Applied Mathematical Sciences, Vol. 17, 2023, no. 6, 273 - 280
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ams.2023.917392

Heat Transfer in Viscous Flow Through a Channel Bounded by a Flat Deformable Sheet and a Naturally Permeable Bed

Kishan Singh Shekhawat

S.K. Govt. Girls College, Sikar, Rajasthan-332001, India
(Pandit Deendayal Upadhaya Shekhawati University Sikar Rajasthan India)

Subhash Yadav

Assistant Director (Commissionerate)
College Education of Rajasthan, Jaipur-302004, India

Abstract

In the present study, two-dimensional flow and heat transfer of a viscous fluid through a channel bounded by a flat deformable sheet and a naturally permeable bed is considered when the upper sheet and the lower bed are maintained at different temperatures. The results for temperature distribution, and rate of heat transfer have been obtained and discussed.

Keywords: Deformable Sheet, Permeability, Porous medium, Viscous fluid, Heat Transfer

1. Introduction

The studies of the flow of fluids in the presence of naturally permeable boundaries are important in view of their many scientific and engineering applications. Beavers and joseph (1) suggested a slip condition at the fluid-porous medium interface. Saffman(2) gave theoretical justification for this slip condition of Beavers and Joseph and further modified it for the case when the permeability of the porous medium is very small to calculate the outer free fluid flow in the vicinity of the porous boundary. Many researchers (3-7) used these conditions extensively in subsequent porous medium fluid problems. Heat transfer in viscous flow through porous media was much investigated by Nield and Bejans. However in a stretching channel, the effects of heat transfer are also important in the presence of a naturally permeable boundary, such problems are investigated with or without porous boundaries. Borkakoti and Bharali (9) studied MHD flow and heat transfer between two horizontal plates is a stretching sheet where the lower plate is a stretching sheet Bhattacharyya and Gupta(10) Mcleod and Rajagopal (11) discussed stability and uniqueness of flow due to a stretching boundary. Chauhan and Agarwal (12) studied flow and heat transfer in a channel bounded by a plate with a porous layer and a shrinking sheet. Xu Hang et al. (13) developed a mathematical model for the flow in micro channel driven by the upper stretching wall. Shekhawat K.S. (14) Studied flow of a viscous incompressible Fluid in the presence of a naturally permeable boundary.

In this paper, the flow of viscous fluid and heat transfer through a channel bounded by a flat deformable sheet and a naturally permeable bed of very small permeability is considered when the sheet is stretched in its own plane with an outward velocity proportional to the distance from a point on it. Two boundaries of the channel are maintained at different temperatures. In the absence of any external pressure gradient and small permeability of the porous medium, the flow inside the porous medium is assumed to be zero, and the effect of the permeability on the outer flow in the channel comes through the Saffman’s (2) simplification to the Beavers and Joseph (1) slip condition. The expressions for the temperature distribution and the rates of heat transfer at the lower permeable bed and the upper sheet, have been obtained and discussed.

2. Formulation

The lower Permeable bed and the upper stretching sheet are maintained at different temperatures, T_0 and T_1 respectively. The flow of a viscous incompressible fluid through a channel bounded by a flat deformable sheet and a naturally permeable bed is considered when the sheet is stretched in its own plane with an outward velocity proportional to the distance from a point $(0, y)$ on it.

The energy equation is

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+2\rho v\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right], \qquad (1)$$

where T is the temperature in the channel ; C_p is the specific heat at constant pressure, and k is the thermal conductivity.

The boundary conditions are

$$at\ \ y=0, T=T_0\ ,\ and\ \ at\ \ y=h, T=T_1 \qquad (2)$$

3. Method of solution

Let the expression for temperature T

$$T=T_0+\left(\frac{c^2h^2}{RC_p}\right)\left[\phi(\eta)+\frac{x^2}{h^2}F(\eta)\right],\ \eta=\frac{y}{h} \qquad (3)$$

the equation (1), using $u=cxf^{'}(\eta), v=-chf(\eta)$ and $\eta=\dfrac{y}{h}$ and comparing coefficients of and terms independent of x, reduces to

$$F^{''}=\Pr R\left[\left(2f^{'}F-fF^{'}\right)-f^{''2}\right], \qquad (4)$$

and

$$\phi^{''}=-\left[2F+\Pr R\left(4f^{'2}+f\phi^{'}\right)\right], \qquad (5)$$

Where $\Pr=\dfrac{C_p\rho v}{k}$ (Prandtl number).

