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Common Fixed Point Theorems for Compatible Mappings of Type(C) in BA-Cone Metric Spaces

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Abstract

The purpose of this paper is to establish common fixed point theorems for four self-mappings using the concepts of compatible mappings of type(C) and weakly compatibility of pairs through rational expressions in a BA-cone metric space.

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1 Introduction

The Banach contraction principle is one of the cornerstone results of nonlinear functional analysis. Due to its usefulness and applications, it has become a very crucial and popular tool in solving existence and uniqueness problems in many different fields of mathematics. This principle has been extended to other kinds of contraction principle, such as contractive conditions involving

product, rational expressions and many others. Because of its importance, the Banach contraction principle has been investigated by a lot of authors either defining contractive mapping in the notion of complete metric space or introducing generalization of metric spaces. A cone metric space is one of the these generalizations and introduced by Huang and Zhang [4]. And also, they have given fundamental fixed point theorems on these spaces using a normal cone. Later, various authors have proved some fixed point theorems with normal and non-normal cones in these spaces.
The concept of compatibility was improved and extended in various directions by many authors. Firstly, Jungck *et al.* [3] defined compatible mapping of type (A), then Pathak and Khan [7] introduced compatible mappings of type (B) and finally, Pathak *et al.* [8] generalized compatible mapping of type (A) defining compatible mapping of type (C).
Since the concept of rational expressions is meaningless in cone metric space, then a kind of cone metric space over Banach algebra, which is called BA-cone metric space, has been introduced in [6]. In this paper, we present a common fixed point theorem for four self-mappings with compatible mappings of type(C) through rational expression and give a result using the concept of weakly compatibility for the pairs in BA-cone metric spaces.

2 Preliminary Notes

Let B be a real Banach space and K be a subset of B. Then K is called a cone if and only if

i. K is closed, nonempty and $K \neq \{0\}$,

ii. $a, b \in \mathbf{R}$, $a, b \geq 0$, $x, y \in K$, then $ax + by \in K$,

iii. $x \in K$ and $-x \in K$ then $x = \theta$.

In this definition, taking a Banach algebra instead of Banach space, K has been called a BA- cone in [6].
Given a cone $K \subset B$, we define a partial ordering $\leq$ with respect to K by $x \leq y$ if and only if $y - x \in K$. We write $x < y$ if $x \leq y$ but $x \neq y$; $x \ll y$ if $y - x \in intK$, where $intK$ is the interior of K. The cone K is called a non-normal if and only if there exists sequences $\{x_n\}, \{y_n\} \in K$ such that

$$0 \leq x_n \leq x_n + y_n, \quad \lim_{n\to\infty} (x_n + y_n) = 0,$$

but $\lim_{n\to\infty} x_n \neq 0$.

Definition 2.1 [4] *Let X be nonempty set, B be a real Banach space and $K \subset B$ be a cone. Suppose the mapping $d : X \times X \to B$ satisfies the following conditions,*

d_1. *$\theta < d(x,y)$ for all $x, y \in X$ and $d(x,y) = \theta$ if and only if $x = y$;*

d_2. *$d(x,y) = d(y,x)$ for all $x, y \in X$;*

d_3. *$d(x,y) \leq d(x,z) + d(z,y)$ for all $x, y, z \in X$,*

then d is called a cone metric on X and the pair (X,d) is called cone metric space. It is obvious that the concept of a cone metric space is more general than of a metric space. Replacing the Banach space with a Banach algebra in the definition of cone metric space, BA- cone metric space has been defined in [6].

Example 2.2 [6] *Let $C^2_{\mathbf{R}}([0,1])$ be the space of all real function on $[0,1]$ whose second derivative is continuous. We recall that for $a, b > 0$, the space $C^2_{\mathbf{R}}([0,1])$ with the norm*

$$\|f\| = \|f\|_\infty + a\|f'\|_\infty + b\|f''\|_\infty$$

is a Banach space, where $\|f\|_\infty = \sup_{t\in[0,1]} |f(t)|$. This space is a Banach algebra if and only if $2b \leq a^2$.

If we take $X = B = C^2_{\mathbf{R}}([0,1])$ with the above norm and $K = \{u \in B : u \geq 0\}$, then (X,d) becomes a cone metric space where

$$d(x,y) = \left(\sup_{t\in[0,1]} |x(t) - y(t)|\right) f(t)$$

and $f : [0,1] \to \mathbf{R}$, $f(t) = e^t$. But if we take $2b > a^2$ then B is not Banach algebra, hence (X,d) is not a BA-cone metric space.

