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A Genetic Algorithm for Option Pricing: The American Put Option

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Abstract

The search for a better option pricing model continues to find the one that outperforms the existing ones in the financial market. In this paper, we present a Genetic Algorithm (GA) to price a fixed term American put option when the underlying asset price is Geometric Brownian Motion. The Genetic Algorithm has a better approximation of the relationship between the option price and its contract terms. Our method produces a perfect and a minimum option price that outperforms other models like the Black-Scholes under the same conditions. The method requires minimum assumptions and can easily adapt to changes and uncertainties in the financial environments.

Keywords: Options, Geometric Brownian Motion, Genetic Algorithm, Black-Scholes Model

1 Introduction

Options are contract that gives the holder the right but not under obligation to trade an underlying asset at a strike price before or at the maturity date. If the seller of the option exercises the option then the buyer must fulfil the obligation to the transaction. The buyer pays a premium to the seller for the right of the transaction that has taken place. Options give a world of opportunities to investors and provides power to the investor to adjust the position of the situation in the market. It can be conserved or speculated depending on the investor, who can choose to protect the position from falling from the movement of a particular stock in the market. It can also be hedged to minimize risk of the movement of assets in the market.

GAs are numerical optimization techniques which are inspired by both natural and artificial genetics. Using population of solutions to solve practical optimization problems was brought up several times in the 1950s and 1960s. John Henry Holland in 1960s invented GA [6]. This brought up many insight in using GA to solve practical problems. Recently researchers use Holland's GA to research in image processing, facial recognition, laser technology, medicine, spacecraft trajectories, analysis of time series, robotics, jobshop scheduling, stock prediction and some problems in other fields of study.

S-H. Cheng and W-C. Lee (1997) demonstrated how GA was helpful in dealing with option pricing [3]. They tested the performance of GA in determining the prices of European call options whose exact solution was known from the Black Scholes option pricing theory. They used GENESIS 5.0 software which was developed by John Grefenstette for the study of GA for function optimisation [3]. They noticed that the boundary conditions using the GA was arbitrarily imposed and it only satisfied the case when the stock price was greater than the exercise price. Moreover, when the stock price was less than the exercise price at the date of expiry, some of the call prices appeared to be negative.

Many researches have been done on pricing options which did not consider pricing a fixed term American Put options with GA. We seek to examine and compare the famous Black-Scholes model and GA in relation to the way it affects option pricing and to determine the best method that gives the perfect

price of a fixed term American put option under the same conditions.
We propose that our method will help decision makers who invest in options to find the optimal time to stop and exercise the contract for them to gain and also reduce risk in the financial market. In addition, it will help the investors determine how much they are worth so that the assets can be bought and sold with confidence.

2 Preliminary Notes / Materials and Methods

Definition 2.1 *We consider a standard Brownian motion as a random process $W = \{W_t : t \in [0, \infty)\}$ with a space $\mathbb{R}$ that satisfies the following properties:*

- $W_0 = 0$
- *W has stationary increments, $\forall s, t \in [0, \infty)$ with $s < t$, the distribution depends only on the difference $W_t - W_s$ that is (W_{t-s})*
- *W has independent increments, $t_1, t_2, \cdots t_d \in [0, \infty)$ with $t_1 < t_2 < \cdots t_d$, then $W_{t_1}, W_{t_2} - W_{t_1}, \cdots, W_{t_d} - W_{t_{d-1}}$ are independent.*
- *W_t is normally distributed with mean 0 and variance t, $\forall\, t \in [0, \infty)$*
- *W_t is continuous on $[0, \infty)$.*

We note that models are important for investors and those who trade in the financial world. They use it to predict accurately the future behaviour of prices of an asset and fluctuations of their prices in the market so that when they intend to buy or sell, they do that at the right time. Geometric Brownian Motion is a good model for market prices because it is everywhere positive and it is with probability of 1.

