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Finite time Control of Chaotic Cellular Neural Network with Uncertain Parameters

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Abstract

This paper solved the problem of control chaotic cellular neural network with finite time and the uncertain parameters. Non-linear control law is designed to ensure achieving the desired state in finite time. Theoretical results are demonstrated by strict Lyapunov stability theory. Simulation results performed on Matlab environment shows the effectiveness of proposed control law.

Mathematics Subject Classification: 34H10, 34H15, 37D45, 37K45, 68Q05

Keywords: Chaotic system, Chaos Control, Cellular Neural Network, Lyapunov function, Stability finite time

1 Introduction

Chaotic system capable of robust applications in such areas as communications security, image processing, bio-chemical technology. So it has attracted much attention of research scientists [1]-[12]. During the application process, often arise chaos control and chaos synchronization control problems. Thither is a difficult problem because the characteristics of the chaotic system are specially sensitive to modest alterations in system parameters and initial conditions. Before the 90s, many scientists still consider that chaos control can not achieve the desired goals. And so the pioneering paper by Ott, Gre Bogi, and Yorke [4]; Pecora and Carroll [5] has completely denied this aspect.

Since then, many methods have been applied to chaos control and chaos synchronization such as sliding mode control [5], adaptive control [1]-[3], [12], state feedback control [4], [9]. Recently, more and more scientists are interested in a finite time stability because this is the key issue in the deployment of real applications. In [16], Wang et al using the terminal slide mode control for synchronizing finite time Duffing chaotic system. In [17]-[18], Wu et al using adaptive control method to control finite time unified chaotic system.In [19], M. Aghababa proposed nonlinear adaptive control scheme for finite time synchronization of chaotic systems with uncertain parameters.

Cellular neural network (CNN) was invented by Leon Chua in 1988 [13]. CNN has the ability to generate chaotic signals and can easy implement circuit [11]-[12], [14]. The study of chaotic characteristics of CNN and the ability to apply them leads to control chaotic CNN, synchronization chaotic CNN and other problems (finite time, have multiple inputs and parameters, uncertain parameters ...). This paper solves a problem in that, which is state controlled chaos CNN achieve desired goals in finite time.

2 Preliminary Notes

CNN was invented by Leon Chua and Ling Yang 1988 [13]. It has the parallel processing capability with high speed and has many applications in image processing, pattern recognition, and secure communication. In this report, we consider CNN 3 cells fully connected with state equations:

$$\dot{x}_j = -x_j + a_j f(x_j) + \sum_{k=1,k\neq j}^{3} a_{jk} f(x_k) + \sum_{k=1}^{3} S_{jk} x_k + I_j \; (j = 1, 2, 3) \tag{1}$$

where x_j and $f(x_j)$ are the state variables and output function of j-th cell, respectively; a_j, a_{jk}, S_{jk} are the real parameters and I_j is the threshold. Output $f(x_j)$ is the piecewise linear function defined as:

$$f(x_j) = \frac{1}{2}\left(|x_j + 1| - |x_j - 1|\right) \tag{2}$$

Set the value of the parameters:

$a_{12} = a_{13} = a_2 = a_{23} = a_{32} = a_3 = a_{21} = a_{31} = 0;$
$s_{13} = s_{31} = s_{22} = s_{21} = s_{23} = s_{33} = I_1 = I_2 = I_3 = 0; s_{21} = s_{23} = s_{33} = 1;$

Then CNN 3 cell model as follows:

$$\begin{cases} \dot{x}_1 = -x_1 + a_1 f(x_1) + s_{11}x_1 + s_{12}x_2 \\ \dot{x}_2 = -x_2 + x_1 + x_3 \\ \dot{x}_3 = s_{32}x_2 \end{cases} \tag{3}$$

Study the behavior of dynamical systems [15], we have some parameters satisfying (3) is chaotic systems:

$$\begin{aligned} &(a)\, a_1 = -7.717, s_{11} = 1.3443, s_{12} = -4.925, s_{32} = 3.649 \\ &(b)\, a_1 = 3.86, s_{11} = -1.55, s_{12} = 0.98, s_{32} = -14.26 \\ &(c)\, a_1 = 4.0279, s_{11} = -1.6856, s_{12} = 9.4, s_{32} = -16 \\ &(d)\, a_1 = -3.6805, s_{11} = 2.2179, s_{12} = 8.342, s_{32} = -11.925 \end{aligned}$$

Fig.1 show the chaotic attractors of CNN (3) with respectively parameters.

