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Fréchet-Weibull Mixture Distribution: Properties and Applications

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Abstract

We propose a new mixture of continuous models called Fréchet-Weibull mixture exponential distribution with variety of statistical properties. Cumulative and density functions of its order statistics were derived along with limit distribution of its maximum and minimum. Method of maximum likelihood was used for determining estimated parameters and its performance was evaluated by biases and mean squared errors with the aid of randomly generated data set. The superiority of the proposed model is illustrated by an application on three real-world data sets.

Mathematics Subject Classification: 62H12, 60E05, 62H10

Keywords: Mixture distribution, Weibull distribution, Fréchet distribution, Moment generating function, Entropy, Order statistics

1 Introduction

In statistics, a mixture distribution is expressed as a combination of other probability distributions. The distributions that mixed are called the component of the mixture. The weight themselves include a probability distribution called the mixing distribution. Thus a mixture is a probability distribution as a property of weights. Probability distribution of this type uses when observed phenomenon can be the consequence of two or more related but usually unobserved phenomena, each of which leads to a different probability distribution. Applications of mixture distributions play an important role in reliability theory, insurance risk theory and oil industry.

A continuous random variable X has a mixture distribution if at least one parameter of the distribution of it is also a random variable. Let $g(x;\theta)$ be probability density function (PDF) of X, where θ is a parameter of the distribution of X. If θ is a random variable with PDF $h(\theta)$, then X has a mixture distribution with PDF $f(x) = \int_\theta g(x;\theta)h(\theta)d\theta$.

Pearson [4] was the first one highlighted the field of mixture distributions by derived a mixture of two normal distributions and studied the parameter estimation of this mixture distribution. After many years Robbins [5] presented some basic properties of mixture distributions. Roy et al. ([8], [9], [10], [11], [7], [12] and [6]) introduced and derived Poisson, binomial, negative binomial, gamma, chi-square and Erlang mixtures of some standard distributions. Rezaul et al. [3] introduced Rayleigh mixture distribution with various weight functions, a two correlated Rayleigh random variables have been determined.

This paper is organized as follows. In section 2 we presented a new mixture distribution called Fréchet-Weibull mixture exponential distribution by its forms of cumulative and probability functions along with survival, hazard and reverse hazard functions. Mode, quantile function, moments, moment generating function, mean residual life function, Entropy and Lorenz, and Bonferroni curves were derived in section 3. In section 4 we introduced its probability and cumulative order statistics functions along with the limit distribution of its minimum and maximum order statistics. Method of maximum likelihood was used to estimate unknown model parameters in section 5. In section 6 we examined the efficiency of model parameter's estimators by randomly generated data, three real-world data sets were applied for showing the superiority of proposed model.

2 Fréchet-Weibull mixture exponential distribution

In this section we introduced a new mixture distribution as a result of mixing re-parameterized Fréchet-Weibull distribution with exponential distribution, it is called Fréchet-Weibull mixture exponential distribution (FWMED).

The PDF and the cumulative distribution function (CDF) for the re-parameterized Fréchet-Weibull distribution ($w = \beta^\alpha$) are given by

$$f(x) = \alpha k w \lambda^{\alpha k} x^{-1-\alpha k} \exp(-w(\frac{\lambda}{x})^{\alpha k}) \ , \ F(x) = \exp(-w(\frac{\lambda}{x})^{\alpha k}), \quad x > 0,$$

respectively, where α and k are shape parameters, λ and w are scale parameters.

If a random variable X follows the re-parameterized Fréchet-Weibull distribution, by taking one of its four parameters (w) as a random variable follows exponential distribution, then it is said to have FWMED with four parameters ($a, \alpha, \lambda, k > 0$) when its PDF and CDF are defined as following for $x > 0$

$$f(x) = \frac{a\alpha k\lambda^{\alpha k} x^{-\alpha k-1}}{\left(a + \left(\frac{\lambda}{x}\right)^{\alpha k}\right)^2}, F(x) = \frac{a}{a + \left(\frac{\lambda}{x}\right)^{\alpha k}}, \tag{1}$$

respectively, where α and k are shape parameters, a and λ are scale parameters.

