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Computational Modeling of Conjugated Aerodynamic and Thermomechanical Processes in Composite Structures of High-speed Aircraft

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Abstract

A conjugated problem statement for aerogasdynamics, internal heat-and-mass transfer and thermal strength of heat shield structures of hypersonic vehicles is formulated. A method for numerical solving of the problem is suggested which is based on the iterative solution of the three types of detached problems: a gasdynamics problem for viscous heat-conducting flows, internal heat-and-mass transfer and thermoelasticity of shell constructions. An example of the numerical solution of the conjugated problem is given. It is shown that due to the high temperatures of the aerodynamic heating of the structure made of a polymer composite material there can appear a polymer phase thermodecomposition and intensive internal gas generation into the structure of the material.

Keywords: conjugated simulation, computational fluid dynamics, aerothermodynamics, hypersonic flows, thermomechanics, polymer composites, thermodecomposition structures, heat shield

1 Introduction

The learning of hypersonic speeds is one of the most promising complex problems of the high-tech development. This problem can be identified such components as: the study of hypersonic aerodynamics of flights, study of the heat transfer on surfaces of aircraft constructions, thermophysics research of constructional materials, thermal strength study, analysis and development of constructional materials for hypersonic aircrafts, and the problems of hypersonic aeroelasticity, control and others. Significant amount of work (eg [1]) described researches of hypersonic aerodynamics conditions, less studied problems of heat transfer [2] under hypersonic speeds. The more difficult problem is the high–temperature behavior of composite materials based on heat–resistant filaments and masters [5]. The complex conjugated problem of aerothermodynamics, heat transfer, thermal physics and thermal strength of hypersonic constructions is still practically uninvestigated, and there are relatively recent work [3, 4] that studied aeroelasticity of constructions under hypersonic speeds. However under the actual operating conditions of hypersonic vehicles the problems of aerothermodynamics, heat transfer and thermophysics of the constructions are conjugated through the boundary conditions on the surface of the constructions, so the parameters of the heat flux acting on the materials depend on the properties of these materials. By turn, thermal properties of the materials at high temperatures may depend on the mode of deformation of the constructions. So the significant level of thermal stresses in composite materials leads to the filament microcracking long before the full macrodestruction of constructions, thereby the gas permeability and thermal conductivity materials and temperature fields in the constructions are changed. Thus the development of methods for solving the conjugated problem for the study of real processes taking place in the constructions of hypersonic vehicles is necessary.

The general formulation of the conjugated problem of aerothermodynamics and thermomechanics consists of the three systems of equations: the Navier-Stokes equations of an external gas flow, the internal heat-and-mass transfer equations, and the equations of thermoelasticity of a shell.

2 System of Gasdynamics Equations

The system of equations of a viscous heat–conducting compressible gas consists of the continuity equation, momentum equations and energy equation [1, 6].

The boundary conditions on the solid surface, which is the interface of the gas and solid domains, are as follows

$$\mathbf{v} = \mathbf{0}, \quad -\lambda \nabla\theta \cdot \mathbf{n} + \varepsilon_{\mathrm{g}}\sigma\theta_{\max}^4 = -\lambda_{\mathrm{s}}\nabla\theta_{\mathrm{s}} \cdot \mathbf{n} + \varepsilon_{\mathrm{s}}\sigma\theta_{\mathrm{s}}^4, \quad \theta_{\mathrm{s}} = \theta,$$

where $\mathbf{v}$ is the velocity vector, θ is the gas temperature, θ_{s} is the temperature of the solid surface, $\theta_{\max}$ is the maximum temperature into the boundary layer, $\nabla\theta_{\mathrm{s}}$ is the temperature gradient on the solid wall from the construction, ε and ε_{s} are the emissivity of the heated gas and solid surface respectively and σ is the Stefan–Boltzmann constant.

