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Computational Complexity for Sum with Products of Stirling Numbers, Factorials and Binomial Coefficients

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Abstract

We consider the representation of the sum of the same powers as products of Stirling numbers of the second kind, factorials, binomial coefficients. This representation is reduced to the polynomial of the maximum of the used argument, and this achieves the desired expression of computational complexity.

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1 Introduction

Let we have a general expression for the sum of the same powers:

$$\Phi(p,\nu)=\sum_{\lambda=1}^{p}\lambda^{\nu}\,, \tag{1}$$

where $p, \nu \in \mathrm{N}$.

We also have the following initial representation of the sum of the same powers

[1]: $$\Phi(p,\nu) = \sum_{\iota=1}^{\max\iota} S(\nu,\iota)(\iota!)C_{p+1}^{\iota+1}. \qquad (2)$$

Let us now describe the transformation of expression (1) in the general case [2]. First, we select the components of the presented version of the sum of the same powers. We align the coefficients of the selected components and take the aligned multipliers out of the common bracket. Equalizing the coefficients means bringing their levels to the maximum values.

We group the terms remaining in the selected components of the considered sum after taking out the aligned factors, and we obtain the polynomial, which corresponds to the desired expression of computational complexity [3, 4].

With regard to the considered variant of the sum of equal powers, it can be stated more specifically that the overall coefficient in the transformation of the considered original expression is the maximum value in the product of the used Stirling number of the second kind, factorial and binomial coefficient.

2 Transformations and complexity representation

The transformation of the sum (2) with Stirling numbers, factorials and binomial coefficients has the following form:

$$\sum_{\iota=1}^{\max\iota} S(\nu,\iota)(\iota!)C_{p+1}^{\iota+1} \leq$$

$$m\sum_{\iota=1}^{\max\iota}(\iota^{\max\iota-\iota+1})\cdot(1+\delta_{\max\iota\,\iota}(\max\iota-1))\cdot(1) = mA; \qquad (3)$$

$\delta_{\max\iota\,\iota}$ — Kroneker delta;

$$m = m_1 m_2 m_3,$$

$$m_1 = S(\nu, \iota_0),$$

ι_0 — index of the boundary of the end of the increase and the beginning of the decrease of the value $S(\nu, \iota)$ [5],

$$S(\nu,1) < S(\nu,2) < \mathrm{K} \le S(\nu,\iota_0) > S(\nu,\iota_0+1) > \mathrm{K} > S(\nu,\max\iota);$$

$$m_2 = \prod_{\iota=1}^{\max\iota-1} \iota;$$

$$m_3 = C_{p+1}^{\iota+1} \,|_{E((\max\iota+1)/2)};$$

here E (•) denotes the integer part of the number in parentheses, which corresponds to the maximum value of the binomial coefficient in the row of Pascal's triangle [6];

accordingly, the value from the right side of expression (3) has the form

$$A = \sum_{\iota=1}^{\max\iota} (\iota^{\max\iota-\iota+1}) \cdot (1 + \delta_{\max\iota\,\iota}(\max\iota - 1)) \cdot (1).$$

We reduce the influence of $\max\iota$ on the value of m_2 and, accordingly, on the value of the desired computational complexity by introducing $\max\iota$ inside the expression

$$1 + \delta_{\max\iota\,\iota}(\max\iota - 1).$$

The desired computational complexity is the quantity

$$O(\max \iota^{\max \iota}) = \begin{cases} O(p^p): & p \le v, \\ O(v^v): & v \le p. \end{cases} \quad (4)$$

3 Conclusion

The considered variant of sums of the same powers has a certain computational complexity, which is expressed through the asymptotic representation O(·).When choosing options for calculating this complexity, one should be guided by the above provisions: we single out the components of the presented variant of the sum of the same powers; we align the coefficients of the selected components and take the aligned factors out of the common bracket, which makes it possible to obtain a certain sum of unequal degrees, i.e. polynomial, which corresponds to the desired expression of computational complexity.

The resulting expression for computational complexity includes two cases for $p \le v$ and for $v \le p$. Based on the considerations presented in Section 2, here we can choose a case with less complexity when calculating sums of the same powers. So, for case $p \le v$, this may be the top line of the record of the value (4), and for case $v \le p$, the bottom line of the indicated record.

The present work is, in essence, a continuation of the well-known article [1] and, in the future, its topic can, in turn, be continued by us in relation to the description of various variants representing sums of the same powers.

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