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Probabilistic Modeling and Predicting Mean Recurrence Time of Major Earthquakes in Bangladesh

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Abstract

Earthquake is one of the most dreadful natural disasters that causes huge amount of destruction of the properties and human lives. That is why, researchers are attempting to find out best model to explain and predict the pattern of earthquake so that the losses of lives and wealth can be minimized. In this paper, a statistical analysis of the earthquake data of Bangladesh has been taken into accounts which were recorded from 1918 to 2016. Considering the large earthquakes (magnitude level $\geq$ 6.0), the interval (in days) between two successive large earthquakes have been taken as the random variable and the data have been examined by fitting Weibull, Lognormal, Exponential, Laplace and Inverse Gaussian Distributions among which the Weibull Distribution fits the best. The derived parameters of the distribution may particularly be useful in predicting the occurrence of a future large earthquake precisely in Bangladesh as well as explaining the pattern of the mean recurrences time of earthquake.

Keywords: Weibull distribution, large earthquake, mean recurrence time, magnitude level, AIC

1. Introduction

Bangladesh is geographically as well as geologically important country of South Asia and is often experienced by many devastating natural disasters for many years. But in the recent past, earthquake has become the most threatening one among them because of the frequent occurrence of earthquakes and poor infrastructure condition of the country. The high density of the population makes the matter one step worse. There are several numbers of tectonic blocks around the geographical region of Bangladesh which are assumed to be responsible for many earthquakes in the past. Calcutta, Assam, Tripura are the three very earthquake prone regions that are joined with Bangladesh in the borders of the Western, Northern and North-Eastern part respectively. If we consider the tectonics and geology, five major faults are active in this country that are significant for the occurrences of devastating earthquakes which are [1],[2]: (i) Bogra fault zone, (ii) Tripura fault zone, (iii) Sub-Dauki fault zone,(iv) Shillong plateau and (v) Assam plateau. For the presence of these major faults, Bangladesh is facing a dreadful threat. Although our country is extremely vulnerable to seismic activity but the nature and the level of this activity is yet to be defined. The Meteorological Department of Bangladesh established a seismic observatory at Chittagong in 1954 which is the only observatory in the country. Accurate historical information on earthquakes is very important in evaluating the seismicity of the country. Also it is very important for us to have the idea- when the next large earthquake may strike us. Statistical modeling especially probabilistic modeling of earthquake may provide a deeper and more reliable insight of this problem. Several researches have been conducted on the earthquake incidents of this region. Some of them are based on all kinds of seismic activities history (i.e. minor, medium, large and severe levels of earthquakes) of this region[3]. Taking into account the geographical background of Bangladesh; earthquake with magnitude level 6.0 or greater is considered to be a big level of earthquake which is our main concern. In the previously done researches, ‘the time interval between two successive large earthquakes’ counted in year; had been considered as the random variable to model the data. That is why prediction about the next large earthquake with concerning probabilities has become confined in years only. It’s not possible to predict from such studies that ‘approximately how many days we have to wait to observe the next large earthquake’ where there are even strong evidences of large earthquakes and aftershocks within the time interval of 0-3 days for the last three to five years.

2. Methodology

B.K. Chakravorti *et al.* (2015) and Shongkour Roy (2014); have worked on the previous earthquake data of Bangladesh up to 2008 and its adjacent regions with spatial distribution and have tried to find out the probable earthquake risk with magnitude ($m_s \geq 6.0$) in this region with an aim to forecast the waiting times for the next earthquake of level ms ≥ 6.0 in Bangladesh and its surrounding regions

[4],[2]. For the prediction of the next strong earthquake, they have proposed Weibull, Log-normal and exponential distribution among which Weibull is the best fitted one. In this paper, considering the history of large earthquake (magnitude level ≥ 6.0), the time interval (in days) between two successive large earthquakes have taken into account as the random variable based on the seismic data recorded from 1918 to 2016 by Bangladesh Meteorological Department (BMD) and 5 different types of continuous distributions have been proposed which are mostly used for meteorological data e.g. Weibull, Inverse Gaussian, Laplace or Double exponential and Lognormal distribution.