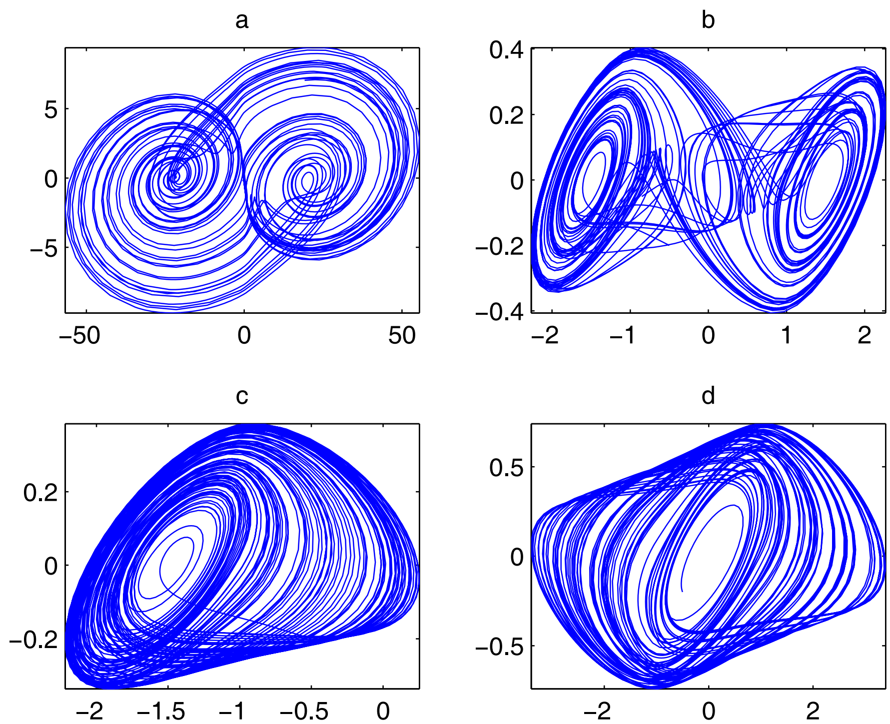

Figure 1: Chaotic Attractor of CNN

The Lyapunov Exponent of system (3) with above case, respectively:

$$\begin{aligned} &(a)\, \lambda_1 = 0.1345, \lambda_2 = -0.0017, \lambda_3 = -1.058 \\ &(b)\, \lambda_1 = 0.3203, \lambda_2 = -0.0018, \lambda_3 = -2.6167 \\ &(c) \lambda_1 = 0.2472, \lambda_2 = -0.0018, \lambda_3 = -2.4525 \\ &(d) \lambda_1 = 0.2985, \lambda_2 = -0.0011, \lambda_3 = -1.401 \end{aligned}$$

Stability in finite time at an equilibrium point of the system means that the state of a dynamical system converges to the equilibrium point (in the general case, the original routine coordinates) during finite time.

Definition 2.1 *[2] Consider the following dynamic system:*

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}) \tag{4}$$

where $\mathbf{x}(t) \in R^n$ is the system state. If there exists a constant $T > 0$ (T may depend on the initial state $x(0)$) such that: $\lim_{t\to T} \|\mathbf{x}(t)\| = 0$ and $\|\mathbf{x}(t)\| \equiv 0$ if $t \geq T$ then system (4) is finite time stable.

Lemma 2.2 *[3] Assumer that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:*

$$\dot{V}(t) \leq -cV^{\eta}(t), \forall t \geq t_0, V(t_0) \geq 0 \tag{5}$$

where $c > 0$ and $0 < \eta < 1$ are constants. Then for any intinial time t_0, $V(t)$ satisfies:

$$\begin{aligned} & V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0) \\ & t_0 \leq t \leq t_1 \end{aligned} \tag{6}$$

and $V(t) \equiv 0, \forall t \geq t_1$, with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)} \tag{7}$$

