Applied Mathematical Sciences, Vol. 16, 2022, no. 2, 59 - 70
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ams.2022.916745

Adomian Decomposition Method Applied to Covid-19 Model

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Abstract

The Adomian decomposition method has been applied to solve the system of ordinary differential equations arising from the COVID-19 model. The results are compared with the modified Adomian decomposition method and findings indicate that there is an agreement between the two methods.

Keywords: COVID-19 Model, Adomian Decomposition Method, Modified Adomian Decomposition Method. System of Nonlinear Equations

1 Introduction

The Adomian Decomposition Method (ADM) that was founded by George Adomian in the 1980's is used for solving a wide range of equations,[1], [2], [3], [4]. The method breaks down the unknown function $u(x)$ into infinite number of components $u_0, u_1, u_2, \cdots$. In this method, the nonlinear terms of the function are decomposed into polynomials which are referred to as Adomian polynomials. The polynomials are denoted by A_n and they depend on the nonlinearity. The solution is then expressed as an infinite series of the form,

$$u(x) = \sum_{n=0}^{\infty} u_n(x). \tag{1}$$

The solution series (1) generally converges very rapidly in real physical problems. The convergence of the series has been investigated by several authors, for example in [6], [7] and [16]. The rest of the paper is arranged as follows: Section 2 gives the theoretical presentation of the Adomian Decomposition method, Modified Adomian Decomposition method and application of the two methods to the Covid-19 model. Section 3 illustrates the results and discussion and section 4 gives the conclusion.

2 Theoretical Presentation of the Adomian Decomposition Method

Consider an Initial Value Problem (IVP) in the form,

$$Lu + Ru + Nu = g, \tag{2}$$

where L is the linear operator to be inverted, N represents the nonlinear term, R is the linear remainder operator and g is the source term. We choose $L = \frac{d}{dx}$ and we assume that its inverse $L^{-1} = \int_0^x (\cdot)dx$ exists. Solving for u by applying L^{-1} on both sides of equation (2) and considering the initial value we get the following equation,

$$u = \varphi(x) + L^{-1}g - L^{-1}[Ru + Nu], \tag{3}$$

The ADM decomposes the solution in the form of equation (1), and the nonlinear term Nu is decomposed into a series

$$Nu = \sum_{n=0}^{\infty} A_n, \tag{4}$$

where the A_n's are called the Adomian polynomials and are calculated using the fomula,

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\sum_{k=0}^{\infty} u_k\lambda^k\right]_{\lambda=0}$$

for $n = 0, 1, 2, \cdots$.

Upon substituting equations (1) and (4) into equation (3) it gives the following equation,

$$\sum_{n=0}^{\infty} u_n = \varphi(x) + L^{-1}g - L^{-1}\left[R\sum_{n=0}^{\infty} u_n + \sum_{n=0}^{\infty} A_n\right]. \tag{5}$$

The solution components $u_n(x)$ can be determined by the recursive scheme,

$$\begin{cases} u_0(x) = \varphi(x) + L^{-1}g \\ u_{n+1} = -\, L^{-1}\left[Ru_n + A_n\right], n \geq 0. \end{cases} \tag{6}$$

The $n-term$ approximation of the solution is given by,

$$\phi_n(x) = \sum_{k=0}^{n-1} u_k(x). \tag{7}$$

Since its introduction, the ADM has been modified several times. Some of the modifications are given in [25] and [11]. Here we describe one of the modifications to the ADM termed as the Modified Adomian Decomposition Method (MADM) [18], [21], [23], [22], [25]. The basic idea of the MADM is to insert the expression $L^{-1}\left[\sum_{n=0}^{\infty} a_n x^n\right] - pL^{-1}\left[\sum_{n=0}^{\infty} a_n x^n\right]$ in equation (5) where p is an artificial parameter and for all $i \in N \cup \{0\}$, a_i are unknown coefficients, [10], [1], [17]. We thus obtain the following equation,

$$\begin{cases} \displaystyle\sum_{n=0}^{\infty} u_n = \varphi(x) + L^{-1}\left[\sum_{n=0}^{\infty} a_n x^n\right] - pL^{-1}\left[\sum_{n=0}^{\infty} a_n x^n\right] \\ \qquad\qquad + L^{-1}g - L^{-1}\left[R\left(\displaystyle\sum_{n=0}^{\infty} u_n\right) + \displaystyle\sum_{n=0}^{\infty} A_n\right]. \end{cases} \tag{8}$$