3 System of Equations of Internal Heat-and-Mass Transfer

We consider the typical element which is made of a heat–resistant composite material consisting of a polymer master with heat-resistant filaments. There are physical and chemical processes of thermodecomposition in such composite master under high temperatures of aerodynamic heating. In these processes the gaseous products of thermal decomposition are generated, then they are accumulated in the pores of the material and filtered into the outer gas flow, as well as a new solid phase is formed. It is the phase of pyrolytic master which has significantly lower elastic-strength properties than the original polymer phase. The four-phase model to describe the internal heat-and-mass transfer and deformations of such composite is proposed in [5]. This model consists of the equation of change of mass of polymer master phase, the equation of filtration of gaseous products of thermodestruction in pores of the composite material and the heat transfer equation in the thermodestruction composite

$$\begin{gathered} \rho_{\mathrm{b}} \frac{\partial \varphi_{\mathrm{b}}}{\partial t} = -J; \\ \frac{\partial \rho_{\mathrm{g}} \varphi_{\mathrm{g}}}{\partial t} + \nabla \cdot \rho_{\mathrm{g}} \varphi_{\mathrm{g}} \mathbf{v} = J\Gamma; \\ \rho c \frac{\partial \theta}{\partial t} = -\nabla \cdot \mathbf{q} - c_{\mathrm{g}} \nabla \theta \cdot \rho_{\mathrm{g}} \varphi_{\mathrm{g}} \mathbf{v} - J \triangle e^0; \end{gathered} \tag{1}$$

where φ_{b}, φ_{g} are the volume concentrations of the phase of the initial polymer master and gas phase; ρ_{b} is the density of the phase of the initial polymer master which is assumed to be constant; ρ_{g} is the average pore density of the gas phase (variable); c_{g} is the specific heat of the gas phase at constant volume, ρ and c are the density and specific heat of the composite as a whole, $\mathbf{q}$ is the heat flux vector, θ is the composite temperature for all phases in common; $\mathbf{v}$ is the velocity vector of the gas phase in the pores; $\triangle e^0$ is the specific heat of the master thermodecomposition; J is the mass velocity of the master thermodecomposition and Γ is the gas–producing factor of the master.

The equations (1) are added the relations between the heat flux vector $\mathbf{q}$, velocity vector of the gas in pores $\mathbf{v}$ with the temperature gradient $\nabla\theta$ and pressure gradient ∇p using the Fourier and Darcy laws, as well as the Arrhenius relation for the mass velocity of the master thermodecomposition J and Mendeleev-Clapeyron equation for the pore pressure of the gas phase p

$$\mathbf{q} = -\mathbf{\Lambda} \cdot \nabla\theta, \qquad \rho_{\mathrm{g}}\varphi_{\mathrm{g}}\mathbf{v} = -\mathbf{K} \cdot \nabla p, \qquad J = J_0 e^{-E_A/R\theta}, \qquad p = \rho_{\mathrm{g}} \frac{R}{\mu_{\mathrm{g}}} \theta,$$

where J_0 is the pre–exponential factor, E_A is the activation energy of the thermodecomposition process, μ_{g} is the molecular weight of the gas phase.

And $\mathbf{\Lambda}$ is the thermal conductivity tensor, $\mathbf{K}$ is the permeability tensor of the composite. They depend on the phase concentration.

The conditions for the equations (1) on the heated surface of the construction are as follows:

$$p = p_e, \qquad \theta = \theta_e,$$

where p_e, θ_e are the pressure and temperature of the flow on the surface.

4 System of Equations of Thermoelasticity

The system of equations in curvilinear coordinates $Oq_1q_2q_3$ associated with the middle surface of an thermoelastic shell composite structure consists of [5]