2.1 Weibull Distribution

The probability density function of a Weibull random variable is[5]:

$$f(x;\lambda,k)=\begin{cases}\frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}\exp(-(x/\lambda)^{k})\;; & x\geq 0\\ 0\;; & x<0\end{cases} \tag{2.1}$$

where, $k>0$ is the shape parameter and $\lambda>0$ is the scale parameter of the distribution. The form of the density function of the Weibull distribution changes drastically with the value of *k.*

2.2 Inverse-Gaussian Distribution

The probability density function of this distribution is given by[6]:

$$f(x;\mu,\lambda)=\sqrt{\left[\frac{\lambda}{2\pi x^{3}}\right]}\exp\{\frac{-\lambda(x-\mu)^{2}}{2\mu^{2}x}\}\;;\; x>0 \tag{2.2}$$

where, μ is the mean and λ is the shape parameter and $\lambda>0,\mu>0$ and $0<x<+\infty$.

2.3 Exponential Distribution

The probability density function (pdf) of exponential distribution is:

$$f(x;\lambda)=\lambda e^{-\lambda x} \tag{2.3}$$

here, $\lambda>0$ is the parameter of the distribution often called the rate parameter. The distribution is supported on the interval $[0,\infty)$.

2.4 Log-Normal Distribution

The probability density function (pdf) of Log-Normal distribution is given by[7]:

$$f(\ln x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(\ln x-\mu)^{2}}{2\sigma^{2}}\right]\;;\; x>0 \tag{2.4}$$

where, $-\infty<\mu<+\infty$ and $\sigma>0$.

2.5 Laplace Distribution

The Laplace distribution, also called the double exponential distribution, is the distribution of differences between two independent variates with identical exponential distributions. A random variable has a Laplace (μ,b) distribution if its probability distribution function (pdf) is:

$$f(x;\mu,b) = \frac{1}{2b}\exp(-|x-\mu|/b) \tag{2.5}$$

here, μ is the location parameter and $b > 0$ which is sometimes referred to as the diversity, is the scale parameter.

3. Data and Data source

Bangladesh Meteorological Department (BMD) also known as *Abohawa Office* (Weather Office) is the national meteorological organization of Bangladesh, working under Ministry of Defence of the Government of the People's republic of Bangladesh and according to their recorded database source, 28 major earthquakes out of a total of 452 recorded earthquakes that have occurred during the period of 1918 to 2016 in Bangladesh area (which had magnitude level≥6.0). Table 1 show the time periods when such major earthquakes occurred with their corresponding earthquake level.

Table 1. Major earthquakes in Bangladesh between the time periods 1918 to 2016.

Date	Magnitude Level (6 and above)	Date	Magnitude Level (6 and above)	Date	Magnitude Level (6 and above)
08/07/1918	7.6	21/09/2009	6.4	25/04/2015	7.5
09/09/1923	7.1	04/02/2011	6.4	25/04/2015	6.4
02/09/1930	7.1	24/03/2011	6.8	26/04/2015	6.6
24/03/1932	7.4	18/09/2011	6.8	12/05/2015	7.1
27/03/1932	7.4	11/04/2012	8.7	12/05/2015	6
09/11/1932	7.4	11/04/2012	8.1	04/01/2016	6.6
06/03/1933	7.6	15/04/2012	6.2	13/04/2016	7.2
21/05/1935	6.3	21/04/2012	6.1	18/04/2016	7.1
21/01/1941	6.8	11/11/2012	6.6	-	-
23/02/1954	6.5	21/05/2014	6.1	-	-

In this paper, the random variable is; "*X=time interval in days between two major earthquakes having magnitude level 6 or above*". Table 2 shows the observations of *X* (random variable). (***Data source: Bangladesh Meteorological Department)

Table 2. Observations of Random variable '*X*' from 1918-2016.

x(in days)	x(in days)	x(in days)	x(in days)
1859	2073	0.124	1
2345	4782	4	17
570	20299	6	0.342
3	502	205	238
228	49	557	101
118	179	340	134
807	207	0.25	203

4. Result

Considering all the 452 earthquake history in Bangladesh recorded from 1918 to 2016, it reveals that the most of the earthquakes that occurred are of magnitude level 4.0 to 4.9; which has been declared as ‘light’ earthquake by the Bangladesh Meteorological Department (BMD).