We expand ϕ and F in power of R as

$$\phi=\sum_{n=0}R^n\phi_n\ , \qquad (6)$$

$$F=\sum_{n=0}R^nF_n\ . \qquad (7)$$

Using (7) in equations (4) and (5), and comparing coefficients of like powers of R, we have

$$F_0^{''}=0, \qquad (8)$$

$$\phi_0^{''}+2F_0=0, \qquad (9)$$

$$F_1'' = \Pr\left[2 f_0' F_0 - f_0 F_0' - f_0''^2\right], \tag{10}$$

$$\phi_1'' = -\left[2F_1 + \Pr\left(4 f_0'^2 + f_0 \phi_0'\right)\right], \text{ etc.} \tag{11}$$

the boundary conditions (29) reduce to

$$at\ \eta = 0, \phi_n = 0 = F_n \text{ for all } n \text{, and}$$

$$at\ \eta = 1, \phi_0 = \frac{\left(T_1 - T_0\right) RC_p}{C^2 h^2}$$

$$F_0 = 0, \phi_n = F_n = 0 \text{ for all } n > 0. \tag{12}$$

4. Solutions

On solving equations (8) to (11), subject to the boundary conditions (12), the non-dimensional expression for temperature is obtained as

$$\theta = \frac{T - T_0}{T_1 - T_0} = \eta + \Pr E_c R^2 \left[\phi_1(\eta) + \xi^2 F_1(\eta)\right] \tag{13}$$

Where $E_c = \dfrac{\nu^2}{C_p\left(T_1 - T_0\right)h^2}$ (Eckert number), $\xi = \dfrac{x}{h}$, and

$$\phi_1(\eta) = \left[\frac{\eta^6}{30} D_5 + \frac{\eta^5}{20} D_4 + \frac{\eta^4}{12} D_3 + \frac{\eta^3}{6} D_2 - 2B_2^2 \eta^2 + D_1 \eta\right], \tag{14}$$

$$F_1(\eta) = \left[c_1 \eta - \left(\frac{\eta^4}{12} A_0^2 + \frac{\eta^2}{2} B_1^2 + \frac{\eta^3}{3} A_0 B_1\right)\right] \tag{15}$$

The constants of integration are

$$C_1 = \left(\frac{1}{12} A_0^2 + \frac{1}{2} B_1^2 + \frac{1}{3} A_0 B_1\right),$$

$$D_1 = \left(2B_2^2 - \frac{1}{30} D_5 - \frac{1}{20} D_4 - \frac{1}{12} D_3 - \frac{1}{6} D_2\right),$$

$$D_2 = -\left(2C_1 + 8B_1 B_2 + \frac{1}{RE_c} B_2\right),$$

$$D_3 = -\left(3B_1^2 + 4A_0 B_2 + \frac{1}{RE_c} B_1\right),$$

$$D_4 = -\left(\frac{10}{3} A_0 B_1 + \frac{1}{6} \frac{1}{RE_c} A_0 \right),$$

$D_5 = -\frac{5}{6} A_0^2$, and the constants A_0, B_1 *and* B_2 are given by (14).

5. Rate of Heat Transfer

The rates of heat transfer at the lower porous interface and the upper stretching sheet,respectively, are given by

$$\left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0} = \left[1 + \Pr E_c R^2 \left(D_1 + \xi^2 C_1 \right) \right], \tag{16}$$

and

$$\left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=1} = \left\{ 1 + \Pr E_c R^2 \left[\left(\frac{D_5}{5} + \frac{D_4}{4} + \frac{D_3}{3} + \frac{D_2}{2} - 4B_2^2 + D_1 \right) + \left(C_1 - \frac{A_0^2}{3} - B_1^2 - A_0 B_1 \right) \xi^2 \right] \right\} \tag{17}$$

6. Discussion and Conclusions

Fig. 1 shows the temperature distribution in the channel for various values of the parameters, when the stretching parameter $R = 0$, the upper wall becomes stationary, and the heat flows from the upper wall to the fluid towards lower wall linearly, which is the same as in the case of a fluid at rest with no friction heat generated. However, when $R > 0$ a distribution which is due to the heat generated through friction is superimposed on this linear temperature distribution, and for a given temperature difference of distribution, and for a given temperature difference of the two walls $(T_1 - T_0) > 0$, heat flows from upper wall to the fluid as long as the stretching velocity does not exceed a certain value. A reversal of the direction of the flow of heat at the upper plate occurs when the temperature gradient at it changes sign. The temperature at large x in the channel is much greater than the temperature near the mouth. It is found that the temperature in the channel increases by increasing R or $\Pr$ or Ec, but by increasing $\overline{K}$ it decreases near the upper plate and increases at the lower permeable bed. In fact due to the stretching of the upper sheet whose temperature is T_1, the fluid near it is thrownout axially and an adverse pressure gradient is developed at a large distance causing back flow near the lower permeable surface whose temperature is T_0, and since the flow increases by increasing the permeability, more warm fluid rushes near the lower permeable bed causing increase in temperature near it.

