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Parametric Identification of Inertial Parameters

Gennady I. Melnikov

Department of Theoretical and Applied Mechanics
University ITMO, St. Petersburg, Russia

Natalia A. Dudarenko

Department of Control Systems and Informatics
University ITMO, St. Petersburg, Russia

Vitaly G. Melnikov

Department of Theoretical and Applied Mechanics
University ITMO, St. Petersburg, Russia

Aleksandr S. Alyshev

Department of Theoretical and Applied Mechanics
University ITMO, St. Petersburg, Russia

Abstract

The moments of inertia of solids are usually experimentally determined on irregular axial rotations and the inertia tensor of the body at its point - on six axial rotation, or on non-spherical motions. Frictional forces in bearings of the actuator adversely affect the accuracy of parametric identification. The paper proposes a method of parametric identification of the inertia tensor matrix of a rigid body and the coordinates of its centre of mass at a spherical motion of the particular form. The proposed method can be used on devices with essential friction.

Keywords: inertia tensor, centre of mass, identification

1 Introduction

The problem of moments of inertia and tensors of inertia identification is one of the fundamental problems of engineering. The inertia parameters have a strong effect on mechanical system dynamics [11, 12]. Therefore a knowledge of the inertia parameters is actual for different applications in transport [1, 9, 13, 22], in spacecraft and aircraft engineering [2–4, 8, 16, 20, 21], in sports and biomechanics [18, 19]. The moments of inertia of solids can be experimentally determined on the irregular rotation around the axis. The inertia tensor of the body at the point can be determined on six axial rotation, or on spherical motions [10]. A survey of current experimental identification methods could be found in [5, 14, 15, 17, 24]. A common practice for non-experimental estimation of rigid body inertia parameters is computation of them using CAD software [23]. The paper is laid out as follows. First, the description of reverse symmetrical semi-program spherical motions and solid precessions are given. Then, a general case of the biaxial gimbal with two gear ratio and hybrid torsion-electromotor actuator is considered and dynamic equations and calculation formulas for axis moments of inertia are obtained as the main result. Thereafter, the result is supported by an example with the simulation result. The paper is finished with some concluding remarks and acknowledgements.

2 Reverse symmetrical semi-program spherical motions and solid precessions

A spherical motion of a body in a biaxial gimbal suspension around a point O in an inertial system $Ox_1y_1z_1$, comprising the steps of braking rotation in limited ranges of variation of the Euler angles and the stage of accelerated rotation of the body at the same intervals is called *reverse-symmetrical spherical motion (RSP)* if its accelerated rotation part repeats in reverse order the braking rotation part. The motion is called *semi-program* if its first part is a measured free uncontrolled motion, and the second part is a controlled motion to satisfy symmetry drawn from previous measurements. *RSP-precession* will be reverse-symmetrical semi-program spherical body motion with a constant angle of nutation $\theta = \pi/2$ and holonomic linear relation between the angles of precession and proper rotation $\psi = \lambda\varphi, \lambda = const$ of the biaxial gimbal suspension. Also we assume that the precession axis Oz_1 is vertical, and its own axis of rotation Oz is horizontal. Let us accept φ and $\Omega = \dot{\varphi}$ for generalized coordinates and generalized velocities of the motion of a body having one degree of freedom. The loose axoid of RSP-precession is a circular cone with an axis Oz, and an angle $\beta = \arctan\lambda$ between the element of a cone and its axis. Cone

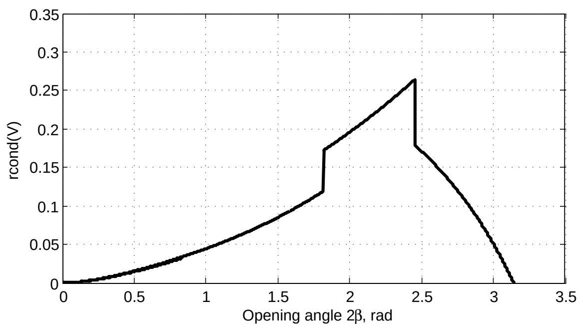

Figure 1: Dependence of inversion of the condition number of matrix V from opening angle of the cone 2β