Lemma 2.3 *[3] Let $0 < c < 1$. Then for positive real numbers a and b, the following inequality holds:*

$$(a+b)^c \leq a^c + b^c \tag{8}$$

3 Main Results

3.1 Chaos control for the certain CNN

Consider chaos CNN (3) with $a_1, s_{11}, s_{12}, s_{32}$ are certain parameters. The goal is to design controllers to achieve global stability in the original equilibrium point in finite time.
System (3) is rewritten as follows:

$$\begin{cases} \dot{x}_1 = -x_1 + a_1 f(x_1) + s_{11}x_1 + s_{12}x_2 + u_1 \\ \dot{x}_2 = -x_2 + x_1 + x_3 + u_2 \\ \dot{x}_3 = s_{32}x_2 + u_3 \end{cases} \tag{9}$$

where $\mathbf{u} = (u_1, u_2, u_3)^T$ is control vector. Design procedure is divided into two steps:

Step 1: Let $u_1 = -a_1 f_1(x_1) - s_{12}x_2 - L_1 x_1 - \mathrm{sgn}(x_1)|x_1|^\beta$, where $\beta \in (0, 1)$, $\mathrm{sgn}(x)$ is the sign function and L_1 is the control parameter. L_1 is chosen to satisfy:

$$L_1 > s_{11} - 1 \tag{10}$$

By the controller u_1, the first equation of (9) is:

$$\dot{x}_1 = -x_1 + s_{11}x_1 - L_1 x_1 - \mathrm{sgn}(x_1)|x_1|^\beta \tag{11}$$

Consider the candidate Lyapunov function $V_1 = \frac{1}{2}{x_1}^2$. The time derivative of V_1 along the trajectory of (11) is

$$\dot{V}_1 = x_1\left(-x_1 + s_{11}x_1 - L_1x_1 - \mathrm{sgn}(x_1)|x_1|^\beta\right) = -\left(L_1 + 1 - s_{11}\right){x_1}^2 - |x_1|^{\beta+1}$$

$$\leq -|x_1|^{\beta+1} = -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(\frac{1}{2}{x_1}^2\right)^{\frac{\beta+1}{2}} = -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}} {V_1}^{\frac{\beta+1}{2}}$$

From Lemma 2.2, the state x_1 will reach $x_1 = 0$ at a finite time T_1.

Step 2: Let $u_2 = -x_3 - \mathrm{sgn}(x_2)|x_2|^\beta$; $u_3 = -s_{32}x_2 - \mathrm{sgn}(x_3)|x_3|^\beta$. If $t > T_1$ then $x_1 \equiv 0$. Substituting $x_1 = 0$ into the second and the third equations of system (9), it yields

$$\begin{cases} \dot{x}_2 = -x_2 - \mathrm{sgn}(x_2)|x_2|^\beta \\ \dot{x}_3 = -\mathrm{sgn}(x_3)|x_3|^\beta \end{cases} \tag{12}$$

Choose the Lyapunov function for (12) as follows:

$$V_2 = \frac{1}{2}{x_2}^2 + \frac{1}{2}{x_3}^2 \tag{13}$$

The time derivative of V_2 along the trajectory of (12) is

$$\dot{V}_2 = x_2\left(-x_2 - \mathrm{sgn}(x_2)|x_2|^\beta\right) + x_3\left(-\mathrm{sgn}(x_3)|x_3|^\beta\right)$$

$$= -{x_2}^2 - |x_2|^{\beta+1} - |x_3|^{\beta+1} \leq -|x_2|^{\beta+1} - |x_3|^{\beta+1}$$

Because of $0 < \frac{\beta+1}{2} < 1$, application Lemma 2.3 we have

$$\dot{V}_2 \leq -|x_2|^{\beta+1} - |x_3|^{\beta+1} = -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(\left(\frac{1}{2}{x_2}^2\right)^{\frac{\beta+1}{2}} + \left(\frac{1}{2}{x_3}^2\right)^{\frac{\beta+1}{2}}\right)$$

$$\leq -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(\frac{1}{2}{x_2}^2 + \frac{1}{2}{x_3}^2\right)^{\frac{\beta+1}{2}} = -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}} {V_2}^{\frac{\beta+1}{2}}$$

From Lemma 2.2, the state x_2 and x_3 will converge to 0 after finite time T_2 Thus, with control law:

$$\begin{aligned} u_1 &= -a_1 f_1(x_1) - s_{12}x_2 - L_1 x_1 - \mathrm{sgn}(x_1)|x_1|^\beta \\ u_2 &= x_3 - \mathrm{sgn}(x_2)|x_2|^\beta \\ u_3 &= -s_{32}x_2 - \mathrm{sgn}(x_3)|x_3|^\beta \end{aligned} \tag{14}$$

the control of chaos CNN with certain parameters in finite time problem has solved.