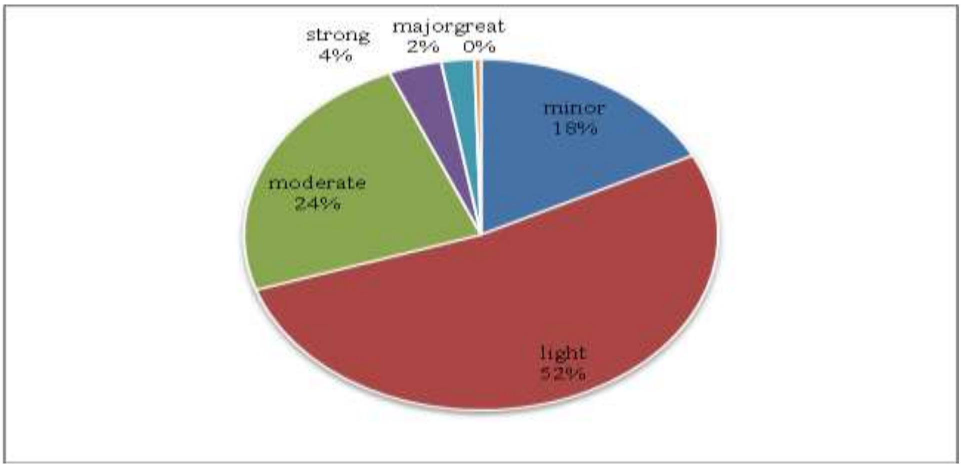

Figure 1. Pie chart showing occurrences of different levels of earthquakes.

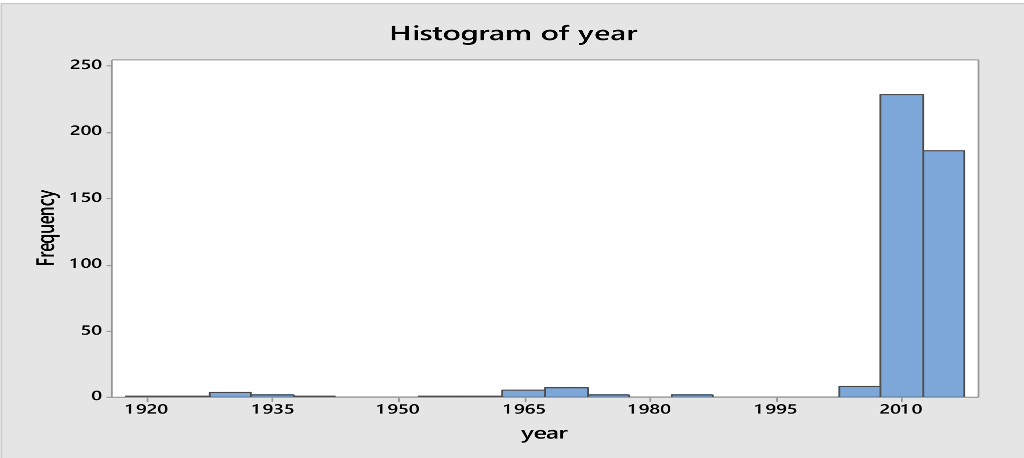

Figure 2. Histogram showing the frequency of earthquake year wise.

Figure 1 shows that the occurrence of earthquake is very common and frequent in Bangladesh which is not as much devastating as those of level 6 and above. Still this might be a significant warning for our country as the occurrences of earthquakes in Bangladesh have dramatically been increased in the last decade compared to the early 90’s that are shown in figure 2. It also indicates that some considerable changes in the geographical structures and seismic patterns might have taken place in this region; which is responsible for the frequent occurrences of earthquakes. According to Vevsel Yilmaz *et. al*. (2004), in modeling earthquake data the most appropriate distribution was proposed as the Weibull distribution[8]. But there are some other distributions also that can serve the similar purpose and that is why four other probability distributions including Lognormal (Nishenko and Buland[9]), Laplace or double exponential (Utsu[10]), Exponential and Inverse Gaussian have been proposed. Table 3 provides the estimated values of the parameters of the candidate distributions.