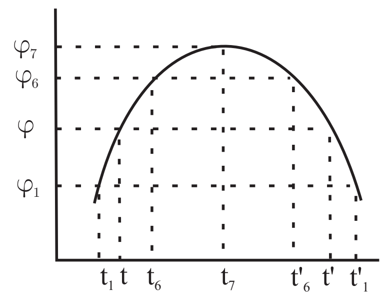

Figure 2: RSP precession

rolls over the fixed cone having an axis Oz_1, and the angle between the element of a cone and its axis $\alpha = \pi/2 - \beta$. We will also consider two RSP-precessions corresponding to the two values of the coefficient $\lambda_1 = 0.76, \lambda_2 = 5.24$, which form the one spherical motion with a single switching gear. These constants are selected such that the three axes of the icosahedron conventionally associated with a body are located on the one loose axoid and the other three axes - on the other axoid. Six axes of the icosahedron are evenly distributed around the center O. This ensures a well-conditioned algebraic problem (the condition number $\mu = 1.58$) and therefore the minimal impact of the experimental error on the accuracy of calculation.

To avoid a gear switch in a biaxial gimbal suspension and therefore to simplify the implementing device we can use the five axes with regular distribution on one cone and the sixth axis located along the one of the axes x, y, z. We define the position of axes on the cone and its opening angle according to the condition of well-conditioned matrix V with inversion of its condition number [6, 7]. Let the five axes be located regularly on the cone with the opening angle 2β and the sixth axis be located along the axis z. The optimal value of the opening angle corresponds to the maximum of inversion of the condition number (rcond) of the transformation matrix V (Fig. 1).

It follows from Fig.1 that the optimal value of the opening angle is 140^0, that relates to the conditional number $\mu = 1.4899$.

Let the results of measurements of a free uncontrolled braking motion in the final angular range is approximated by a kinematic equation:

$$\varphi = f(t) \quad \text{at} \quad \varphi \in [\varphi_1, \varphi_7], t \in [t_1, t_7], \varphi_7 - \varphi_1 \geq 4\pi \tag{1}$$

Then assigning to parameter t the value of $2t_7 - t'$, where t' is a new specification of time, we get the equation of the reverse-symmetrical motion

$$\varphi' = f(2t_7 - t') \quad t' \in [t_7, t_1' = 2t_7 - t_1] \tag{2}$$

We get RSP-motion as a union of a two-revolve motion according to equation (1) in the interval $[t_1, t_6]$ at $t_6 \leq t_7, \varphi \in [\varphi_1, \varphi_6 = \varphi_1 + 4\pi]$ and a two-revolve reverse motion determined by equation (2) in the interval $[t_6' = 2t_7 - t_6, t_1']$ (Fig. 2). Denote by $\Omega = p(t)$ the approximation of an angular velocity Ω of non-program braking motion during the time period $[t_1, t_6]$. We get the design formulas for the angular velocity $\Omega(t')$ and the angular acceleration $\varepsilon(t')$ of the symmetric reverse motion at $t_6' \geq t_6$ as:

$$\begin{aligned} &\Omega(t') = -p(t) \quad \text{at} \quad t = t_6' + t_6 - t',\ t' \in [t_6', t_1'] \\ &\varepsilon(t') = d\Omega(t')/dt', \quad \varphi' = \int_{t_6'}^{t'} \Omega(t')dt' \end{aligned} \tag{3}$$

3 Dynamic equations and calculation formulas for axis moments of inertia

Here we consider a general case of the biaxial gimbal with two gear ratio and hybrid torsion-electromotor actuator. Let the biaxial gimbal with the tested body perfoms the RSP-precession at the gear ratio $\lambda_1 = \mathrm{tg}\ \beta_1$ in the range $\Phi^{(1)} = [\varphi_1, \varphi_6]$, $\varphi_1 = 0, \varphi_6 = 10\pi/3$ with intermediate equidistant nodes $\varphi_k = \varphi_1 + (k-1)h, k = 2, 3, 4, 5; h = 2\pi/3$. Let us highlight three single-turn intersecting subintervals in the two-turn interval:

$$\Phi_1^{(1)} = [\varphi_1, \varphi_1 + 2\pi] \equiv [\varphi_1, \varphi_4], \quad \Phi_2^{(1)} = [\varphi_2, \varphi_5], \quad \Phi_3^{(1)} = [\varphi_3, \varphi_6],$$