The price of the underlying asset S_t is given by a Geometric Brownian Motion Model,

$$\frac{dS_t}{S_t} = \rho dt + \sigma dW_t. \tag{1}$$

We determine the underlying asset price using Itô Lemma.

Lemma 2.2 *We consider an Itô formula on S_t which is a stochastic process and $f(a,t)$ is a measurable function with a continuous partial derivatives up to the second order term then,*

$$df(t,S_t) = \frac{\partial f}{\partial t}(t,S_t)dt + \frac{\partial f}{\partial a}(t,S_t)dS_t + \frac{1}{2}\frac{\partial^2 f}{\partial a^2}(t,S_t)(dS_t)^2.$$

We assume $W(t)$ as a one-dimensional standard Brownian motion with respect to the risk neutral measure Q and $dW_t \sim N[0,dt]$. The volatility rate, σ is constant, ρ is the constant force of interest. We note that S_t is the underlying asset price at time t.

Let W_t, $0 \leq t \leq T$ be a Brownian motion on a probability space $(\Omega, \mathcal{F}, Q)$ and $\{\mathcal{F}(t), 0 \leq t \leq T\}$ be a filtration for this Brownian motion where T is a fixed final time.

From (1) we have,

$$\frac{dS_t}{S_t} = \rho\, dt + \sigma\, dW_t\,, \qquad \text{assuming } W_0 = 0\,.$$

$$dS_t = S_t\,[\rho\, dt + \sigma dW_t]\,.$$

Solving for S_t we apply Itô formula to $d\ln S_t$,

$$\begin{aligned} d\ln S_t &= \frac{1}{S_t}dS_t - \frac{1}{2S_t^2}dS_t^2, \\ d\ln S_t &= \frac{1}{S_t}S_t[\rho\, dt + \sigma dW_t] - \frac{1}{2S_t^2}S_t^2[\sigma^2 dW_t^2], \\ d\ln S_t &= \rho\, dt + \sigma dW_t - \frac{1}{2}\sigma^2 dt. \end{aligned}$$

Integrating and applying fundamental theorem of calculus we have,

$$\ln S_t - \ln S_0 = \left(\rho - \frac{\sigma^2}{2}\right)t + \sigma W_t,$$

which gives the price of the underlying asset as,

$$S_t = S_0 \exp\left[\left(\rho - \frac{\sigma^2}{2}\right)t + \sigma W_t\right]. \tag{2}$$

2.1 Expectation of the Underlying asset

From (2) we have,

$$E[S_t] = E\left[S_0 \exp\left[\left(\rho - \frac{\sigma^2}{2}\right)t + \sigma W_t\right]\right],$$
$$E[S_t] = S_0 \exp\left[\left(\rho - \frac{\sigma^2}{2}\right)t\right] E[e^{\sigma W_t}],$$

but $W_t = W_t - W_0$ follows a normal distribution $N(0, t)$, then,

$$E[e^{\sigma W_t}] = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{\sigma x} e^{-\frac{x^2}{2t}} dx.$$

We have,

$$-\frac{(x - \sigma t)^2}{2t} = -\frac{x^2}{2t} + \sigma x - \frac{\sigma^2 t}{2},$$
$$e^{\sigma x} e^{-\frac{x^2}{2t}} = e^{-\frac{(x-\sigma t)^2}{2t}} e^{\frac{(\sigma^2 t)}{2}},$$

which gives,

$$E[e^{\sigma W_t}] = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-\sigma t)^2}{2t}} e^{\frac{\sigma^2 t}{2}} dx,$$
$$E[e^{\sigma W_t}] = e^{\frac{\sigma^2 t}{2}} \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-\sigma t)^2}{2t}} dx,$$