2.1 Survival and hazard functions

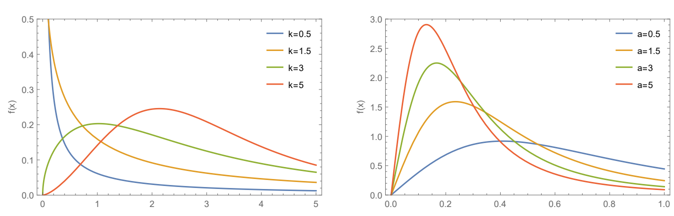
The survival function and its correlated functions are useful to study examples of any lifetime phenomenon. The survival, hazard and reverse hazard functions of FWMED is defined as following

$$S(x) = \frac{1}{a\left(\frac{x}{\lambda}\right)^{\alpha k} + 1}, h(x) = \frac{a\alpha k x^{-\alpha k-1}\left(a x^{\alpha k} + \lambda^{\alpha k}\right)}{\left(a + \left(\frac{\lambda}{x}\right)^{\alpha k}\right)^2}, r(x) = \frac{\alpha k}{a x\left(\frac{x}{\lambda}\right)^{\alpha k} + x},$$

respectively.

2.2 The impact of changing parameters values

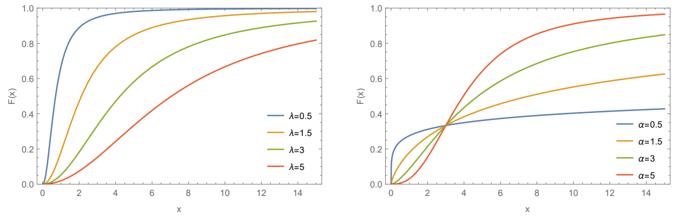
In this subsection we displayed the impact of changing parameters values on drawing PDF, CDF, S(x), h(x) of FWMED, which displayed in the following figures, respectively.



(a) (b)

Figure 1: The effect of k and a parameters on the PDF of FWMED

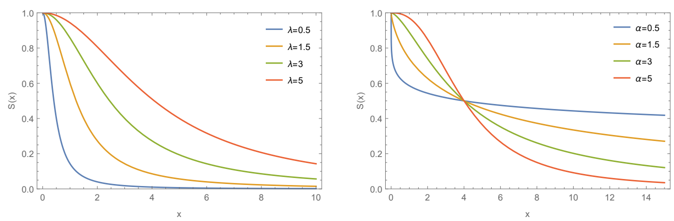
Figure (1a) explains the behavior of the PDF of FWMED affected by changing the parameter k, where $\alpha = 0.5$, $\lambda = 3$ and $a = 1$, and also figure (1b) explains its behavior affected by changing the parameter a, where $\alpha = 2$, $\lambda = 0.5$ and $k = 1$.



(a) (b)

Figure 2: The effect of λ and α parameters on the CDF of FWMED

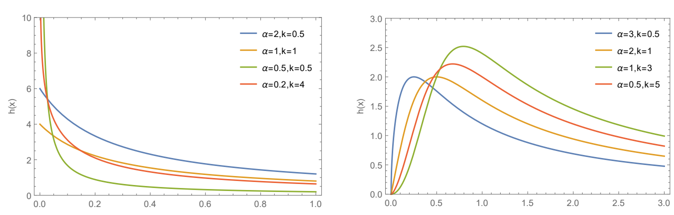
Figure (2a) shows the CDF behavior affected by changing the parameter λ, where $\alpha = 2$, $a = 0.5$ and $k = 0.5$, while figure (2b) shows the behavior of the CDF affected by changing the parameter α, where $\lambda = 3$, $a = 0.5$ and $k = 0.5$.