Let the two-turn precession consists of the non-programmed measured break motion at the interval $\Phi^{(1)}$ and the programmed reverse symmetrical motion (Fig. 2). We divide it into three intersecting one-turn precessions. Using the theorem of change of kinetic energy for the body-device system on the brake turns and reverse motions for intervals $\Phi_k, k = 1, 2, 3$, we obtain six equations:

$$\begin{aligned} &E_{k+3} - E_k = A_k + B_k + V_k, \\ &E_k - E_{k+3} = A_k' + B_k' + V_k', \quad k = 1, 2, 3. \end{aligned} \tag{4}$$

Here $E_k = T_k + \Pi_k$ are the node values of mechanical energy; Π_k is potential energy of torsions; A_k, A_k' are motor operations on the turns; B_k, B_k' are negative operations of internal friction forces in the torsion with the resistance forces of the environment; V_k, V_k' are negative operations of friction forces in kinematic pairs of a device and in motor bearings. The gravity force operation of the body is equal to zero due to complete turn rotation by φ and the vertical axis of precession. We assume that the structures resistance is invariant to the direction of motion such that $B'_k \approx B_k$ and the operations of friction forces on braking and reverse acceleration are approximately equal $V'_k = V_k$.

By term by term subtracting the equations (4), we obtain the following:

$$2E_k - 2E_{k+3} = A'_k - A_k, \qquad k = 1, 2, 3 \tag{5}$$

By term by term summing of equations (4) we obviously have the equations $B_k + V_k = -(A'_k + A_k)/2$ for an evaluation of the dissipative forces operation on turns.

Denote by $T = (J(\varphi) + I\lambda_1^2)\Omega^2/2$ the kinetic energy of the system, where $J(\varphi)$ is the moment of inertia of the body adduced to phase vector $[\Omega, \varphi]$, and $I = \text{const}$ is the moment of inertia relative to the axis Oz_1 of the outer frame. From (5) it follows that the formulas for the moments of inertia reduced to Ω for three axes of the icosahedron $J_k \equiv J(\varphi_k)$:

$$J_k = (2\Pi_{k+3} - 2\Pi_k + A'_k - A_k)(\Omega_k^2 - \Omega_{k+3}^2)^{-1} - I\lambda_1^2, \quad k = 1, 2, 3. \tag{6}$$

Here Π_k are the nodal values of the potential energy of torsion; A_k, A'_k are motor operations which expended on changing of the mechanical motion and to overcome the dissipative forces at full throttle in the positive and negative directions.

Further, the execution of the precession at a gear ratio λ_2 is performed and we consider the intervals $\Phi_1^{(2)}, \Phi_2^{(2)}, \Phi_3^{(2)}$.

For these intervals, we get the following three equivalent moments of inertia:

$$J_k = (2\Pi_{k+3} - 2\Pi_k + W'_k - W_k + \delta_k - \delta'_k)(\Omega_k^2 - \Omega_{k+3}^2)^{-1} - I\lambda_2^2, \quad k = 4, 5, 6 \tag{7}$$

The axial moments of inertia of the body are determined by $J_k^0 = J_k(1 + \lambda_{1,2}^2)^{-1} - I_1$, where I_1 are the cylinder moments of inertia relative to the instantaneous axis OL_1 or OL_2 are constant due to the circular symmetry of the system.

