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Controlling *Aedes Aegypti* Mosquitoes by Using Ovitraps: A Mathematical Model

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Abstract

We have proposed and analyzed a mathematical model for the population growth dynamics of the *Aedes aegypti*, including control by applying ovitraps. This model is based on non-linear ordinary differential equations. The local stability analysis shows that the equilibrium points without and with mosquitoes are asymptotically stable, with restrictions for the control. Also, we have done a local sensitivity analysis of the mosquito population growth threshold and the equilibrium point with mosquitoes as function of each parameter. The dynamic system is assessed using the Maple software.

Keywords: *Aedes aegypti*, Ovitraps, Mathematical model, local sensitivity, local stability, population growth threshold

1 Introduction

It is well known that Dengue is a virus transmitted by vectors, principally by the *Aedes aegypti* ones. After a person gets infected the common symptomatology is: fever, muscle ache, nausea, and purple bruises. Sometimes, infection caused by Dengue may cause death [10].

According to World Health Organization (WHO) data, it is reported at least 50 million cases each year, with 24000 deaths because of the infection [3, 8]. As well, it is estimated that around 3000 million people live in places where Dengue is endemic [6, 8, 7]. The high infection and mortality rates generate problems to the public health as well as to the economy. Therefore, it is necessary to know the mosquito dispersion to propose control mechanisms against them.

The *Aedes aegypti* population's sustained control by implementing sanitation and environmental campaigns as well as a gradual employment of insecticide are some of the applied measures which are available nowadays.

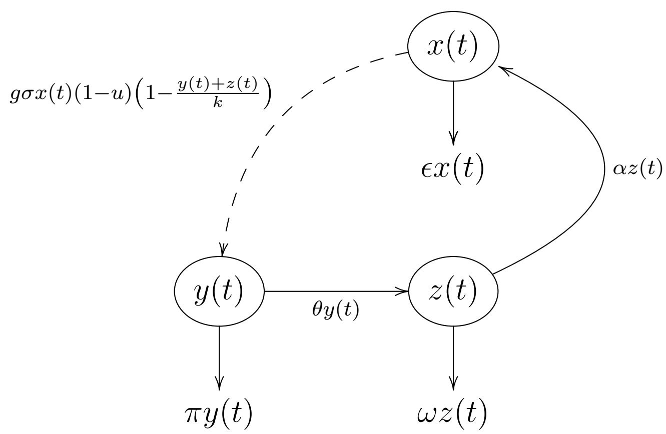
However, not all the breeding places can be eliminated at all neither the insecticides are 100% effective. These facts make necessary the proposal of new control actions that of course imply low costs and also laying the groundwork to develop new programs that involve the community. In this sense, ovitraps development may be in practice a good technique for vector control.

Ovitraps were constructed for the first time in United States (1965) as vigilance tools [8], since then, these ones have been used around the world [11]. In Brasil, the ovitraps use is a strategy to study the mosquito dispersion [7]. In Peru, the National Health Institute is developing studies to implement this quickly detection strategy against to oviposition [2].

In the first section we have done a short introduction about Dengue and previous realized studies. In the second section, it is treated the description of the model, the variables and parameters and the local stability analysis respecting to the population growth threshold of the mosquito. Finally, in the third section we discuss the results and conclusions.

2 Model

We have formulated a model to treat the *Aedes aegypti* mosquito population growth, including the eggs and pupae of the aquatic stage, constant control using ovitraps, and a regulatory function to the immature stages.



The integral equations system is,

$$x(t) = x(0) + \alpha \int_0^t z(\tau)d\tau - \epsilon \int_0^t x(\tau)d\tau \tag{1}$$

$$y(t) = \int_0^t x(\tau)\left(1 - \frac{y(\tau)+z(\tau)}{k}\right)d\tau - (\theta+\pi)\int_0^t y(\tau)d\tau \tag{2}$$

$$z(t) = z(0) + \theta \int_0^t y(\tau)d\tau - (\omega+\alpha)\int_0^t z(\tau)d\tau \tag{3}$$

where $\Omega = y(0) + g\sigma(1-u)$ and their associated ordinary differential equations system is,

$$\frac{dx(t)}{dt} = \alpha z(t) - \epsilon x(t) \tag{4}$$

$$\frac{dy(t)}{dt} = g\sigma(1-u)x(t)\left(1 - \frac{y(t)+z(t)}{k}\right) - (\theta+\pi)y(t) \tag{5}$$

$$\frac{dz(t)}{dt} = \theta y(t) - (\omega+\alpha)z(t) \tag{6}$$

In this system all the parameters are positive, and the initial conditions $x(0) = x_0$, $y(0) = y_0$ and $z(0) = z_0$, with σ the oviposition rate, θ the developing rate from egg to larva, α is the developing rate from pupa to adult stage, π is the eggs' unviability rate, ϵ the death rate of the female mosquitoes, ω the death rate of the pupae, g the eggs fraction that turns into female mosquitoes, k charge capacity of the breeding places, u the economic cost to diminish the breeding places, $x(t)$ average number of female mosquitoes at time t, $y(t)$ average number of eggs at time t, and $z(t)$ the average number of pupae at time t.