From equation (7), we can define the following recursive scheme:

$$\begin{cases} u_0 = \varphi(x) + L^{-1}\left[\displaystyle\sum_{n=0}^{\infty} a_n x^n\right], \\ u_1 = L^{-1}g - pL^{-1}\left[\displaystyle\sum_{n=0}^{\infty} a_n x^n\right] - L^{-1}\left[R(u_0) + A_0\right], \\ u_{n+1} = -L^{-1}\left[R(u_n) + A_n\right], for \quad n = 1, 2, \cdots. \end{cases} \tag{9}$$

To avoid calculation of A_n, $n = 0, 1, 2, \cdots$, we determine the coefficients a_n, for $n = 0, 1, 2, \cdots$, by setting $u_1 = 0$ and immediately we verify $u_n = 0$ for all $n \geq 1$, and set $p = 1$, to find the solution in the form:

$$u(x) = \varphi(x) + L^{-1}\left[\sum_{n=0}^{\infty} a_n x^n\right]. \tag{10}$$

MADM increases the rate of convergence by reducing the number of iterations as only u_0 and u_1 are calculated, [1], [17], [11].

2.1 Solution Of The System Of Nonlinear Equations

Covid-19 caused by the novel corona virus is an airborne disease. It is believed to have originated from Wuhan China in December 2019, [12], [9], [24]. It eventually spread worldwide.

Several studies on COVID-19 have been done, [9], [13], [20], [24]. We adopt and modify a model by [24] by solving it using ADM and MADM.The main advantage of using both ADM and MADM is that these two methods solve all types of differential equations with less complications. The description of variables and parameters used in the model are found in the appendices.

$$\begin{cases} \frac{dX_s}{dt} = -\Lambda X_s \\ \frac{dX_e}{dt} = \Lambda X_s - (\delta_e + \mu_r)X_e \\ \frac{dX_p}{dt} = \delta_e X_e + \mu_r X_i + \delta_m X_m - (\delta_a + \delta_s)X_p, \\ \frac{dX_a}{dt} = \delta_a X_p - (\tau_a + \mu_a)X_a, \\ \frac{dX_m}{dt} = \nu_m X_i - \delta_m X_m, \\ \frac{dX_w}{dt} = \delta_s X_p - (\tau_w + (1-b)\nu_w + b\nu_w + \kappa_w)X_w, \\ \frac{dX_i}{dt} = \mu_r X_e + \mu_a X_a + \tau_i X_u + (1-b)\nu_w X_w + \phi_i X_h - (\kappa_i + \nu_m)X_i, \\ \frac{dX_h}{dt} = b\nu_w X_w - (\phi_i + \tau_h + \kappa_h + \pi_h)X_h, \\ \frac{dX_c}{dt} = \pi_h X_h - (\kappa_c + \tau_c)X_c, \\ \frac{dX_u}{dt} = \tau_a X_a + \tau_w X_w - (\tau_i + \tau_s)X_u, \\ \frac{dX_t}{dt} = \pi_h X_h + \tau_c X_c + \tau_s X_u. \end{cases} \tag{11}$$

The force of infections Λ is given by:

$\Lambda = \frac{1}{N}\beta I_T(t)$, where $I_T = X_p + X_a + X_m + X_w + X_i$.