To obtain the elements of the inertia tensor matrix from the moments of inertia about the axis of the icosahedron we have the following design formula:

$$[J_x\ J_y\ J_z\ J_{xy}\ J_{yz}\ J_{xz}] = [J_1, ..., J_6]V^{-1}, \quad \det V = 2, 3, \tag{8}$$

where the transformation matrix contains six column vectors:

$$V = [V_1, ..., V_6], \quad V_k = [e_{kx}^2, e_{ky}^2, e_{kz}^2, 2e_{kx}e_{ky}, 2e_{ky}e_{kz}, 2e_{kx}e_{kz}]^T$$

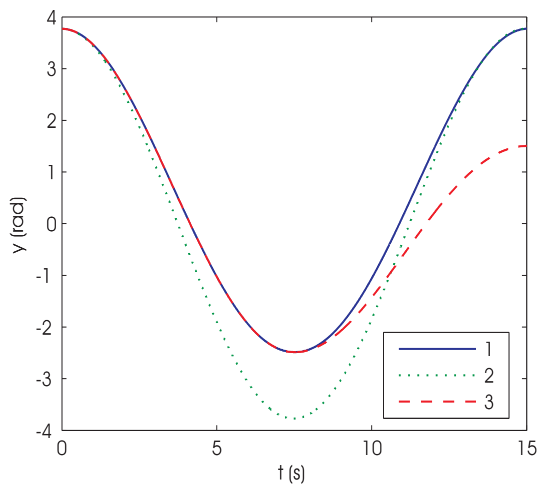

Figure 3: Symmetric semiprogram motion (1), free undamped motion (2), free damped motion (3)

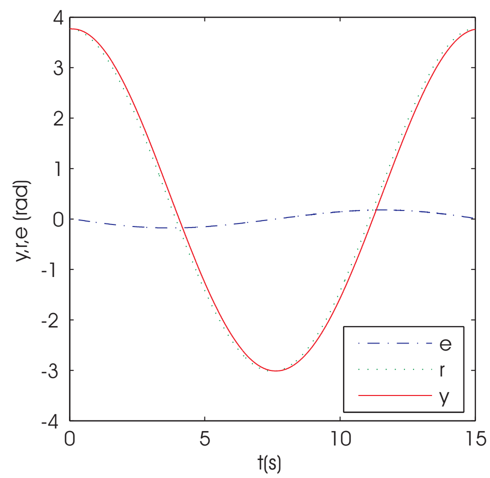

Figure 4: Reference (r), output (y) and error (e) of the actuator at $J = 0.025$ kg m^2

3.1 Simulation results

We consider the dissipative system with standard steel bearings. The model of moment of dissipation in actuator is estimated as follows $M_{dis}(t) = (a_1 + a_3\omega^2(t)) \operatorname{sign}\omega(t) + a_2\omega(t)$ N m, where $a = [a_1, a_2, a_3] = [10^{-4}, 10^{-4}, 10^{-3}]$. The free motion of this system shows (Curve 3, Fig. 3) relatively high damping. The ideal undamped system with the same inertial load is shown in Curve 3. The symmetric semiprogram motion in Curve 1 has a free first part and a controlled symmetric second part. The most used method [25] considers the motion as approximately undamped oscillation $\ddot{\varphi} + k^2\varphi = 0, k = \sqrt{c_t/J} = 2\pi/\tau$ with the design formula for the moment of inertia as follows $J = \frac{c_t}{4\pi^2}\tau^2$. The accuracy of identification using the standard method on the considered dissipative system is 5%. Higher accuracy can be obtained only on systems with low dissipation.

A state feedback regulator provides a semiprogram motion to the system with unknown inertia load and unknown friction disturbance (Fig 4). Simulation result shows a high accuracy of inertia identification on a dissipative system using the developed method, that is $1 \cdot 10^{-2}$%. Note also that for the proposed identification algorithm we need the high symmetry of the tested motion, while the following error may be nonzero.

4 Conclusion

The aim of this paper was to present a method for identifying the inertia tensor of a mechanical systems on a semiprogram precession motion. The tensor is defined on rotational and spherical motions using the work-energy principle. The paper gives the presentation of the proposed identification method and data processing procedure with sensitivity analysis and error estimation using proposed and classical methods. The method can be used on devices with essential dissipation.

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