2.1 Stability analysis

In equations (4) - (6) we make $\frac{dx}{dt} = 0$, $\frac{dy}{dt} = 0$ and $\frac{dz}{dt} = 0$, to have the expression $\alpha z = \epsilon x$, or $x = \frac{\alpha z}{\epsilon}$ and $y = \frac{(\omega+\alpha)}{\theta}z$. Therefore, from that equation we have $x = 0$, and the first equilibrium point is $E_0(0,0,0)$. On the other hand, if $x \neq 0$ and given that $y = \frac{(\omega+\alpha)}{\theta}z$ we obtain,

$$(1-u)g\sigma(1 - z\frac{\omega+\alpha+\theta}{\theta k}) = \frac{\epsilon(\theta+\pi)(\omega+\alpha)}{\theta\alpha}.$$

For $\varphi(u) = \frac{\theta\alpha\sigma(1-u)g}{\epsilon(\theta+\pi)(\omega+\alpha)}$ it is obtained

$$\hat{x} = \frac{\alpha\theta k(\varphi(u)-1)}{\epsilon\varphi(u)(\alpha+\omega+\theta)},\ \hat{y} = \frac{k(\alpha+\omega)(\varphi(u)-1)}{\varphi(u)(\alpha+\omega+\theta)},\ \hat{z} = \frac{\theta k(\varphi(u)-1)}{\varphi(u)(\alpha+\omega+\theta)}$$

After that, the other equilibrium point is $E_1(\hat{x},\hat{y},\hat{z})$.

Now we carry out the analysis for E_0

$$J_0 = \begin{pmatrix} -\epsilon & 0 & \alpha \\ (1-u)g\sigma & -(\theta+\pi) & 0 \\ 0 & \theta & -(\alpha+\omega) \end{pmatrix}$$

and their associated characteristic equation is,

$$\lambda^3+(\omega+\alpha+\theta+\pi+\epsilon)\lambda^2+[(\epsilon+\theta+\pi)(\omega+\alpha)+\epsilon(\theta+\pi)]\lambda+(1-u)g\sigma\theta(\frac{1}{\varphi(u)}-1) = 0.$$

Theorem 2.1 *If $\varphi(u) < 1$ or $u > 1 - \frac{\epsilon(\theta+\pi)(\omega+\alpha)}{\theta\alpha\sigma g}$ then, the equilibrium point E_0 of the system (4) - (6) is local and asymptotically stable.*

By the Descartes signs rule, all the roots of the characteristic polynomial are negative if all the polynomial coefficients are positive, and to make it happen, it is necessary to accomplish that $\varphi(u) < 1$ and so, the equilibrium point E_0 is local and asymptotically stable.

Analysis for E_1

$$J_1 = \begin{pmatrix} -\epsilon & 0 & \alpha \\ (1-u)g\sigma(1-\frac{\hat{y}+\hat{z}}{k}) & -(\theta+\pi+\frac{(1-u)g\sigma\hat{x}}{k}) & \frac{-(1-u)g\sigma\hat{x}}{k} \\ 0 & \theta & -(\alpha+\omega) \end{pmatrix}$$

and the associated characteristic polynomial for J_1 is,

$$\lambda^3 + (A+C+\epsilon)\lambda^2 + (AC+\epsilon C+\epsilon A+\theta B) + \epsilon AC + \epsilon\theta B - \frac{(1-u)g\sigma\alpha\theta}{\varphi(u)} = 0$$

where $A = \theta + \pi + B$, $B = \frac{(1-u)g\sigma\hat{x}}{k}$, $C = \alpha + \theta$ y $D = \frac{(1-u)g\sigma}{\varphi(u)}$.