Throughout the paper we take B as a Banach commutative division algebra. Recall that, a division algebra is an algebra with identity e, in which every non-zero element is a unit, where the identity is a non-zero element such that $xe = ex = x$ for all x and in any algebra with identity e, an element which has an inverse is called a unit, i.e. x is a unit if and only if there exists an inverse y such that $xy = yx = e$. We write $y = x^{-1}$ and observe that x^{-1} is unique when it exists in [5].

Followings are the definitions of compatible mappings of type(C) and the concept of weakly compatibility.

Definition 2.3 [2] *A pair of mappings (T,S) on a non-empty set X is said to be weakly compatible if they commute at their coincidence points; i.e., if $Tx = Sx$ for some $x \in X$, then $TSx = STx$.*

Definition 2.4 [12] *Let A and S be mappings from a complete cone metric space X into itself. The mappings A and S are said to be compatible of type (C) if*

$$\lim_{n\to\infty} d\left(ASx_n, SSx_n\right) \leq \frac{1}{3}\left[\lim_{n\to\infty} d\left(ASx_n, At\right) + \lim_{n\to\infty} d\left(At, AAx_n\right) + \lim_{n\to\infty} d\left(At, SSx_n\right)\right]$$

and

$$\lim_{n\to\infty} d\left(SAx_n, AAx_n\right) \leq \frac{1}{3}\left[\lim_{n\to\infty} d\left(SAx_n, St\right) + \lim_{n\to\infty} d\left(St, SSx_n\right) + \lim_{n\to\infty} d\left(St, AAx_n\right)\right]$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.

We need the following propositions in the sequel.

Proposition 2.5 [1] *Let E be a real Banach space with cone K in E and $x, y, z \in X$. Then:*

i. if $x \leq y$ and $y \ll z$, then $x \ll z$,

ii. if $x \ll y$ and $y \ll z$, then $x \ll z$,

iii. if $\theta \leq x \leq y$ and $a \geq 0$, where a is real number, then $\theta \leq ax \leq ay$,

iv. if $\theta \leq x_n \leq y_n$, for $n \in \mathbf{N}$ and $\lim_n x_n = x$, $\lim_n y_n = y$, then $\theta \leq x \leq y$.

Proposition 2.6 [12] *Let A and S be mappings from a complete metric space (X, d) into itself. If a pair (A, S) is compatible of type(C) on X and $At = St$ for $t \in X$, then*

$$ASt = SAt = AAt = SSt.$$

Proposition 2.6 is also valid for cone metric spaces.

3 Main Results

Initially, we give following proposition which is necessary to prove our main theorem.

Proposition 3.1 *Let A and S be mappings from a complete BA-cone metric space (X, d) into itself. If a pair (A, S) is compatible of type(C) on X and $d\left(Ax_n, t\right) \ll c, d\left(Sx_n, t\right) \ll c$ for some $t \in X$, then we have*

i. $d\left(SSx_n, At\right) \ll c$ as $n \to \infty$ if A is continuous at t;

ii. $d(AAx_n, St) \ll c$ *as* $n \to \infty$ *if* S *is continuous at* t;

iii. $ASt = SAt$ *and* $At = St$ *if* A, S *are continuous at* t.

Proof. The proof can be obtained easily, so we omit.

At the beginning of this section we give an example related to one of our concept which we investigate. This example illustrates that the pair (A, S) is compatible of type(C) in BA-cone metric space.

Example 3.2 *Let* $X = B = \mathbf{R}^2$, $K = \{(x, y) : x, y \geq \theta\}$ *with the metric* $d(x, y) = (|x - y|, \alpha|x - y|)$, *where* $\alpha \geq 0$ *is a constant. Then* X *is a* BA*-cone metric space since* $\mathbf{R}^2$ *is a real commutative Banach algebra. Suppose that* A *and* S *be self mappings of* (X, d) *as follows:*

$$A(x, y) = \left(3x^2 - 2, 3y^2 - 2\right) \quad \text{and} \quad S(x, y) = (x, y).$$

Let $x_n = \left(1 + \frac{1}{2n}, 1 + \frac{1}{n}\right)$, *for* $n \geq 1$, *then*

$$\lim_{n\to\infty} d(ASx_n, SSx_n) \leq \frac{1}{3}\left[\lim_{n\to\infty} d(ASx_n, At) + \lim_{n\to\infty} d(At, AAx_n) + \lim_{n\to\infty} d(At, SSx_n)\right].$$

Hence, A *and* S *are compatible mappings of type(C).*

Theorem 3.3 *Let* $A, B, S, T : X \to X$ *be continuous self-mappings of a complete* BA*-cone metric space* (X, d) *with a non-normal cone* K *satisfying the following conditions:*

i. $S(X) \subseteq B(X)$ *and* $T(X) \subseteq A(X)$;

ii.

$$d(Sx, Ty) \leq \alpha d(Ax, By) + \beta \frac{d(Ax, Sx)\, d(By, Ty)}{d(Ax, Ty) + d(By, Sx) + d(Ax, By)} \tag{1}$$

where α, $\beta > 0$ *and* $\alpha + \beta < 1$;

iii. one of the mappings A, B, S *or* T *is continuous;*

iv. the pairs (A, S) *and* (B, T) *are compatible of type(C) on* X.