(a) (b)

Figure 3: The effect of λ and α parameters on the S(x) of FWMED

Figure (3a) shows S(x) plots changing as the parameter λ increasing, where the parameters $\alpha = 2$, $a = 1.5$ and $k = 1$ are fixed, while figure (3b) shows the behavior of S(x) with changing the parameter α, where $\lambda = 4$, $a = 1$ and $k = 0.5$.



(a) (b)

Figure 4: The effect of k and α parameters on the h(x) of FWMED

The behavior of h(x) was studied for different values of parameters k and α, and fixed $a = 1$ and $\lambda = 1$ by Glaser's lemma (see [2]) as following:

- if $\alpha k \leq 1$, then h(x) has a decreasing failure rate which was satisfied in figure (4a).

- if $\alpha k > 1$, then h(x) has upside down bathtub failure rate which was satisfied in figure (4b).

3 Statistical Properties

3.1 Mode and quantile function

By differentiating the logarithm of the PDF of FWMED with respect to x and equating to zero, we get the mode of FWMED as following:

$$x_0 = \lambda \left(\frac{a(\alpha k + 1)}{\alpha k - 1} \right)^{-\frac{1}{\alpha k}}, \alpha k > 1,$$

and if $\alpha k \leq 1$, then f(x) is a decreasing function (no mode), which also shown in figure (1a). The quantile function $Q(p)$ of FWMED is given by:

$$Q(p) = \inf\{x \in R : F(x) \geq p\} = \lambda \left(\frac{a}{p} - a \right)^{-\frac{1}{\alpha k}}, \tag{2}$$

by setting p = 0.25, 0.5 and 0.75 we obtain the first, second and third quartiles of FWMED, respectively.

3.2 Moments

The r^{th} moments μ'_r about the origin of FWMED is defined as

$$\mu'_r = \int_{x=0}^{\infty} x^r f(x) dx = \frac{\pi r \lambda^r a^{-\frac{r}{\alpha k}} \csc\left(\frac{\pi r}{\alpha k}\right)}{\alpha k},$$

by setting r = 1, 2, 3 and 4 we obtain the first four moments about the origin of FWMED, respectively.

By using the moments about origin, we can determine the first four central moments about the mean of FWMED which are given by the following relations

$$\begin{aligned}
\mu_1 &= 0, \mu_2 = \frac{\pi \lambda^2 a^{-\frac{2}{\alpha k}} \left(2\alpha k \csc\left(\frac{2\pi}{\alpha k}\right) - \pi \csc^2\left(\frac{\pi}{\alpha k}\right)\right)}{\alpha^2 k^2}, \\
\mu_3 &= = \frac{\pi \lambda^3 a^{-\frac{3}{\alpha k}} \left(3\alpha^2 k^2 \csc\left(\frac{3\pi}{\alpha k}\right) + 2\pi^2 \csc^3\left(\frac{\pi}{\alpha k}\right) - 6\pi \alpha k \csc\left(\frac{2\pi}{\alpha k}\right) \csc\left(\frac{\pi}{\alpha k}\right)\right)}{\alpha^3 k^3}, \\
\mu_4 &= \frac{\pi \lambda^4 a^{-\frac{4}{\alpha k}}}{\alpha^4 k^4} [4\alpha^3 k^3 \csc\left(\frac{4\pi}{\alpha k}\right) - 3\pi^3 \csc^4\left(\frac{\pi}{\alpha k}\right) \\
&- 12\pi \alpha^2 k^2 \csc\left(\frac{3\pi}{\alpha k}\right) \csc\left(\frac{\pi}{\alpha k}\right) + 6\pi^2 \alpha k \csc^3\left(\frac{\pi}{\alpha k}\right) \sec\left(\frac{\pi}{\alpha k}\right)],
\end{aligned}$$

respectively, which will be used to determine coefficients of skewness, kurtosis and variation as following