then,

$$E[e^{\sigma W_t}] = e^{\frac{\sigma^2 t}{2}}.$$

$$E[S_t] = S_0 e^{(\rho - \frac{\sigma^2}{2})t} e^{\frac{\sigma^2 t}{2}},$$

which gives the expectation of the underlying asset as,

$$E[S_t] = S_0 e^{\rho t}. \qquad (3)$$

2.2 The variance of the Underlying asset

From (3) we have,

$$(E[S_t])^2 = S_0^2 e^{2\rho t}.$$

$$E[S_t^2] = E\left(\left(S_0 \exp\left[\left(\rho - \frac{\sigma^2}{2}\right)t + \sigma W_t\right]\right)^2\right),$$
$$E[S_t^2] = \left(S_0 \exp\left(\rho - \frac{\sigma^2}{2}\right)t\right)^2 E[e^{2\sigma W_t}].$$

$$E[e^{2\sigma W_t}] = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{2\sigma x} e^{-\frac{x^2}{2t}} dx.$$
$$-\frac{(x-2\sigma t)^2}{2t} = -\frac{x^2}{2t} + 2\sigma x - 2\sigma^2 t,$$

then,

$$e^{2\sigma x} e^{-\frac{x^2}{2t}} = e^{-\frac{(x-2\sigma t)^2}{2t}} e^{2\sigma^2 t},$$

which gives,

$$E[e^{2\sigma W_t}] = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-2\sigma t)^2}{2t}} e^{2\sigma^2 t} dx,$$
$$E[e^{2\sigma W_t}] = e^{2\sigma^2 t} \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(x-2\sigma t)^2}{2t}} dx.$$

Then,

$$E[e^{2\sigma W_t}] = e^{2\sigma^2 t}.$$

$$E[S_t^2] = \left(S_0 e^{(\rho - \frac{\sigma^2}{2})t}\right)^2 e^{2\sigma^2 t},$$
$$E[S_t^2] = S_0^2 e^{2\rho t} e^{-\sigma^2 t} e^{2\sigma^2 t},$$
$$E[S_t^2] = S_0^2 e^{(2\rho + \sigma^2)t}.$$

$$Var(S_t) = E[S_t^2] - (E[S_t])^2,$$
$$Var(S_t) = S_0^2 e^{(2\rho+\sigma^2)t} - S_0^2 e^{2\rho t},$$

which gives the variance of the underlying asset as

$$Var(S_t) = S_0^2 e^{2\rho t} \left(e^{\sigma^2 t} - 1\right).$$

In simulating the stock price in equation (2), the following initial numerical values are assigned: initial underlying asset price S_0= \$ 100, the volatility rate $\sigma = 0.35$, the interest rate $\rho = 10\%$ and the maturity time $T = 4$ years.