Then A, B, S *and* T *have a unique common fixed point in* X.

Proof. Let x_0 be any arbitrary element of X, then by (1) there exists $x_1 \in X$ such that $Ax_1 = Tx_0$ and for x_1 there exists $x_2 \in X$ such that $Bx_2 = Sx_1$ and so on. Continuing this process we can define a sequence $\{y_n\}$ in X such that

$$y_{2n+1} = Ax_{2n+1} = Tx_{2n} \quad \text{and} \quad y_{2n} = Bx_{2n} = Sx_{2n-1}.$$

By condition (1), we have

$$\begin{aligned} d(y_{2n}, y_{2n+1}) &= d(Sx_{2n-1}, Tx_{2n}) \leq \alpha d(Ax_{2n-1}, Bx_{2n}) \\ &+\beta \frac{d(Ax_{2n-1}, Sx_{2n-1})\, d(Bx_{2n}, Tx_{2n})}{d(Ax_{2n-1}, Tx_{2n}) + d(Bx_{2n}, Sx_{2n-1}) + d(Ax_{2n-1}, Bx_{2n})} \\ &= \alpha d(y_{2n-1}, y_{2n}) \\ &+\beta \frac{d(y_{2n-1}, y_{2n})\, d(y_{2n}, y_{2n+1})}{d(y_{2n-1}, y_{2n+1}) + d(y_{2n}, y_{2n}) + d(y_{2n-1}, y_{2n})} \\ &= \alpha d(y_{2n-1}, y_{2n}) + \beta \frac{d(y_{2n-1}, y_{2n})\, d(y_{2n}, y_{2n+1})}{d(y_{2n-1}, y_{2n+1}) + d(y_{2n-1}, y_{2n})}. \end{aligned}$$

As $d(y_{2n}, y_{2n+1}) \leq d(y_{2n}, y_{2n-1}) + d(y_{2n-1}, y_{2n+1})$, we get

$$d(y_{2n}, y_{2n+1}) \leq \alpha d(y_{2n-1}, y_{2n}) + \beta \frac{d(y_{2n-1}, y_{2n})\, d(y_{2n}, y_{2n+1})}{d(y_{2n}, y_{2n+1})}.$$

Then, $d(y_{2n}, y_{2n+1}) \leq \lambda d(y_{2n-1}, y_{2n})$ is obtained, where $\lambda = (\alpha + \beta) < 1$. Continuing this way, the following relation holds;

$$d(y_n, y_{n+1}) \leq \lambda d(y_{n-1}, y_n) \leq \lambda^2 d(y_{n-2}, y_{n-1}) \leq \ldots \leq \lambda^n d(y_0, y_1)$$

for all $n \in \mathbf{N}$. It is obvious that the following inequality holds for $m > n$,

$$\begin{aligned} d(y_n, y_m) &\leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \ldots + d(y_{m-1}, y_m) \\ &\leq \left(\lambda^n + \lambda^{n+1} + \lambda^{n+2} + \ldots + \lambda^{m-1}\right) d(y_0, y_1) \leq \frac{\lambda^n}{1-\lambda} d(y_0, y_1). \end{aligned}$$

Let $\theta \ll c$ be given. Choose $\delta > 0$ such that $c + N_\delta(\theta) \subseteq K$, where $N_\delta(0) = \{y \in E : \|y\| \leq \delta\}$. Also, choose a natural number N, such that $\frac{\lambda^n}{1-\lambda} d(y_0, y_1) \in N_\delta(0)$, for all $n \geq N_1$. Then $\frac{\lambda^n}{1-\lambda} d(y_0, y_1) \ll c$, for all $n \geq N_1$. Thus, $d(y_n, y_m) \frac{\lambda^n}{1-\lambda} d(y_0, y_1) \ll c$, for all $m > n$. Therefore, $\{y_n\}_{n \geq 1}$ is a Cauchy sequence in (X, d). Since, X is a complete BA-cone metric space, there exists $z \in X$ such that $y_n \to z$, $n \to \infty$. Consequently, the subsequences $\{Ax_{2n+1}\}$, $\{Tx_{2n}\}$, $\{Bx_{2n}\}$, $\{Sx_{2n-1}\}$ converge to z.