$$\beta_1 = -\frac{\left(3\alpha^2k^2\csc\left(\frac{3\pi}{\alpha k}\right)+2\pi^2\csc^3\left(\frac{\pi}{\alpha k}\right)-6\pi\alpha k\csc\left(\frac{2\pi}{\alpha k}\right)\csc\left(\frac{\pi}{\alpha k}\right)\right)^2}{\pi\left(\pi\csc^2\left(\frac{\pi}{\alpha k}\right)-2\alpha k\csc\left(\frac{2\pi}{\alpha k}\right)\right)^3},$$

$$\beta_2 = \frac{4\alpha^3k^3\csc\left(\frac{4\pi}{\alpha k}\right)-12\pi\alpha^2k^2\csc\left(\frac{3\pi}{\alpha k}\right)\csc\left(\frac{\pi}{\alpha k}\right)-3\pi^3\csc^4\left(\frac{\pi}{\alpha k}\right)+6\pi^2\alpha k\csc^3\left(\frac{\pi}{\alpha k}\right)\sec\left(\frac{\pi}{\alpha k}\right)}{\pi\left(\pi\csc^2\left(\frac{\pi}{\alpha k}\right)-2\alpha k\csc\left(\frac{2\pi}{\alpha k}\right)\right)^2},$$

$$CV = \sin\left(\frac{\pi}{\alpha k}\right)\sqrt{\frac{2\alpha k\csc\left(\frac{2\pi}{\alpha k}\right)}{\pi}-\csc^2\left(\frac{\pi}{\alpha k}\right)}\times 100,$$

respectively.

3.3 Moment generating function

The moment generating and characteristic functions of FWMED are given by

$$M(t)=\frac{\pi}{\alpha k}\sum_{m=0}^{\infty}\frac{m(\lambda t)^m a^{-\frac{m}{\alpha k}}\csc\left(\frac{\pi m}{\alpha k}\right)}{m!},\ \phi(t)=\frac{\pi}{\alpha k}\sum_{m=0}^{\infty}\frac{m(\lambda it)^m a^{-\frac{m}{\alpha k}}\csc\left(\frac{\pi m}{\alpha k}\right)}{m!},$$

respectively.

3.4 Mean residual life function

The mean residual life function of a continuous random variable X follows FWMED is given by

$$\mu(x) = \left(a\left(\frac{x}{\lambda}\right)^{\alpha k}+1\right)\left(\frac{\pi\lambda a^{-\frac{1}{\alpha k}}\csc\left(\frac{\pi}{\alpha k}\right)}{\alpha k}-x\,{}_2F_1\left(1,\frac{1}{k\alpha};1+\frac{1}{k\alpha};-a\left(\frac{x}{\lambda}\right)^{k\alpha}\right)\right),$$

where ${}_2F_1\left(1,\frac{1}{k\alpha};1+\frac{1}{k\alpha};-a\left(\frac{x}{\lambda}\right)^{k\alpha}\right)$ is hyper geometric function.

3.5 Entropy

Entropy describes the amount of uncertainty related with a random variable. It determines the randomness found in a probability distribution and it is a basic concept in information theory and cryptography.

Renyi, Tsallis and Shannon entropies for FWMED are given by

$$R_r(X) = \frac{1}{1-r}\log\int_{x=0}^{\infty}f^r(x)dx=\frac{1}{1-r}\log\left(\frac{\left(\lambda^{1-r}(\alpha k)^r\right)B\left(\frac{1-r}{k\alpha}+r,\frac{r-1}{k\alpha}+r\right)}{a^{\frac{1}{\alpha k}-\frac{r}{\alpha k}}}\right), r>0, r\neq 1,$$

$$T_r(X) = -R_r(X)-1,$$

$$S_H(X) = -\int_{x=0}^{\infty}f(x)\log f(x)dx = 2-\log\left(\frac{\alpha k a^{\frac{1}{\alpha k}}}{\lambda}\right),$$

respectively, where $B\left(\frac{1-r}{k\alpha}+r, \frac{r-1}{k\alpha}+r\right)$ is beta function.