2.1.1 Solutions by ADM

We employ the ADM to find the solution of equation (11). Since the system of equations has first-order ordinary differential equations with initial values, we apply equation (5) and obtain the recursive scheme in the form of equation (6). After long calculations the solutions are obtained by replacing initial and parameter values given in tables 1 and 2 respectively in the recursive scheme. Using SageMath 8.6 software the solutions are evaluated as follows:

$$X_s(t) = 170000000 - 2809.96401208488\,t - 1326.03950734871\,t^2 + \cdots$$
$$X_e(t) = 24685 - 11968.5460567397\,t + 4504.93633906886\,t^2 + \cdots$$
$$X_p(t) = 789 + 12104.9606105320\,t - 10213.2150645609\,t^2 + \cdots$$
$$X_a(t) = 3272 - 629.745065576000\,t + 2847.35382840878\,t^2 + \cdots$$
$$X_m(t) = 3930 - 261.378220000000\,t + 476.221193685099\,t^2 + \cdots$$
$$X_w(t) = 324 + 301.697142996000\,t + 2705.58088922844\,t^2 + \cdots$$
$$X_i(t) = 5771 + 2889.04820374000\,t - 2483.83651305502\,t^2 + \cdots$$
$$X_h(t) = 435 - 151.777779420000\,t + 50.1772824134033\,t^2 + \cdots$$
$$X_c(t) = 46 - 14.5466669260000\,t + 1.82616307671704\,t^2 + \cdots$$
$$X_u(t) = 1998 - 96.0423441400000\,t - 13.3255802876208\,t^2 + \cdots$$
$$X_t(t) = 2479 + 311.347106346000\,t - 12.2565475880364\,t^2 + \cdots$$

2.1.2 Solutions by MADM

In order to compare the results obtained using ADM, the system equation (11) is solved using MADM. By using equation (8) we rewrite equation system (11) and get the recursive scheme of the form of equation (9). Solutions are obtained by using equation (10) and replacing parameter and initial values in the result to get the following equations:

$$X_s(t) = 170000000 - 2809.96400878753\,t + \cdots$$
$$X_e(t) = 24685 - 11968.5460583883\,t + 3544.53094621983\,t^2 + \cdots$$
$$X_p(t) = 789 + 12104.9606105320\,t + 474.222072714268\,t^2 + \cdots$$
$$X_a(t) = 3272 - 629.745065576000\,t - 58.8706679852206\,t^2 + \cdots$$
$$X_m(t) = 3930 - 261.378220000000\,t - 26.1827790538400\,t^2 + \cdots$$
$$X_w(t) = 324 + 301.701771336000\,t + 34.4793405410897\,t^2 + \cdots$$
$$X_i(t) = 5771 + 2889.04820374000\,t + 450.537540736801\,t^2 + \cdots$$
$$X_h(t) = 435 - 151.777779420000\,t + 24.2657253465530\,t^2 + \cdots$$
$$X_c(t) = 46 - 14.5466669260000\,t + 6.40053344744000\,t^2 + \cdots$$
$$X_u(t) = 1998 - 96.0423441400000\,t + 15.1732725490146\,t^2 + \cdots$$
$$X_t(t) = 2479 + 311.347106346000\,t + 20.7564735488353\,t^2 + \cdots$$

3 Results and discussion

Figures 1 and 2 show the comparisons of the solutions of ADM and MADM in each class of the model. The graphs in $(1a)$, $(1b)$, $(1i)$ and $(1k)$ show a close relationship between ADM and MADM. The rest of the graphs show a close relationship between ADM and MADM for the first few minutes and tend to diverge from each other for the remaining time. In genaral, one can deduce that there is an agreement between ADM and MADM.

4 Conclusion

The COVID-19 model with systems of ordinary differential equations is solved using the Adomian Decomposition Method. The results are compared with the Modified Adomain Decomposition Method. It is noticed that the first two terms of the series solutions in both MADM and ADM are similar. The rest of the terms tend to be different due to errors. The solutions are compared graphically and shows the variations of different populations classified in the model with time for both MADM and ADM. The graphs show an agreement between ADM and MADM.

Variable	Value	References
X_{s_0}	$1,700,000$	Assumed
X_{e_0}	24685	[26]
X_{p_0}	789	[26]
X_{a_0}	3272	[26]
X_{m_0}	3930	[26]
X_{w_0}	324	[26]
X_{i_0}	5751	[26]
X_{h_0}	435	[26]
X_{c_0}	46	[26]
X_{u_0}	1998	[26]
X_{t_0}	2479	[26]

Table 1: Variables used in the model and their initial values.