3.2 Chaos control for the CNN with uncertain parameters

Consider the following chaotic CNN with uncertain parameters:

$$\begin{cases} \dot{x}_1 = -x_1 + a_1 f(x_1) + (s_{11} + \Delta_1)x_1 + (s_{12} + \Delta_2)x_2 + u_1 \\ \dot{x}_2 = -x_2 + x_1 + x_3 + u_2 \\ \dot{x}_3 = (s_{32} + \Delta_3)x_2 + u_3 \end{cases} \tag{15}$$

where $\Delta_i, i = 1,2,3$ denote the bounded uncertain parameters s_{11}, s_{12}, s_{32}, $|\Delta_i| \le \rho_i, i = 1,2,3$. In the following, we will design a controller to stabilize the uncertain CNN (15). Control law proposed:

$$\begin{cases} u_1 = -a_1 f(x_1) - L_1 x_1 - L_2 x_2 - \mathrm{sgn}(x_1)|x_1|^\beta \\ u_2 = -x_1 - x_3 - \mathrm{sgn}(x_2)|x_2|^\beta \\ u_3 = -\mathrm{sgn}(x_3)|x_3|^\beta - L_3 x_2 \end{cases} \tag{16}$$

where $\beta \in (0,1)$, $\mathrm{sgn}(x)$ is the sign function and L_1, L_2, L_3 are the control parameters satisfy:

$$\begin{aligned} &L_1 \ge S_{11} + \rho_1 - \tfrac{1}{2} \\ &s_{12} + \rho_2 - 1 \le L_2 \le s_{12} + \rho_2 + 1 \\ &s_{32} + \rho_3 - 1 \le L_3 \le s_{32} + \rho_3 + 1 \end{aligned} \tag{17}$$

Theorem 3.1 *Control law proposed (16) ensures system (15) finite time asymptotic stability at equilibrium $O(0,0,0)$.*

Proof With control law (16), system (15) become:

$$\begin{cases} \dot{x_1} = -x_1(1 + L_1 - (s_{11} + \Delta_1)) - x_2(L_2 - (s_{12} + \Delta_2)) - \mathrm{sgn}(x_1)|x_1|^\beta \\ \dot{x_2} = -x_2 - \mathrm{sgn}(x_2)|x_2|^\beta \\ \dot{x_3} = -x_2(L_3 - (s_{32} + \Delta_3)) - \mathrm{sgn}(x_3)|x_3|^\beta \end{cases} \tag{18}$$

Choose Lyapunov function as follow:

$$V(t) = \frac{1}{2}{x_1}^2 + \frac{1}{2}{x_2}^2 + \frac{1}{2}{x_3}^2 \tag{19}$$

The time derivative of V along the trajectory of (18) is

$$\dot{V}(t) = x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 = -{x_1}^2\left(1 + L_1 - (s_{11} + \Delta_1)\right) + x_1x_2\left(s_{12} + \Delta_2 - L_2\right)$$

$$-|x_1|^{\beta+1} - {x_2}^2 - |x_2|^{\beta+1} + x_3x_2\left(s_{32} + \Delta_3 - L_3\right) - {x_3}^2 - |x_3|^{\beta+1} \tag{20}$$

We have

$$x_1x_2\left(s_{12} + \Delta_2 - L_2\right) \le \frac{{x_1}^2}{2} + \frac{{x_2}^2}{2}(s_{12} + \Delta_2 - L_2)^2$$

$$x_2x_3\left(s_{32} + \Delta_3 - L_3\right) \le \frac{{x_2}^2}{2} + \frac{{x_3}^2}{2}(s_{32} + \Delta_3 - L_3)^2 \tag{21}$$

