^2(1 + \alpha^2)^2\lambda^2} \right)}{m^2pV(1 + \alpha^2)^2\lambda^2}
\]

\[
Q_0 = \frac{V}{p} \left( \frac{2\lambda p\tau}{V} - 1 \right)
\]
Substituting the optimal selling horizon expression in equation (11) into equation (5), we obtain the following expression for the optimal profit per period for the firm,

$$
\pi(\tau, T) = \frac{m\lambda T}{pQ_0 + V}
$$

Based on the above expressions for the optimal $Q_0$ and the optimal $\tau$, we observe that the optimal profit per period is increasing in unit profit margin $m$; increasing in unit demand rate $\lambda$; and decreasing in unit price $p$.

4 Summary & Conclusions

In this study, we developed an inventory management model to explain how a fast-fashion retailer improves its profits through deliberate understocking (scarcity effect) in the current selling period to induce demand in the next selling period (demand spillover effect). In our EOQ-based model, we have showed that it is optimal for the retailer to stock-out deliberately before the selling period is over to create demand spillover for the next period. We have developed several sensitivity analysis results using the closed-form expressions for the optimal solutions. Our model shed light on how fast-fashion retailers benefit from deliberate undersupply and demand spillover (energizing) effects. We conclude that when display inventory serves as a tool to motivate customer purchases, the retailers should account for the impact of inventories on demand in developing effective stocking decisions. Our model works with the assumption that the demand rate equals the observed average demand rate in prior inventory cycles. To account for the uncertainty in customer demand fully, an important direction for future research would be to study a dynamic newsvendor model with stochastic demand.

References


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