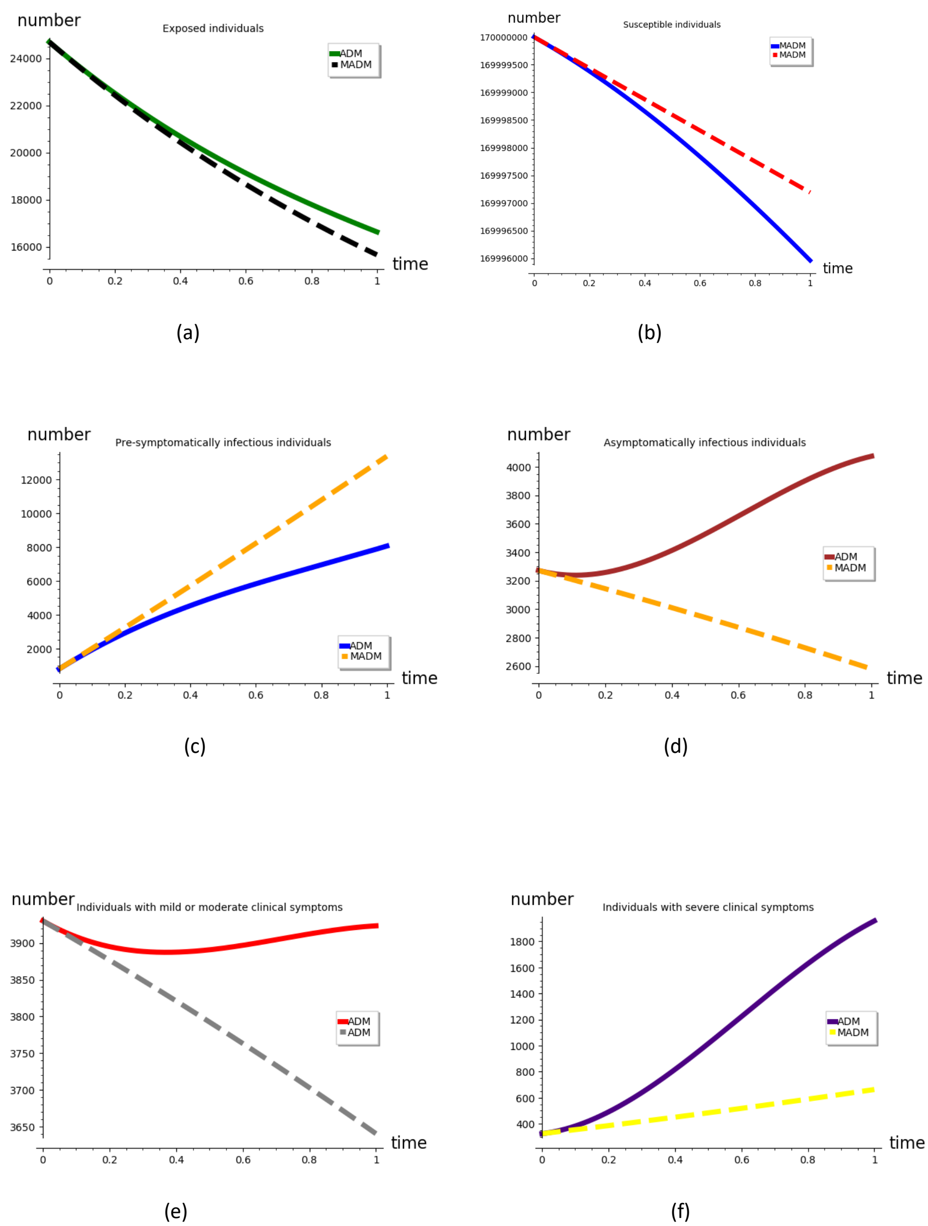

Figure 1: (a)-(f) Comparisons between ADM and MADM

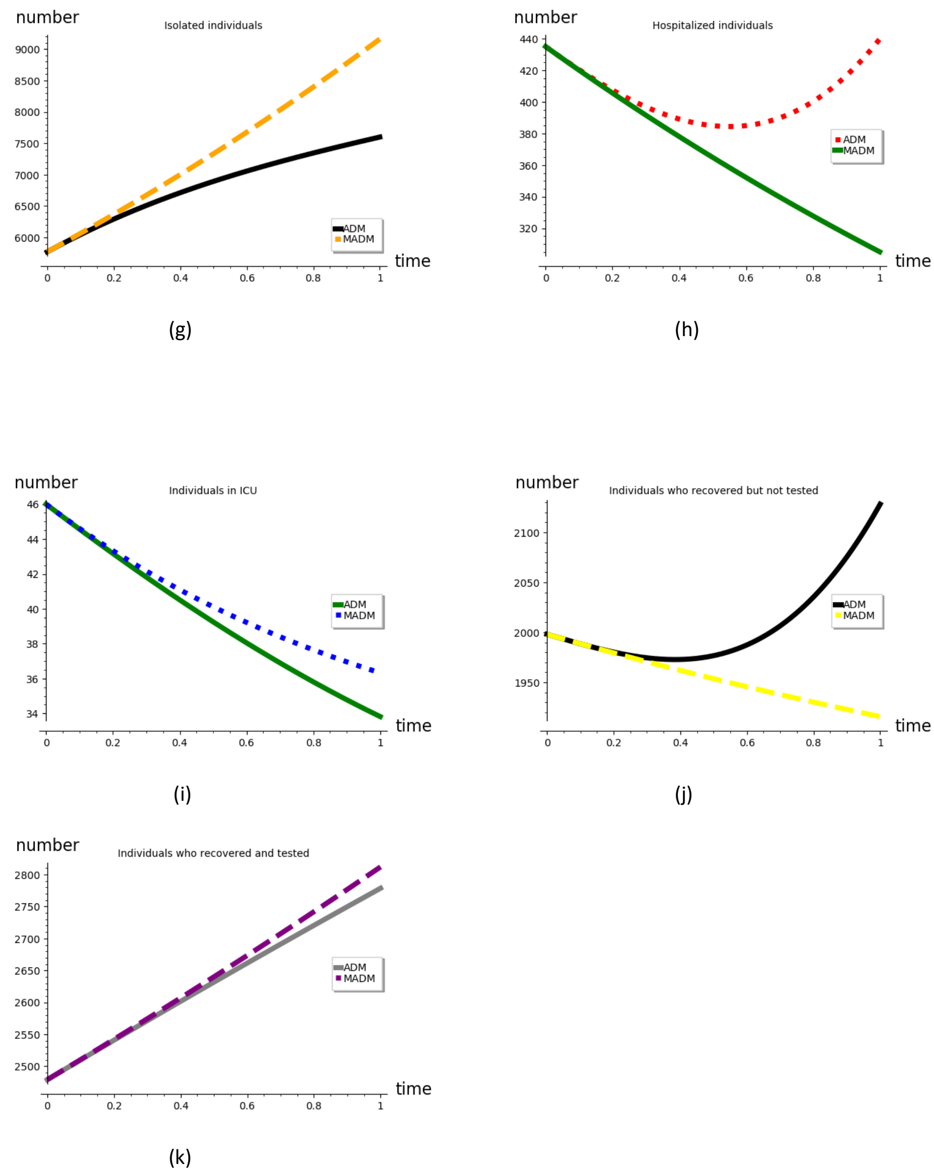

Figure 2: (g)-(k) Comparisons between ADM and MADM

Parametr	Value	Reference
N	$17,043,744$	[8]
β	0.02	Assumed
δ_e	0.4	[19]
μ_r	0.192307692	[15]
δ_a	0.5	[15]
δ_m	0.486044	Assumed
δ_s	0.5	[15]
$\kappa_i, \kappa_h, \kappa_w$	0.015	[15]
κ_c	0.88	[5]
ν_w	0.285714285	[14]
τ_i	0.142857142	[14]
π_h, τ_c	0.066666666	[15]
τ_c, τ_w	0.1	[15]
$\tau_s,$	0.13978	[15]
τ_a	0.133333333	[15]
p	0.32400000	[14]
μ_p	0.1296	Assumed
μ_a	0.1797	[14]
ϕ_i	0.2857	[14]
ν_m	0.2857	[14]
b	0.4	[19]

Table 2: Parameters used in the model and their values.

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Received: February 7, 2022; Published: February 26, 2022