3.6 Lorenz and Bonferroni curves

In financial aspects to gauge the pay and poverty level, Bonferroni and Lorenz curves are frequently used. These two have great linkup to one another and have applications in demography. The Lorenz and Bonferroni curves for FWMED are defined by the following relations

$$\begin{aligned} L_X(p) &= \frac{1}{\mu}\int_0^{x_p} xf(x)dx = \\ &= \frac{\left(\pi a^{1-\frac{1}{\alpha k}}\lambda\csc\left(\frac{\pi}{\alpha k}\right) - \frac{\alpha k\lambda^{\alpha k}x_p^{1-\alpha k}\left(\left(a+\left(\frac{\lambda}{x_p}\right)^{\alpha k}\right){}_2F_1\left(1,1-\frac{1}{k\alpha};2-\frac{1}{k\alpha};-\frac{\left(\frac{\lambda}{x_p}\right)^{k\alpha}}{a}\right)+a(\alpha k-1)\right)}{(\alpha k-1)\left(a+\left(\frac{\lambda}{x_p}\right)^{\alpha k}\right)}\right)}{\mu\alpha k a}, \\ B_X(p) &= \frac{1}{\mu F(x)}\int_0^{x_p} xf(x)dx = \frac{L_X(p)}{F(x)}, \end{aligned}$$

respectively, where ${}_2F_1\left(1,1-\frac{1}{k\alpha};2-\frac{1}{k\alpha};-\frac{\left(\frac{\lambda}{x_p}\right)^{k\alpha}}{a}\right)$ and x_p are hyper geometric function and the quantile function, respectively.

4 Order statistics

4.1 Probability and cumulative functions

Let $X_1, X_2, \ldots, X_n$ is a random sample from FWMED. Let $X_{1:n} < X_{2:n} < \ldots < X_{n:n}$ denote the corresponding order statistics. The PDF and the CDF of the ith-order statistic of FWMED are given by

$$\begin{aligned} f_{i:n}(x) &= \lim_{\delta x\to 0}\left(\frac{P(x < X_{i:n} \leqslant x+\delta x)}{\delta x}\right) = \frac{n!}{(i-1)!(n-i)!}F(x)^{i-1}(1-F(x))^{n-i}f(x) \\ &= \frac{a\alpha k n!\lambda^{\alpha k}x^{-\alpha k-1}\left(\frac{a}{a+\left(\frac{x}{\lambda}\right)^{-\alpha k}}\right)^{i-1}\left(\frac{1}{a\left(\frac{x}{\lambda}\right)^{\alpha k}+1}\right)^{n-i}}{(i-1)!(n-i)!\left(a+\left(\frac{\lambda}{x}\right)^{\alpha k}\right)^2}, \\ F_{i:n}(x) &= P(X_{i:n}\leqslant x) = \sum_{r=i}^{n}\binom{n}{r}F(x)^r(1-F(x))^{n-r} \\ &= \binom{n}{i}\left(\frac{a}{a+\left(\frac{x}{\lambda}\right)^{-\alpha k}}\right)^{i}\left(\frac{1}{a\left(\frac{x}{\lambda}\right)^{\alpha k}+1}\right)^{n-i}{}_2F_1\left(1,i-n;i+1;-a\left(\frac{x}{\lambda}\right)^{k\alpha}\right), \end{aligned}$$

respectively, where ${}_2F_1\left(1, i-n; i+1; -a\left(\frac{x}{\lambda}\right)^{k\alpha}\right)$ is hyper geometric function. By setting $i = 1$ and $i = n$, we obtain the minimum and the maximum of FWMED order statistics, respectively.