– the equilibrium equations of the shell

$$\begin{gathered}
\frac{\partial A_\beta T_{\alpha\alpha}}{\partial q_\alpha} + \frac{\partial A_\alpha T_{\alpha\beta}}{\partial q_\beta} - \frac{\partial A_\beta}{\partial q_\alpha} T_{\beta\beta} + \frac{\partial A_\alpha}{\partial q_\beta} T_{\alpha\beta} + A_\beta \left(A_\alpha k_\alpha Q_\alpha - \frac{\partial P_\mathrm{g}}{\partial q_\alpha} \right) = 0, \\
\frac{\partial A_\beta M_{\alpha\alpha}}{\partial q_\alpha} + \frac{\partial A_\alpha M_{\alpha\beta}}{\partial q_\beta} - \frac{\partial A_\beta}{\partial q_\alpha} M_{\beta\beta} + \frac{\partial A_\alpha}{\partial q_\beta} M_{\alpha\beta} - A_\beta \left(A_\alpha Q_\alpha - \frac{\partial P_\mathrm{g}}{\partial q_\alpha} \right) = 0, \\
-A_1A_2(k_1T_{11} + k_2T_{22} + p_e) + \frac{\partial A_2 Q_1}{\partial q_1} + \frac{\partial A_1 Q_2}{\partial q_2} - (k_1 + k_2)A_1A_2\varphi_\mathrm{g} P_\mathrm{g} = 0;
\end{gathered} \tag{2}$$

– the kinematic relations

$$\begin{gathered}
e_{\alpha\alpha} = \frac{1}{A_\alpha}\frac{\partial U_\alpha}{\partial q_\alpha} + \frac{1}{A_1A_2}\frac{\partial A_\alpha}{\partial q_\beta} U_\beta + k_\alpha W, \quad 2e_{\alpha 3} = \frac{1}{A_\alpha}\frac{\partial W}{\partial q_\alpha} + \gamma_\alpha - k_\alpha U_\alpha, \\
2e_{12} = \frac{1}{A_2}\frac{\partial U_1}{\partial q_2} + \frac{1}{A_1}\frac{\partial U_2}{\partial q_1} - \frac{1}{A_1A_2}\left(\frac{\partial A_1}{\partial q_2} U_1 + \frac{\partial A_2}{\partial q_1} U_2 \right), \\
\kappa_{\alpha\alpha} = \frac{1}{A_\alpha}\frac{\partial \gamma_\alpha}{\partial q_\alpha} + \frac{1}{A_1A_2}\frac{\partial A_\alpha}{\partial q_\beta}\gamma_\beta, \qquad 2\kappa_{\alpha 3} = -k_\alpha \gamma_\alpha, \\
2\kappa_{12} = \frac{1}{A_2}\frac{\partial \gamma_1}{\partial q_2} + \frac{1}{A_1}\frac{\partial \gamma_2}{\partial q_1} - \frac{1}{A_1A_2}\left(\frac{\partial A_1}{\partial q_2}\gamma_1 + \frac{\partial A_2}{\partial q_1}\gamma_2 \right),
\end{gathered} \tag{3}$$

– the defining relationships of thermoelasticity shell

$$\begin{gathered}
T_{\alpha\alpha} = \sum_{\beta=1}^{2} (C_{\alpha\beta} e_{\beta\beta} + N_{\alpha\beta}\kappa_{\beta\beta}) - P_{\mathrm{g}\alpha} - \hat{T}_\alpha, \qquad T_{12} = 2(C_{66}e_{12} + N_{66}\kappa_{12}), \\
M_{\alpha\alpha} = \sum_{\beta=1}^{2} (N_{\alpha\beta} e_{\beta\beta} + D_{\alpha\beta}\kappa_{\beta\beta}) - M_{\mathrm{g}\alpha} - \hat{M}_\alpha, \\
M_{12} = 2(N_{66}e_{12} + D_{66}\kappa_{12}), \qquad Q_\alpha = \bar{C}_{\alpha+3,\alpha+3} e_{\alpha 3}, \quad \alpha = 1, 2;
\end{gathered} \tag{4}$$

where $T_{\alpha\alpha}$, $T_{\alpha\beta}$, $M_{\alpha\alpha}$, $M_{\alpha\beta}$ are the forces and moments in the shell; Q_α are the shear forces; $e_{\alpha\alpha}$, $e_{\alpha 3}, e_{12}$ are the deformations of the middle surface; $\kappa_{\alpha\alpha}$,

$\kappa_{\alpha 3},\ \kappa_{12}$ are the curvatures of the middle surface; $U_\alpha,\ \gamma_\alpha,\ W$ are the displacement, angles of curvature and deflection of the middle surface; $A_\alpha,\ k_\alpha$ are the parameters of the first quadratic form and principal curvatures of the middle surface and $P_{\mathrm{g}},\ M_{\mathrm{g}}$ are the forces and moments of the pore pressure, $\alpha, \beta = 1, 2; \quad \alpha \neq \beta$.