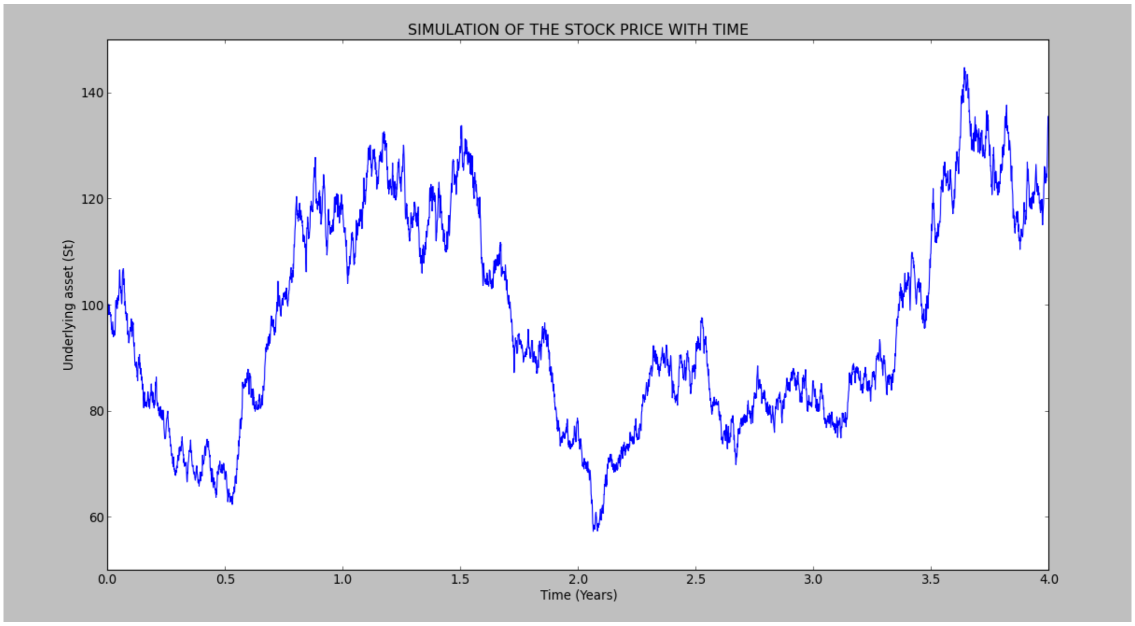

Figure 1: A Simulated Underlying Asset Price that Follows a Geometric Brownian Motion Model.

The graph in (2.2) shows a simulated underlying asset price over the period of $[0, T]$. We observe from the graph that the underlying asset price is positive everywhere and also changes along with time. A put option can be exercised at $t = 3.6$years.

2.3 Black Scholes model

"The Pricing of Options and Corporate Liabilities" was published by Fischer Black and Myron Scholes in 1973 and they derived the Black Scholes equations [1]. Black Scholes model is deduced from the mathematical module that gives a theoretical estimates of the price of a European option. This model has gone through researches and empirical test and it has been noticed that the price of this model is 'fairly close' to the prices observed in the financial market. The Black Scholes model estimates the price of the option over time and this was done to prevent loss in the market. Prevention of loss can be done by means of hedging the option through buying and selling of the underlying asset at the right way and the right time [1].

2.4 Pricing a Fixed term American Put Option

We consider K as the strike price and S_t is the underlying asset price at time t. Then if $K > S_t$, then the investor would exercise a put option and have the payoff of the exercised put option as,

$$P_t = (K - S_t)^+.$$

If $K < S_t$, then the investor would exercise a call option and have the following payoff of the exercised call option;

$$P_t = (S_t - K)^+.$$

Let ρ be the risk-free interest rate, then the present value of the payoff is,

$$e^{-\rho t} P_t = e^{-\rho t}(S_t - K)^+.$$

We assume an equally spaced dates $t_0, t_1, \cdots, t_d$ where $t_0 = 0$, $t_d = T$ which is possible to exercise an American put option. We also assume the underlying asset price S_t is $\mathcal{F}_t$ measurable and S_t is adapted to $\{\mathcal{F}_t\}_{0\leq t\leq T}$.

Since $P_t = (K - S_t)^+$ is the payoff from exercising the American put option, we let $M_t = M(S_t)$ be the option price at time t. At maturity $t_d = T$ the payoff of the option is given as $M_T = P_T$. At t_{d-1} the option holder can exercise immediately and get intrinsic value of $P_{t_{d-1}}$ or wait till the maturity time t_d and exercise it. Then the value of the American put option at that time t_{d-1} is the maximum between intrinsic value and the discounted expected future value of the option. The option price the holder gets at the time before maturity t_{d-1} is,

$$M_{t_{d-1}} = \max(P_{t_{d-1}}, e^{-\rho\delta t} E[M_{t_d} | \mathcal{F}_{t_{d-1}}]),$$

where δt is the length of time from one step to the other.

Let τ be the stopping time for an option price then we have $\{\tau \leq t_j\}$: $\{w|\tau(w) \leq t_j\} \in \mathcal{F}_{t_j}$, $\quad j = 0, 1, \cdots, d$ and also suppose we have $h_{0,T}$ to be the set of all stopping times τ of an option price over the interval $[0, T]$. If $\tau = t_k$ then we have,

$$\tilde{M}^{\tau}_{t_j} := \begin{cases} \tilde{M}_{t_j} & 0 \leq j < k \\ \tilde{M}_{t_k} & k \leq j \leq T \end{cases} \quad ,$$

where $\tilde{M}^{\tau}_{t_j}$ denotes the present value of the option price at time t_j and τ is the time where it is possible to exercise the option. Also, we let $c = \inf\{\tau \in$

$\mathcal{T}_{0,T}|\tilde{M}_\tau = \tilde{P}_\tau\}$ be the optimal stopping time which is the smallest possible time for exercising the option.