4.2 Limiting distribution for maximum order statistics

Suppose that $Z_n = X_{n:n} = max(X_1, X_2, ..., X_n)$ from FWMED and the limiting distribution of Z_n can be obtained as following (for more details see theorem 2.1.1 in [1])

$$\lim_{t\to+\infty} \frac{1-F(tx)}{1-F(t)} = x^{-\alpha k}, \lim_{n\to+\infty} P(Z_n < a_n + b_n x) = \exp(-x^{-\alpha k}),$$

the normalizing constants are $a_n = 0$ and $b_n = F^{-1}(1-\frac{1}{n}) = \lambda\left(\frac{a}{n-1}\right)^{-\frac{1}{\alpha k}}$.

4.3 Limiting distribution for minimum order statistics

Suppose that $W_n = X_{1:n} = min(X_1, X_2, ..., X_n)$ from FWMED and the limiting distribution of W_n can be obtained as following (for more details see theorem 2.1.5 in [1])

$$\lim_{t\to-\infty} \frac{F(\frac{-1}{tx})}{F(\frac{-1}{t})} = x^{-\alpha k}, \lim_{n\to+\infty} P(W_n < c_n + d_n x) = 1-\exp(-x^{\alpha k}),$$

the normalizing constants are $c_n = 0$ and $d_n = F^{-1}(\frac{1}{n}) = \lambda(a(n-1))^{-\frac{1}{\alpha k}}$.

5 Parameters estimation

If $X = (X_1, X_2, ..., X_n)$ is independent random sample having PDF of FWMED, then the likelihood and log-likelihood functions are given by

$$\begin{aligned} L(x) &= \prod_{i=1}^{n} f(x_i) = (a\alpha k\lambda^{\alpha k})^n \prod_{i=1}^{n} \frac{x_i^{-\alpha k-1}}{\left(a+\left(\frac{\lambda}{x_i}\right)^{\alpha k}\right)^2}, \\ \log L(x) &= n\log(a\alpha k\lambda^{\alpha k}) - (\alpha k+1)\sum_{i=1}^{n}\log x_i - 2\sum_{i=1}^{n}\log\left(a+\left(\frac{\lambda}{x_i}\right)^{\alpha k}\right), \end{aligned}$$

respectively. By differentiating the last equation with respect to the four parameters and trying to solve them for getting the estimated parameters $\hat{\alpha}$, $\hat{a}$, $\hat{\lambda}$ and $\hat{k}$ in explicit form is complicated, so these estimates will be obtained numerically.

6 Applications

6.1 Randomly generated data

A number of hundred random samples were generated for each sample size n = 50, 150, 300 and 450 by using equation (2) as $X = \lambda(\frac{a}{u} - a)^{-\frac{1}{\alpha k}}$ with parameters $\alpha = 2$, $a = 0.5$, $\lambda = 1$ and $k = 1.5$, where u is uniformly distributed. Table 1 shows the estimates, biases and mean squared errors (MSEs) of the parameters for each sample size. It is easy to notice that estimates are close to their actual values with small enough MSE. Also MSE gets smaller as the sample size increases for each parameter.

Table 1: Estimates, biases and mean squared errors of $\hat{\alpha}$, $\hat{a}$, $\hat{k}$ and $\hat{\lambda}$

n	Estimates	Bias	MSE	n	Estimates	Bias	MSE
50	$\hat{\alpha} = 2.402642$	0.402642	0.162121	150	$\hat{\alpha} = 2.292757$	0.2927574	0.0857069
	$\hat{a} = 0.2450939$	0.254906	0.064977		$\hat{a} = 0.327579$	−0.172420	0.0297287
	$\hat{k} = 1.326591$	0.173409	0.030071		$\hat{k} = 1.348106$	−0.151894	0.0230718
	$\hat{\lambda} = 0.75913$	0.24087	0.058018		$\hat{\lambda} = 0.852416$	−0.147584	0.0217811
300	$\hat{\alpha} = 2.224214$	0.224214	0.050272	450	$\hat{\alpha} = 2.181179$	0.1811792	0.03282589
	$\hat{a} = 0.3702232$	−0.1297768	0.016842		$\hat{a} = 0.383708$	−0.116291	0.01352365
	$\hat{k} = 1.377781$	−0.1222193	0.014937		$\hat{k} = 1.395675$	−0.104325	0.01088378
	$\hat{\lambda} = 0.8908138$	−0.1091862	0.011921		$\hat{\lambda} = 0.905803$	−0.0941967	0.00887302