The forces and moments of the interphase interaction $P_{\mathrm{g}\alpha}$, $M_{\mathrm{g}\alpha}$ and thermal stresses $\hat{T}_\alpha$, $\hat{M}_\alpha$, depending on the thermal deformations $\hat{\varepsilon}_\alpha$ for the shell are

$$P_{\mathrm{g}\alpha} = \int\limits_{-h/2}^{h/2} p\tilde{f}_\alpha\, dq_3,\ M_{\mathrm{g}\alpha} = \int\limits_{-h/2}^{h/2} p\tilde{f}_\alpha q_3\, dq_3;\ \hat{T}_\alpha = \sum_{\beta=1}^{3} C_{\alpha\beta}\hat{\varepsilon}_\beta^{(0)},\ \hat{M}_\alpha = \sum_{\beta=1}^{3} C_{\alpha\beta}\hat{\varepsilon}_\beta^{(1)},$$

$$\hat{\varepsilon}_\beta^{(j)} = \int\limits_{-h/2}^{h/2} a_{\theta 1}\hat{\varepsilon}_\beta^{(0)} q_3^j\, dq_3, \qquad \hat{\varepsilon}_3^{(j)} = \int\limits_{-h/2}^{h/2} a_{\theta 2}\hat{\varepsilon}_3 q_3^j\, dq_3; \qquad j = 0, 1; \quad \beta = 1, 2;$$

$$\hat{\varepsilon}_\gamma = \left(\alpha_{\mathrm{f}}\varphi_{\mathrm{f}} B_\gamma + \alpha_{\mathrm{b}}\varphi_{\mathrm{b}}\Omega_\gamma\right)\left(\theta - \theta_0\right) + \alpha_{\mathrm{p}}\Omega_\gamma \int\limits_0^t \left(\theta\left(t\right) - \theta\left(\tau\right)\right)\hat{\varphi}_{\mathrm{p}}\, d\tau - \beta_{\mathrm{p}}\varphi_{\mathrm{p}}\Omega_\gamma,$$

where $\tilde{f}_\alpha$ are the coefficients of interphase interaction, $\alpha_{\mathrm{f}}, \alpha_{\mathrm{b}}, \alpha_{\mathrm{p}}$ are the coefficients of the thermal expansion of the filament, polymer and pyrolysis phases, β_{p} is the shrinkage ratio, B_γ, Ω_γ are the coefficients depending on the location of the filaments in the composite [5], $\gamma = 1, 2, 3$. Due to the softening of the polymer master and its thermodecomposition, stiffnesses of the shell are changing when heated. This change for orthotropic composite shells are allowed using 2 functions $a_{\theta 1}$, $a_{\theta 2}$ [5].

Deformations $\varepsilon_{\alpha\beta}$ and stresses $\sigma_{\alpha\beta}$ in the shell

$$\varepsilon_{\alpha\beta} = e_{\alpha\beta} + q_3\kappa_{\alpha\beta}; \quad \alpha, \beta = 1, 2; \quad \varepsilon_{33} = 0, \quad \varepsilon_{\alpha 3} = e_{\alpha 3},$$

$$\sigma_{\alpha\alpha} = -\tilde{f}_\alpha p + a_{\theta 1}\sum_{\beta=1}^{3} C_{\alpha\beta}\left(\varepsilon_{\beta\beta} + q_3\kappa_{\beta\beta} - \hat{\varepsilon}_\beta\right); \qquad \sigma_{12} = a_{\theta 1}C_{66}(\varepsilon_{12} + q_3\kappa_{12})\ .$$