Therefor

$$\dot{V}(t) \le -{x_1}^2\left(\frac{1}{2} - S_{11} - \Delta_1 + L_1\right) - |x_1|^{\beta+1} - {x_2}^2\left(\frac{1}{2} - \frac{(s_{12} + \Delta_2 - L_2)^2}{2}\right)$$

$$-|x_2|^{\beta+1} - {x_3}^2\left(\frac{1}{2} - \frac{(s_{32} + \Delta_3 - L_3)^2}{2}\right) - |x_3|^{\beta+1} \tag{22}$$

From assuming conditions (17) of L_1, L_2, L_3, leads to

$$\frac{1}{2} - S_{11} - \Delta_1 + L_1 \ge 0, \frac{1}{2} - \frac{(s_{12} + \Delta_2 - L_2)^2}{2} \ge 0, \frac{1}{2} - \frac{(s_{32} + \Delta_3 - L_3)^2}{2} \ge 0$$

This inferred

$$\dot{V}(t) \le -|x_1|^{\beta+1} - |x_2|^{\beta+1} - |x_3|^{\beta+1} \tag{23}$$

On the other hand, we have

$$-|x_1|^{\beta+1} - |x_2|^{\beta+1} - |x_3|^{\beta+1} = -\left(\frac{1}{2}\right)^{-\frac{\beta+1}{2}}\left(\left(\frac{1}{2}{x_1}^2\right)^{\frac{\beta+1}{2}} + \left(\frac{1}{2}{x_2}^2\right)^{\frac{\beta+1}{2}} + \left(\frac{1}{2}{x_3}^2\right)^{\frac{\beta+1}{2}}\right)$$

Since $0 < \beta < 1$ then $0 < \frac{\beta+1}{2} < 1$. From Lemma 2.3 we have

$$\dot{V} \le -\left(\frac{1}{2}\right)^{-\left(\frac{\beta+1}{2}\right)}\left(\frac{1}{2}{x_1}^2 + \frac{1}{2}{x_2}^2 + \frac{1}{2}{x_3}^2\right)^{\frac{\beta+1}{2}} = -\left(\frac{1}{2}\right)^{-\left(\frac{\beta+1}{2}\right)}\mathrm{V}^{\frac{\beta+1}{2}} \tag{24}$$

Finally, apply Lemma 2.2 we can conclude that all states x_1, x_2, x_3 of system (15) will converge to 0 at finite time T. It means that the controlled chaos CNN with uncertain parameters is finite time stable.

4 Simulation results

In this section, we will simulate the effect of the proposed control law in two cases above. ODE45 function in Matlab R2012a is used to solve the system of differential equations with time steps size 0.001. The parameters of CNN are $a_1 = -7.717, s_{11} = 1.3443, s_{12} = -4.925, s_{32} = 3.649$. The intinial condition of CNN is alway adopted as $\mathbf{x}^0 = (5, -4, 6)$, $\beta = 0.6 \in (0, 1)$

Fig.2 show the simulation results in the case of certain parameters. The controller gains (14) are chosen as $L_1 = s_{11}$

In the case CNN has uncertain parameters, the uncertainties are adopted as $\Delta_1 = \sin x_1; \Delta_2 = \cos x_2; \Delta_3 = \sin x_3$. Then we may chosen $\rho_1 = \rho_2 = \rho_3 = 1$ and condition (17) satisfies with $L_1 = s_{11} + 1; L_2 = s_{12}; L_3 = s_{13}$. Fig.3 show the simulation result for the uncertain CNN. We can see that the controller has strong robustness to the uncertainties. The controlled uncertain chaotic CNN is finite time stable.