Now, let us suppose that A is continuous. Since S and A are compatible of type(C) on X, then by proposition 3.1, we have

$$d(ASx_{2n}, Az) \ll \left[\frac{1-\alpha}{\alpha+\beta}\left(\frac{c}{4}\right)\right] \quad \text{and} \quad d(SSx_{2n}, Az) \ll \left[\frac{1-\alpha}{\alpha+\beta}\left(\frac{c}{4}\right)\right]$$

for a natural number N_2 such that $n \geq N_2$. Now by (1), we get

$$\begin{aligned} d(Az, z) &\leq d(Az, SSx_{2n}) + d(SSx_{2n}, Tx_{2n}) + d(Tx_{2n}, z) \\ &\leq d(Az, SSx_{2n}) + \alpha d(ASx_{2n}, Bx_{2n}) \\ &+\beta \frac{d(ASx_{2n}, SSx_{2n})\, d(Bx_{2n}, Tx_{2n})}{d(ASx_{2n}, Tx_{2n}) + d(Bx_{2n}, SSx_{2n}) + d(ASx_{2n}, Bx_{2n})} + d(Tx_{2n}, z). \end{aligned}$$

which implies that

$$\begin{aligned} d(Az, z) &\leq d(Az, SSx_{2n}) + \alpha d(ASx_{2n}, Bx_{2n}) \\ &+\beta \frac{d(ASx_{2n}, SSx_{2n})\, d(Bx_{2n}, Tx_{2n})}{d(Tx_{2n}, Bx_{2n})} + d(Tx_{2n}, z), \end{aligned}$$

so we have

$$\begin{aligned} d(Az, z) &\leq \frac{1+\beta}{1-\alpha} d(Az, SSx_{2n}) + \frac{\alpha+\beta}{1-\alpha} d(ASx_{2n}, Bx_{2n}) \\ &+ \frac{\beta}{1-\alpha} d(z, y_{2n}) + \frac{1}{1-\alpha} d(y_{2n+1}, z). \end{aligned}$$

Therefore, $d(Az, z) \ll \frac{c}{i}$ for all $i \geq 1$. Hence, $\frac{c}{i} - d(Az, z) \in K$ for all $i \geq 1$. Since K is closed, $-d(Az, z) \in K$ and so $d(Az, z) = 0$. Thus, $Az = z$.
Now, we show that z is fixed point of S. Using (1),

$$\begin{aligned} d(Sz, z) &\leq d(Sz, Tx_{2n}) + d(Tx_{2n}, z) \\ &\leq \alpha d(Az, Bx_{2n}) + \beta \frac{d(Az, Sz)\, d(Bx_{2n}, Tx_{2n})}{d(Az, Tx_{2n}) + d(Bx_{2n}, Sz) + d(Az, Bx_{2n})} + d(Tx_{2n}, z). \end{aligned}$$

and

$$d(Sz, z) \leq \alpha d(Az, Bx_{2n}) + \beta \frac{d(Az, Sz)\, d(Bx_{2n}, Tx_{2n})}{d(Bx_{2n}, Tx_{2n})} + d(Tx_{2n}, z).$$

which implies that

$$d(Sz, z) \leq \frac{\alpha+\beta}{1-\beta} d(Az, z) + \frac{\alpha}{1-\beta} d(z, y_{2n}) + \frac{1}{1-\beta} d(y_{2n+1}, z).$$

Hence, $d(Sz, z) \ll \frac{c}{i}$ for all $i \geq 1$. Therefore, $\frac{c}{i} - d(Sz, z) \in K$ for all $i \geq 1$. Since K is closed, $-d(Sz, z) \in K$ and so $d(Sz, z) = 0$.Thus, $Sz = z$.

Consequently, z is the common fixed point of A and S.

Since $S(X) \subseteq B(X)$ and $Sz = z$, $z \in B(X)$. Also, B is self map of X, so there exists a point $u \in X$ such that $z = Sz = Bu$. We show that $Tu = z$. By condition (1) we get

$$d(z, Tu) = d(Sz, Tu)$$

$$\leq \alpha d(Az, Bu) + \beta \frac{d(Az, Sz)\, d(Bu, Tu)}{d(Az, Tu) + d(Bu, Sz) + d(Az, Bu)}.$$

Because $z = Sz = Az$ and $Sz = Bu$, we obtain $d(z, Tu) = 0$, i.e. $z = Tu$. Thus, $z = Tu = Bu$. By proposition 2.6, we have $TBu = BTu$. Hence $Tz = Bz$. Here, we illustrate that z is common fixed point of B and T.