The transverse normal stress σ_{33} has the following formulas

$$\sigma_{33} = 6\eta\left(\frac{p_1 + p_2}{2} - \frac{P_{\mathrm{g}1}}{h} + \frac{1}{h}C_{31}\left(a_{\theta 1}^{(0)}e_{11} + a_{\theta 1}^{(1)}\kappa_{11} - \hat{\varepsilon}_1^{(0)}\right) + \right.$$
$$\left. + \frac{1}{h}C_{32}^0\left(a_{\theta 1}^{(0)}e_{22} + a_{\theta 1}^{(1)}\kappa_{22} - \hat{\varepsilon}_2^{(0)}\right) - \frac{1}{h}C_{33}^0\varepsilon_3^{(0)}\right) + \left(p_2 - p_1\right)\xi + \frac{p_1 - p_2}{2} + \varphi_{\mathrm{g}}p,$$
$$\xi = \frac{1}{2} - \frac{q_3}{h}, \qquad \eta = \frac{1}{4} - \left(\frac{q_3}{h}\right)^2,$$

where p_1, p_2 are the pressures on the external surfaces of the shell. The maximum value of the tangential stress is reached on the middle surface.

5 Computational Method

The following method is suggested to solve the conjugated problem. Cycle by 'slow' time $\bar{t} = t/t_0$ corresponding to the heat transfer in the body shell is entered, where t_0 is the characteristic time of the construction heating. Inside this cycle 'fast' time $\tau = t/t_{\rm g}$ is entered, where $t_{\rm g}$ is the characteristic establishment time of the flow. For every fixed moment of the slow time $\bar{t}_n$ the heat flux through the wall $q_{\rm s} = -\lambda_{\rm s}\nabla\theta_{\rm s} \cdot \mathbf{n}$, which is generally unknown, is supposed to be fixed. Then the gas dynamics equations are separated from the heat-and-mass transfer equation (1)–(4) for one step at slow time.

For the solution of the system of gasdynamics equations is used RKDG (Runge-Kutta Discontinuous Galerkin) method. For the solution of the system of differential equations (1) is used a linearization technique and implicit finite–difference method. For the numerical solution of the equations of thermoelasticity for a shell made of a composite material (2)–(4) is used the finite–element method. The input data for this task are pressures on the outer p_1 and inner p_2 surface of the shell, which is calculated after the solution of the aerogasdynamics equations, as well as the distributions of the temperature θ, volume concentrations of the phases $\varphi_{\rm f}$, $\varphi_{\rm b}$, $\varphi_{\rm p}$, $\varphi_{\rm g}$ and the pore pressure p which are calculated by the solving of the equations of internal heat-and-mass transfer (1) for the current time step.

6 Results and Discussion

A numerical solution of the conjugated problem was calculated for a hypersonic flow ($M = 6$) around the nose of a vehicle model flying at the altitude of 15 km.

The Fig. 1 show the distribution of the temperature of the flow near the body. The temperature reaches 1,600 K at the stagnation point on the nose of the vehicle and decreases monotonically as the distance from the stagnation point, but it remains rather high value. At the maximum distance the temperature is about 800 K for the edge and lower generatrix and 1,000 K for the upper generatrix of the vehicle, which has a large value of the cone angle.

The results of numerical calculations of the fields of internal heat-and-mass transfer in the shell element of the hypersonic aircraft are shown in Fig. 2. The results include the distribution of temperature on the outer surface and the thickness maximum pore pressure of the construction at the time of maximum construction heating. Thermodecomposition of the polymer phase of the composite shell leads to the formation of a large amount of gaseous products in the pores of the material. Due to the low permeability of the composite, the produced gases do not have time to percolate into the outer gas flow and create the internal pore pressure.