Since we can rewrite the independent term in the following form,

$$\epsilon B(\alpha + \omega) + \frac{(1-u)g\sigma\theta\hat{x}}{k},$$

We conclude that all the polynomial roots are negative, then, E_1 is a local and asymptotically stable equilibrium point. With this result we have formulated the following theorem:

Theorem 2.2 *If $\varphi(u) > 1$ or $u < 1 - \frac{\epsilon(\theta+\pi)(\omega+\alpha)}{\theta\alpha\sigma g}$ then the equilibrium point E_1 of the system (4) - (6) is local and asymptotically stable.*

3 Results and conclusions

The local sensitivity analysis and all simulations were realized using values for the parameters as reported previously in the literature, see Table 1. The sensitivity indices allow us to measure the relative change in a variable when a parameter changes [5]. The sensitivity coefficient is defined as the variation of a parameter while all the remaining ones stay constant [4].

$$I_p^{\hat{x}_i} = \frac{\partial \hat{x}_i}{\partial p} \times \frac{p}{\hat{x}_i}$$

where, $\hat{x}_i = \hat{x}, \hat{y}, \hat{z}, \varphi$. It is observed that the populations $\hat{x}$, $\hat{y}$ and $\hat{z}$ are sensitive in an inversely proportional way to ϵ , θ and α for $u = 0.3$, respectively. For the case when $u = 0.85$, the populations $\hat{x}$, $\hat{y}$ and $\hat{z}$ are sensitive in an inversely proportional way to ϵ and α, respectively. In both cases, they are proportional to the charge capacity k. The population growth threshold φ is sensitive in an inversely proportional way to ϵ and u, and proportional to σ and g for both values 0.3 and 0.85.

In Figure 1 when the control over ovitraps is $u = 0.3$, the female mosquitoes (dash-dot line) and eggs population show a decrease compared with the control-free population behavior (solid line). The population decrement is improved if the ovitraps control has a value of $u = 0.85$, as shown in the dashed line of the graphic. For the case of Figure 2, when ovitraps control is $u = 0.3$, the graphic (dash-dot line) corresponding to pupae shows a decrease if compared with the control-free population behavior (solid line). If $u = 0.85$ the control is improved as seen in the dashed line of the graphic.

Figure 3 depicts the time dependent behavior of the mosquito population as function of the population growth threshold (φ) of the equilibrium point (E_1),

Table 1: Parameters and sensitivity analysis for $u = 0.3; 0.85$.

Parameter	Value	$I_p^{\hat{x}}$	$I_p^{\hat{y}}$	$I_p^{\hat{z}}$	$I_p^{\hat{\varphi}}$
θ	0.65	0.5 - 0.6	-0.5 - (-0.5)	0.5 - (0.5)	0.1 - 0.1
α	0 .5	0.6 - 0.8	0.5 - (0.6)	-0.2 - (-0.4)	0.02 - 0.02
σ	4	0.1 - 0.1	0.07 - 0.04	0.07 - 0.4	1 - 1
g	0.4	0.1 - 0.1	0.07 - 0.04	0.07 - 0.4	1 - 1
u	0.3; 0.9	-0.03-0.1	-0.03 - (-0.1)	-0.03 - (-0.03)	-0.4 - (-5.8)
ϵ	0.1687	-1.7 - (-1.4)	-0.07 - (-0.04)	-0.02 - (-0.04)	-1 - (-1)
π	0.1	-0.01 - (-0.01)	-0.01 - (-0.06)	0.002 - (-0.1)	-0.1 - (-0.1)
ω	0.01005	-0.01 - (-0.01)	0.01 - (-0.003)	0.01 - (-0.02)	-0.02 - (-0.02)
k	1000	1 - 1.3	1 - 1	1 - 1	0 - 0

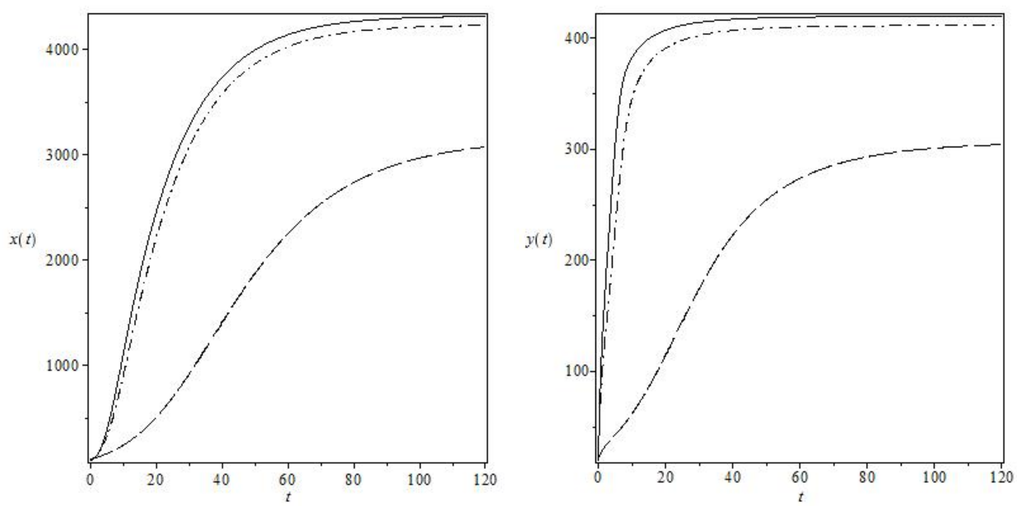


Figure 1: Behavior of the average number of female mosquitoes $x(t)$: and average number of eggs, $y(t)$: without (left part of the graphic) and with (right part of the graphic) control.

where it is possible to observe stability for small values of threshold.