$$d(z, Tz) = d(Sz, Tz) \leq \alpha d(Az, Bz) + \beta \frac{d(Az, Sz)\, d(Bz, Tz)}{d(Az, Tz) + d(Bz, Sz) + d(Az, Bz)},$$

which implies that

$$d(z, Tz) \leq \alpha d(z, Tz).$$

Hence, $z = Tz$ and consequently $z = Tz = Bz$. Therefore, z is common fixed point of A, B, T and S. Similarly, we can prove this theorem by taking one of the mapping B, T or S is continuous.
The uniqueness of common fixed point of A, B, T and S can be obtained easily.

Corollary 3.4 *Let (X, d) be a complete BA-cone metric space with a non-normal cone K. Let the mappings A, B, S and T be self-maps of X such that $S(X) \subseteq B(X)$, $T(X) \subseteq A(X)$ and satisfying inequality (1). Suppose that (A, S) and (B, T) are weakly compatible pairs, then A, B, S and T have a unique common fixed point in X.*

Proof. We can prove that there exists $z \in X$ such that $y_n \to z$, $n \to \infty$ using the same technique as in Theorem 3.3. Then, subsequences $\{Ax_{2n+1}\}$, $\{Bx_{2n}\}$, $\{Sx_{2n-1}\}$ and $\{Tx_{2n}\}$ converge to z. Since $Tx_{2n} \to z$ and $T(X) \subseteq A(X)$, there exists a point $u \in X$ such that $z = Au$. Now, we prove $Su = z$. Then, by (1), we have

$$d(Su, z) \leq d(Su, Tx_{2n}) + d(Tx_{2n}, z)$$

$$\leq \alpha d(Au, Bx_{2n}) + \beta \frac{d(Au, Su)\, d(Bx_{2n}, Tx_{2n})}{d(Au, Tx_{2n}) + d(Bx_{2n}, Su) + d(Au, Bx_{2n})} + d(Tx_{2n}, z)$$

and

$$d(Su, z) \leq \alpha d(Au, Bx_{2n}) + \beta \frac{d(Au, Su)\, d(Bx_{2n}, Tx_{2n})}{d(Au, Su)} + d(Tx_{2n}, z)$$

so that

$$d(Su,z) \leq (\alpha+\beta)\, d(z,y_{2n}) + (1+\beta)\, d(y_{2n+1},z).$$

Choose a natural number N_3 such that $d(y_{2n+1},z) \ll \left[\frac{c}{2}\left(\frac{1}{\alpha+\beta}\right)\right]$ for all $n \geq N_3$. Hence, $d(Su,z) \ll \frac{c}{i}$ for all $i \geq 1$. Therefore, $\frac{c}{i} - d(Su,z) \in K$ for all $i \geq 1$. Since K is closed, $-d(Su,z) \in K$ and so $d(Su,z) = 0$.Thus, $Su = z$. Consequently, we obtain $z = Au = Su$. Since $S(X) \subseteq B(X)$, there exists a point $v \in X$ such that $z = Bv$. Then, again using (1), we have

$$d(z,Tv) \leq d(Su,Tv) \leq \alpha d(Au,Bv) + \beta \frac{d(Au,Su)\, d(Bv,Tv)}{d(Au,Tv) + d(Bv,Su) + d(Au,Bv)}$$

which implies that $d(z,Tv) \leq 0$. Therefore $z = Tv$. Hence $z = Bv = Tv$. Thus $z = Au = Su = Tv = Bv$. Since A and S are weakly compatible maps, then $SAu = ASu$ i.e., $Sz = Az$. Now we show that z is a fixed point of S. If $Sz \neq z$, then by (1), we obtain

$$d(Sz,z) \leq d(Sz,Tv) \leq d(Az,Bv) + \beta \frac{d(Az,Sz)\, d(Bv,Tv)}{d(Az,Tv) + d(Bv,Sz) + d(Az,Bv)}$$

and $d(Sz,z) \leq \alpha d(Sz,z)$ which is a contradiction since $(1-\alpha) < 1$. Therefore $Sz = z$. Hence $z = Sz = Az$. Similarly, since the pair (B,T) is weakly compatible, we have $TBu = BTu$ i.e., $Tz = Bz$. And with the same procedure we get $Tz = z$, and consequently $z = Bz = Tz$. Thus, $z = Sz = Az = Bz = Tz$, i.e. z is common fixed point of A, B, T and S.
Uniqueness can be proved easily, too. Hence A, B, S, T have a unique common fixed point in X, respectively.

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