Table 3. Based on the data estimated parameters of the distribution.

Distribution	Estimated Parameters value
Exponential	λ=7.8152E-4
Inv. Gaussian	λ=139.91, μ =1279.6
Laplace	λ=3.6547E-4, μ=1279.6
Lognormal	σ=3.0277, μ=4.4829
Weibull	α=0.37327, β=301.87

To examine the goodness of fit of the distributions Kolmogorov Smirnov, Anderson Darling and Chi-Square goodness of fit tests have been applied and the ‘Rank’ has been provided to the distributions by the tests based on their goodness of fit. The better the distribution fit the data, the upper the rank has been provided and it has been clearly evident from the results of three types of goodness of fit tests that the Weibull distribution is the best fitted model for the specified data set. Though there is a little variation in the ranking of good fit of the other distributions based on these three test statistics, but there is no doubt about Weibull distribution regarding the best model (see for table 4).

Table 4. Goodness of fit-summary and estimated AIC values of the model.

Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared		AIC
	Statistic	*Rank*	*Statistic*	*Rank*	*Statistic*	*Rank*	
Exponential	0.47313	5	18.7060	4	34.299	5	458.639
Inv. Gaussian	0.28112	3	47.7730	5	2.7294	2	2651.510
Laplace	0.40074	4	6.51210	3	15.562	4	273.17E6
Lognormal	0.19599	2	0.86164	2	9.6260	3	401.4994
Weibull	0.16405	1	0.48154	1	0.08788	1	394.143*

N.B. (*) indicates the lowest AIC value among all.

From the above summary results, we may comment that the random variable ‘*X=time interval (in days) between two successive large earthquakes*’ can be modeled and predicted by using Weibull distribution. Therefore, the probability density function of the random variable X can be written as:

$$f(x;\alpha,\beta)=\frac{0.37327}{301.87}\left(\frac{x}{301.87}\right)^{0.37327-1}\exp\left[-\frac{x}{301.87}\right]^{0.37327}\quad;\quad x>0$$

with fitting parameters and mean recurrence time.

Table 5. Estimated parameter values and mean recurrence time.

Estimated parameter		Mean recurrence time
$\hat{\beta}$	$\hat{\alpha}$	$E(x)$
0.37327	301.87	1228.638

Figure 3 & 4 shows the graphs of the pdf's and the cdf's of the five distributions under study and a comparison among these distributions.

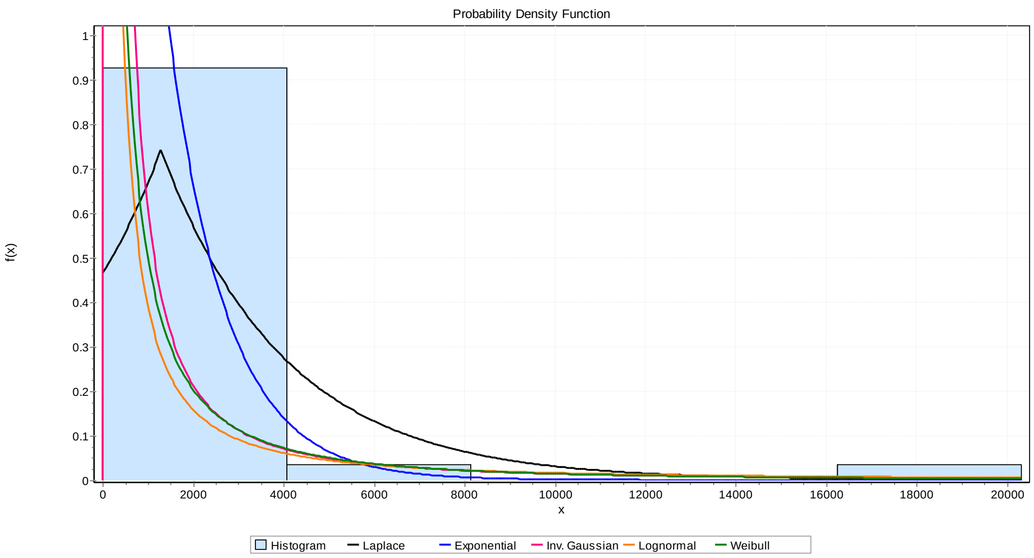

Figure 3. Fitting probability distribution functions of Weibull**,** Lognormal, Exponential, Laplace and Inverse Gaussian Distribution.