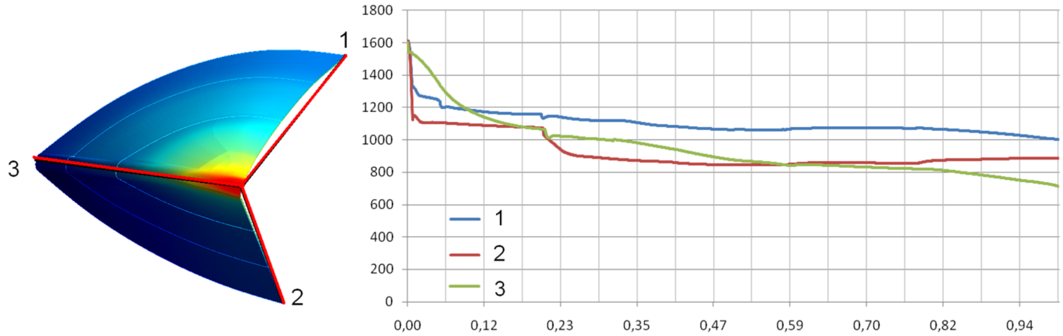

Figure 1: Temperature θ (K) distribution of the gas flow along the longitudinal axis for the upper (1) and lower (2) generatrixes and for edge (3) connecting the two sides of the surface of the hypersonic vehicle

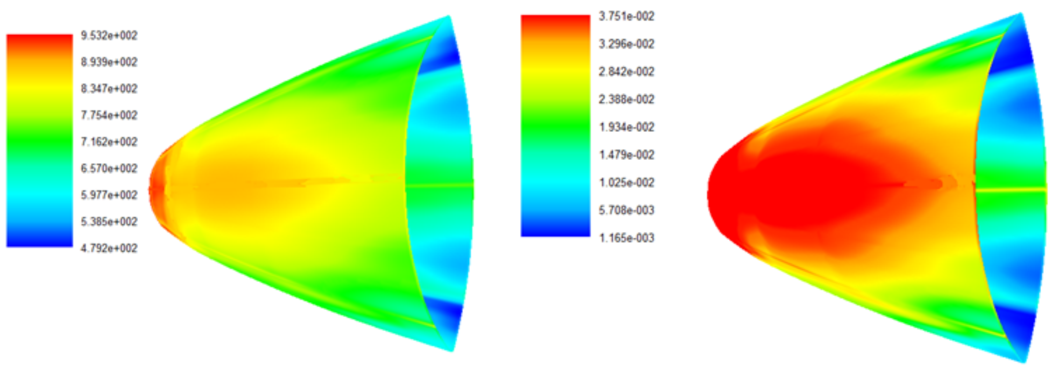

Figure 2: Distribution of the parameters in the shell of the hypersonic vehicle construction: (left) temperature θ (K); (right) maximum pore pressure of the gas phase p (GPa)

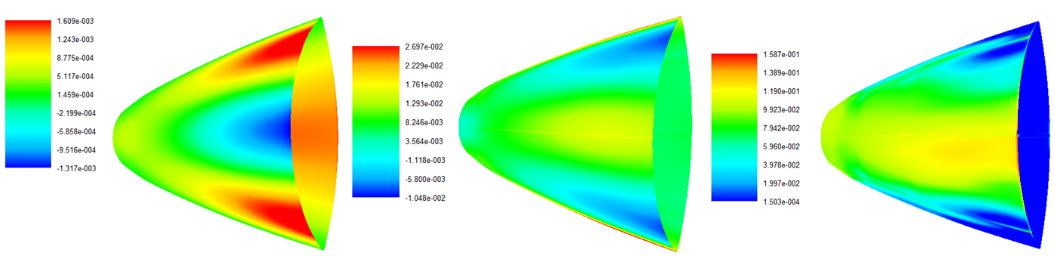

Figure 3: Distribution of the parameters in the shell of the hypersonic vehicle construction: (left) deflection W (m); (center) circumferential stress σ_{22} (GPa); (right) traverse stress σ_{33} (GPa)

Fig. 3 shows the distribution of the deflection of the shell, circumferential and traverse stresses on the outer surface of the shell at the time of the maximum construction heating. During the heating the circumferential compression stresses gradually increase together with the values of the maximum tensile circumferential stresses at the periphery of the shell closer to the edges of the shell. At the time of the maximum construction heating the positive peak of the tensile stresses occurs due to thermodestruction of the composite. As a result the pore pressure increases and the shrinkage deformation of the shell arises. The values of traverse stresses at the bottom of the shell are reached values of 0.13 GPa which is significantly higher than the breaking point of the composite's shell in the transverse direction. The destruction of the bundle type may occur in this part of the shell in which upper layers of the composite's fabric are delaminated from the rest of the material. It should be noted that the results depend strongly on the external gas flow conditions.

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