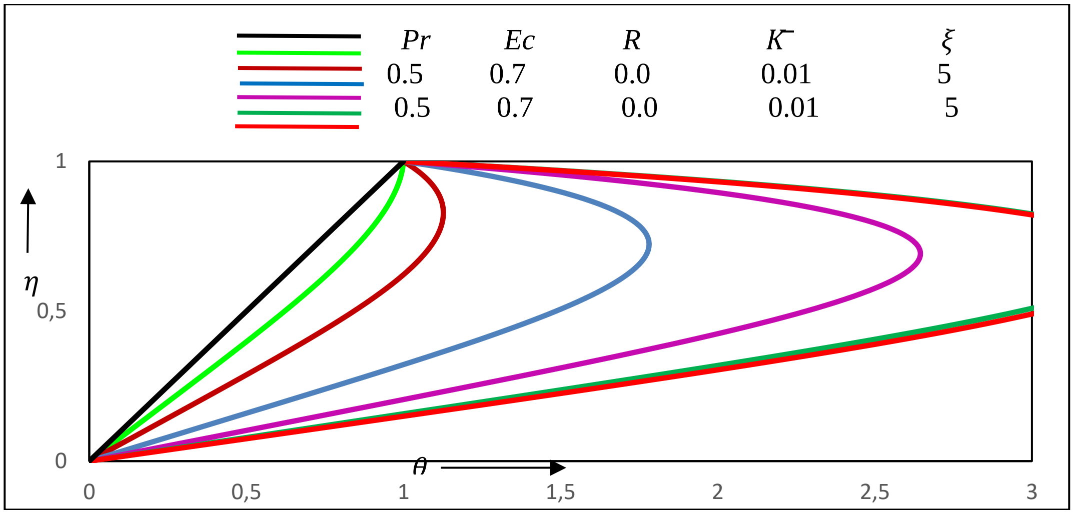

Fig. 1. Temperature distribution for $\alpha = 0.1$

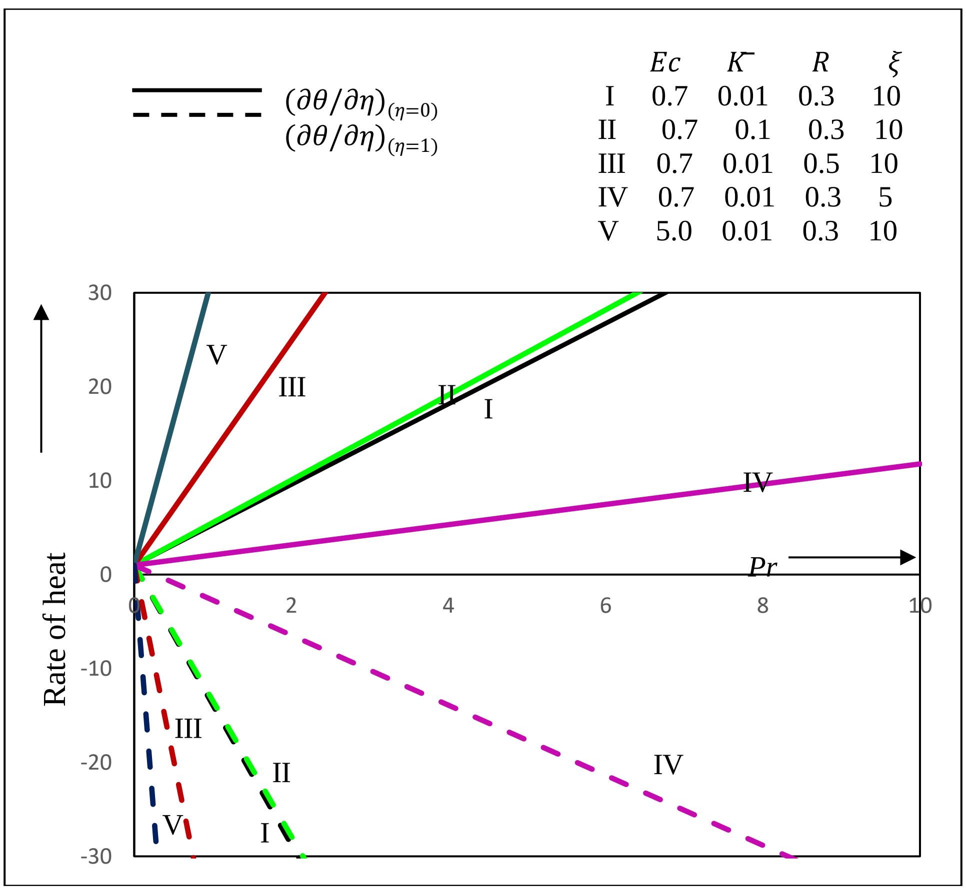

Fig. 2. Rate of heat transfer Vs Pr for $\alpha = 0.1$

Fig. 2 shows the variation of the rate of heat transfer at the upper and the lower wall for different parameters. It is found that the rate of heat transfer at the lower porous bed increases by increasing $\Pr$ or Ec or R or $\overline{K}$ or ξ, whereas it decreases numerically by increasing the permeability parameter $\overline{K}$, at the upper wall, however, it increases numerically by increasing the remaining parameters.

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Received: March 1, 2023; March 25, 2023