6.2 Real-world data

In this sub section we introduced a practical examples to compare the flexibility of FWMED with Fréchet-Weibull distribution (FWD), exponential distribution and Weibull distribution. In order to compare these distributions, we calculated the Akaikes information criterion(AIC), the Bayesian information criterion(BIC), Hannan Quinn information criterion(HQIC), Kolmogorov Smirnov (K-S), Anderson and Darling (A_n^2) and Cramér-Von Mises (W^2). The model with minimum of these statistics values is chosen as the best model to fit the data. The parameters are estimated by using the maximization of the log-likelihood function, the calculations are performed by using the method of Nelder-Mead which is a numerical method used to find the global maximum of an objective function in a multidimensional space and we used Wolfram Mathematica software version 10 to apply this method.

6.2.1 Australian athletes data set

These data were collected in a study of how data on various characteristics of the blood varied with sport body size and sex of the athlete. A data consists of 202 observations on 13 variables (for more details see [13]). we studied the variables rcc (red blood cell count), wcc (white blood cell count) and bmi (body mass index).

Table 2 gives the descriptive statistics, table 3 gives the log-likelihood function, AIC, BIC, HQIC, K-S, A_n^2 and W^2 values, and finally, table 4 gives the maximum likelihood estimates values for each parameter of all compared distributions for the three variables, respectively.

Table 2: Descriptive statistics for athletes data sets

Data	Min	Max	Mean	Variance	Q_1	Q_2	Q_3	Skewness	Kurtosis
rcc	3.8	6.72	4.71861	0.209742	4.37	4.755	5.03	0.416005	3.66295
wcc	3.3	14.3	7.10891	3.24121	5.9	6.85	8.3	0.835234	4.44946
bmi	16.75	34.42	22.9559	8.20211	21.07	22.72	24.47	0.946516	5.18347

Table 3: Log L, AIC, BIC, HQIC, K-S, A_n^2 and W^2 for athletes data sets

red blood cell count data set							
Distribution	Log L	AIC	BIC	HQIC	K-S	A_n^2	W^2
FWMED	−130.535	269.07	282.303	274.424	0.0803007	1.86532	0.325593
FWD	−134.304	276.608	289.841	281.962	0.118276	2.95289	0.498792
Weibull	−148.667	301.333	307.95	304.01	0.0911863	2.86258	0.300524
exponential	−515.406	1032.81	1036.12	1034.15	0.557477	76.1131	16.4875
white blood cell count data set							
Distribution	Log L	AIC	BIC	HQIC	K-S	A_n^2	W^2
FWMED	−396.124	800.247	813.48	805.601	0.040197	0.312482	0.0445505
FWD	−412.801	833.603	846.836	838.957	0.104843	3.2393	0.490626
Weibull	−412.589	829.177	835.794	831.855	0.0997345	2.76539	0.40285
exponential	−598.193	1198.39	1201.69	1199.72	0.424154	53.8674	11.2885
body mass index data set							
Distribution	Log L	AIC	BIC	HQIC	K-S	A_n^2	W^2
FWMED	−487.565	983.13	996.363	988.484	0.0285568	0.188921	0.0173053
FWD	−499.024	1006.05	1019.28	1011.4	0.0757206	2.41841	0.370406
Weibull	−524.521	1053.04	1059.66	1055.72	0.112653	6.64338	0.988893
exponential	−834.982	1671.96	1675.27	1673.3	0.523864	72.7248	15.7116