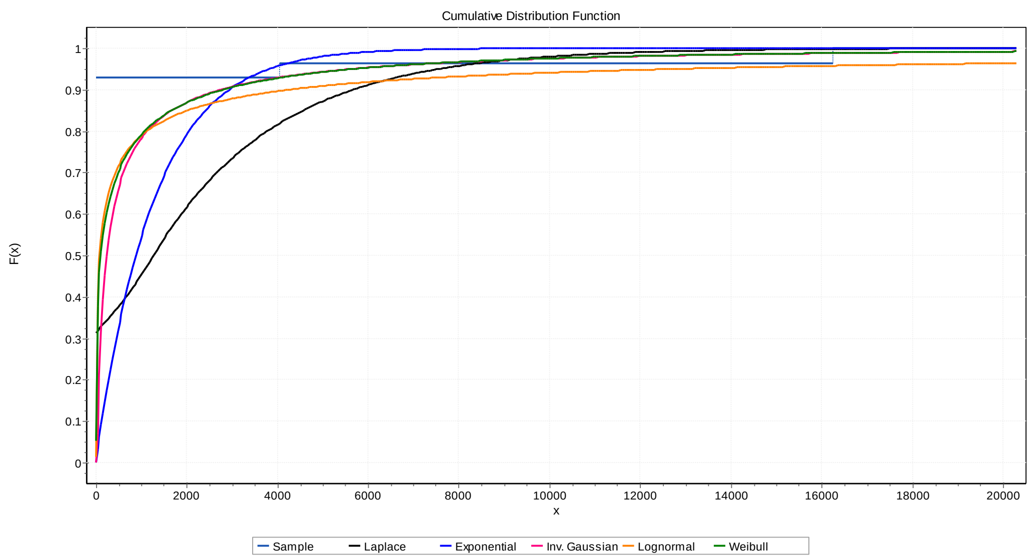

Figure 4. Cumulative density functions of Weibull**,** Lognormal, Exponential, Laplace and Inverse Gaussian Distribution.

5. Discussion and Conclusion

Based on the earthquake data of Bangladesh Meteorological Department (BMD) from 1918 to 2016, we have considered the number of days between two successive large earthquakes (that had magnitude level $\geq$ 6.0) as a random variable and that follow any of the specified distributions. From the analysis we may conclude that the Weibull distribution is the most suitable one for analyzing earthquake data of Bangladesh than the other four distributions. Using this distribution, it has been found that the mean occurrence period of earthquake in Bangladesh with magnitude 6 or above is approximately 1229 days (table 5), which indicates that approximately after 1229 days we may expect that the next large scale earthquake (having magnitude level $\geq$ 6.0) in Bangladesh. Though the results support the findings of Vevsel Yilmaz *et. al*. (2004) and Shongkour Roy (2014) but they have focused on the yearly based time interval between two major earthquakes, whereas in the last few years the occurrences of major earthquakes repeated several times in the same day and year based time interval may fail to provide precise information about that. This is why it important to find the average recurrence time of major earthquakes in days rather in years, so that more specific conclusions may be made. Moreover in the work of Vevsel Yilmaz *et. al*. (2004) and Shongkour Roy (2014), the fact has been ignored that the occurrences of the major earthquakes can be repeated in the same day and it is very important to model all those repeated events as it might help to depict some conclusions about the aftershock of any major earthquakes. To serve the purposes, the time interval has been counted in days which are more reliable and logical. During 1918 to 2016, it has been noticed that most of the earthquakes that had been occurred in Bangladesh during the last 98 years were 'light' earthquake (almost 52%) and the magnitude of these earthquakes are on an average 4.0 to 4.5 level. The frequency of the earthquakes in Bangladesh has been increased dramatically and the frequent occurrence of medium and light earthquake may significantly change the geographical pattern of the fault zones of Bangladesh.

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