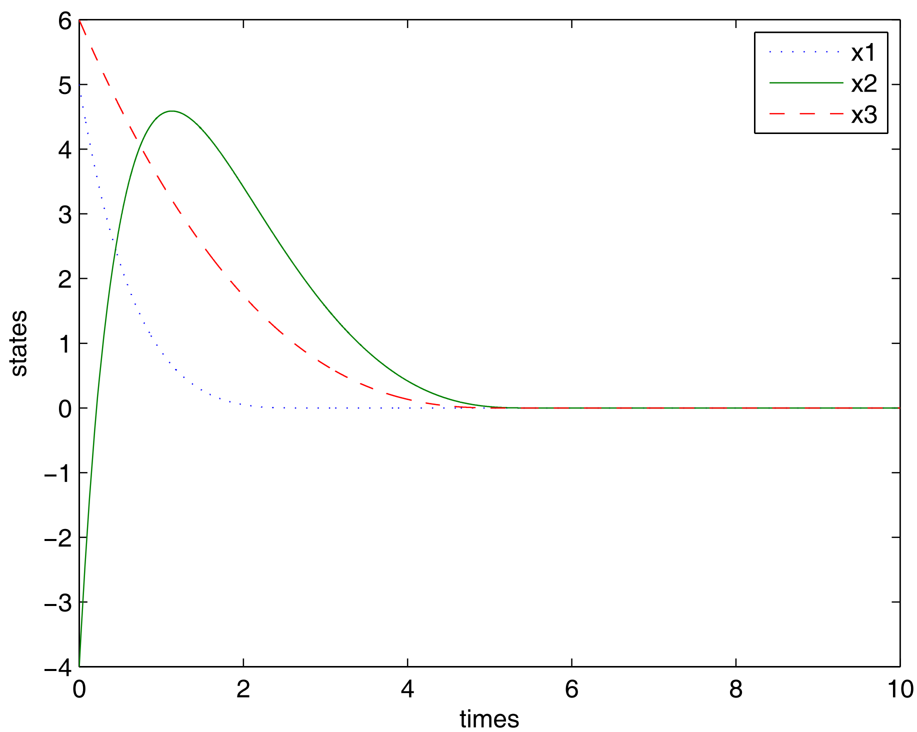

Figure 2: The state responses of controlled certain CNN

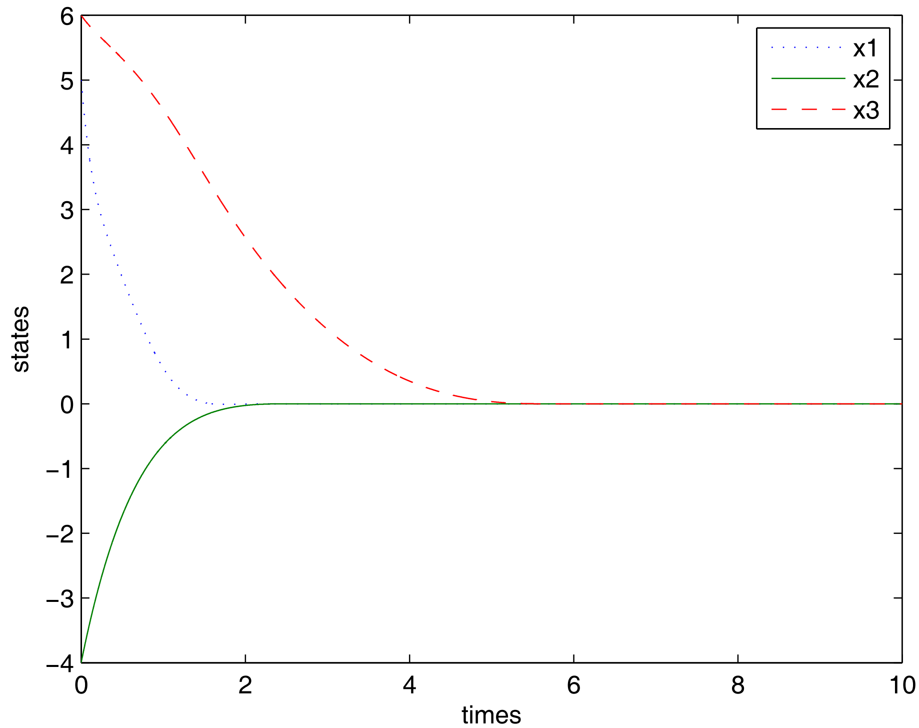

Figure 3: The state responses of controlled uncertain CNN

5 Conclusions

This paper solves the problem of controlled chaos CNN (3) in two cases as well as certain and uncertain parameters. The results of the paper is the design of the control law (14) and (16) respectively. The correctness of the control law were proved via Lyapunov stability theory. The simulation results in Matlab also illustrate the effectiveness of the control law proposed. In the future, the authors will continue to study chaotic synchronization in finite time and application in a number of areas such as encryption, secure communications.

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