Table 4: Estimated parameters for athletes data sets

Distribution	rcc data set	wcc data set	bmi data set
FWMED	$\hat{\alpha} = 2.73086$	$\hat{\alpha} = 1.86063$	$\hat{\alpha} = 2.51816$
	$\hat{a} = 0.763751$	$\hat{a} = 0.594879$	$\hat{a} = 0.0232658$
	$\hat{\lambda} = 4.62999$	$\hat{\lambda} = 6.41519$	$\hat{\lambda} = 17.7075$
	$\hat{k} = 6.48436$	$\hat{k} = 3.81849$	$\hat{k} = 5.99023$
FWD	$\hat{\alpha} = 2.47878$	$\hat{\alpha} = 1.26916$	$\hat{\alpha} = 1.6039$
	$\hat{w} = 3.98424$	$\hat{w} = 3.37751$	$\hat{w} = 8.96207$
	$\hat{\lambda} = 3.95807$	$\hat{\lambda} = 4.48735$	$\hat{\lambda} = 16.7165$
	$\hat{k} = 4.51645$	$\hat{k} = 3.14967$	$\hat{k} = 5.44161$
Weibull	$\hat{\alpha} = 9.79481$	$\hat{\alpha} = 3.98623$	$\hat{\alpha} = 7.28066$
	$\hat{\beta} = 4.92816$	$\hat{\beta} = 7.80407$	$\hat{\beta} = 24.2586$
exponential	$\hat{a} = 0.211927$	$\hat{a} = 0.140669$	$\hat{a} = 0.0435618$

Table 3 shows that our proposed model behaves best for fitting each data set compared with FWD, Weibull distribution and exponential distribution.

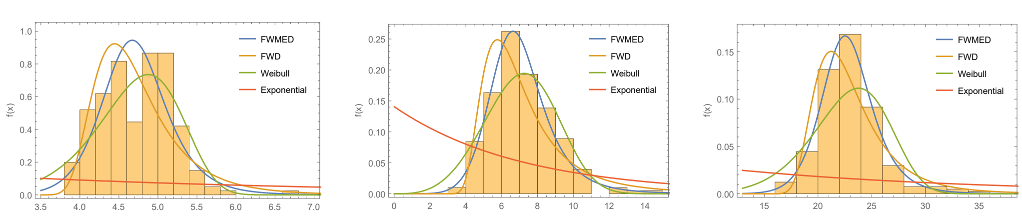


Figure 5: Histogram of data sets compared to the fitted PDFs

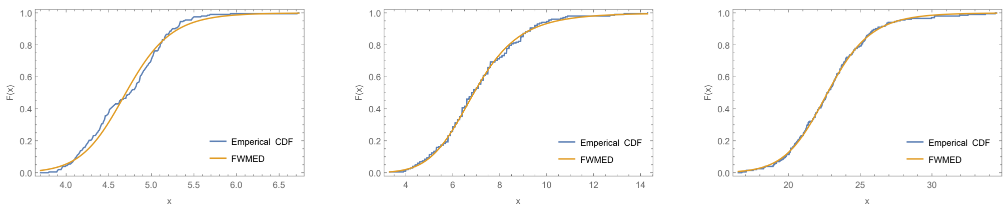


Figure 6: Empirical CDF of data sets and the fitted CDF of FWMED

Figure 5 shows that the proposed model has the superiority to fit the used data sets than other compared models, while figure 6 shows how the suitability of data for proposed model.

7 Conclusion

This paper introduced a new mixture distribution called FWMED along with some of its statistical properties. Estimation of unknown parameters was performed numerically on randomly generated data by maximum likelihood method. Four real-world data sets were applied for confirming the superiority of our model for fitting these kind of data sets than our compared models. Finally, We hope that the proposed model will draw in more extensive applications in different areas, for example, survival and lifetime data, economics (income inequality) and others.

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