For $\varphi(0.3) = 15.38$ the equilibrium point E_1 has components, $\hat{x} = 2619.4$, $\hat{y} = 439.68$ and $\hat{z} = 523.9$, according with the results observed in the left part of Figure 3. The corresponding eigenvalues ($\lambda_1 = -2.78, \lambda_2 = -1.498, \lambda_3 = -0.0065$) are negative; hence, the equilibrium point is local and asymptotically stable, in agreement with theorem 2.2, Figure 1, and Figure 2. In the case, $\varphi(0.85) = 3.296$, the equilibrium point is $E_1(1963.7, 263.47, 302.1)$ and the eigenvalues $\lambda_1 = -0.8854 + 0.4604i$, $\lambda_2 = -0.8854 - 0.4604i$ and $\lambda_3 = -0.059$,

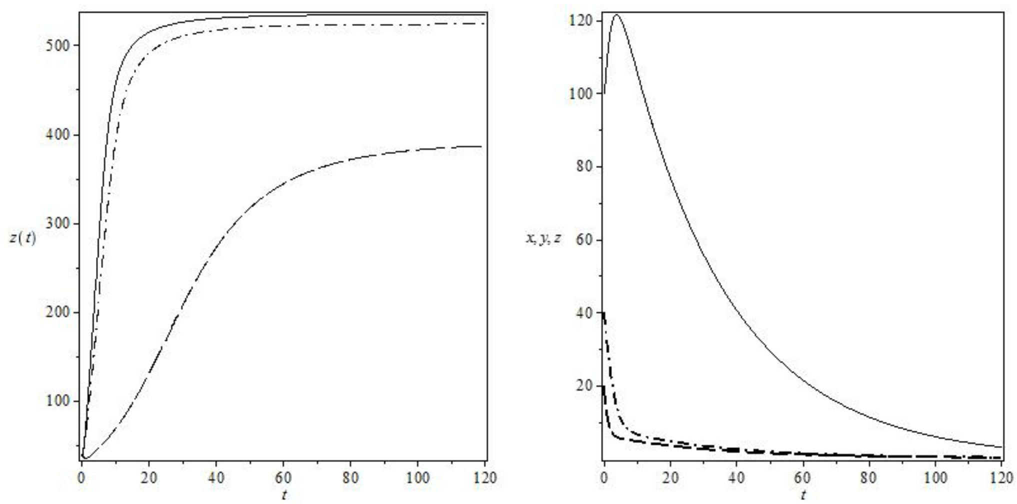


Figure 2: Behavior of the average number of pupas $z(t)$: without (left part of the graphic) and with (right part of the graphic) control.

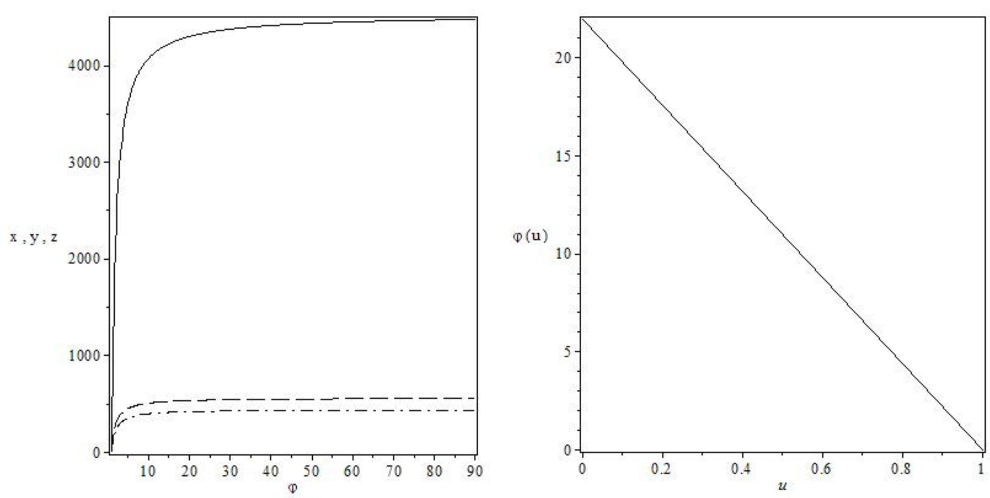


Figure 3: Behavior of the mosquito trajectories in terms of the threshold and threshold in terms of the control.

have negative real part, therefore, the equilibrium point is local and asymptotically stable, as shown in theorem 2.2, Figure 1, and Figure 2.

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