Then, $\{\tilde{M}^c_{t_j}\}_{j=0}^d$ is a martingale. Hence,

$$M_0 = E[\tilde{M}^c_T] = E[\tilde{M}_c] = E[\tilde{P}_c] = \sup_{\tau \in \mathcal{T}_{0,T}} E[\tilde{P}_\tau].$$

The optimal stopping time is the smallest time for a path of the simulated underlying asset price $S_t(w_i)$ at the point where the option holder should exercise the option. When the option value equals the intrinsic value then we obtain the optimal stopping time.

3 Option pricing using Genetic Algorithm

We assign numerical values to the parameters to find the value of the option and the optimal stopping time of a fixed term American put option when the underlying asset price follows the Geometric Brownian Motion model as follows: S_0= \$ 100, K= \$ 120, $\sigma = 0.35$, $\rho = 10\%$ and $T = 4$ years.

A random population of asset prices were generated. A fitness function of $(\max\{K - S_t, 0\})$ was used to select the fittest organisms that will survive the process. Roulette wheel was used to select individuals at random for crossover and mutation. One-point crossover was applied to the individuals selected. Then, flip-bit mutation was applied. The stopping time and the option price were found and calculated.

We note a summary of the GA to price a fixed term American put option when the underlying asset price is Geometric Brownian motion as follows:

i. Generate a random population of asset Prices

ii. Test each individual for fitness using $(\max\{K - S_t, 0\})$.

iii. Using Roulette wheel selection, select individuals from the population generated.

iv. Decode each individual into binary form.

v. Select pairs of individuals and crossover (One-point crossover).

vi. Apply the mutation operator to every individual (flip-bit mutation).

vii. Encode each new generations into real numbers.

viii. Set the option value before or at maturity time to be equal to the intrinsic value before or at maturity time.

ix. Set τ to be the optimal stopping time.

x. Calculate for the Option Prices and find the average.

xi. Calculate for the stopping times and find the average.

3.1 Option pricing function

The code below using python 2.7.3 software price a fixed term American put option using our GA when the underlying asset price is Geometric Brownian Motion.

```
from __future__ import division
import math
import random
import pylab
import numpy as np
S0=100
K=120
p=0.1
sigma=0.35
T=4
m=4
dt = 0.001
n1=10
ut=[]
at=[]
bt=[]
lt=[]
l=[]
qt=[]
v=[S0]
al=[]


for i in range (n1):
```

```
W = np.random.standard_normal(size = n1)
W = np.cumsum(W)*np.sqrt(dt)
X = (p-0.5*sigma**2)*T + sigma*W
Stj=S0*np.exp(X)
l.append(Stj)
Pt= np.max (K-Stj, 0)
ut.append(Pt)
ints = [int(float(num)) for num in ut]
al.append(ints)
for a in al:
for b in al:
a!=b
for h in a:
x1 = bin(h)
for g in b:
y1 = bin(g)
d1 = list(x1)
d2 = list(y1)


del d1[0:2]
del d2[0:2]
if len(d1)%2 != 0:
d1.insert(0,'0')
if len(d2)%2 != 0:
d2.insert(0,'0')

offspring1 = []
offspring2 = []
m1 = len(d1)
m11 = m1/2
m111 = int(m11)
m2 = len(d1)
n11 = len(d2)/2
n111 = int(n11)


for i in range(n111):
offspring1.append(d2[i])

for i in range(m111,m2):
offspring1.append(d1[i])
```

```
for i in range(m111):
offspring2.append(d1[i])

for i in range(n111,len(d2)):
offspring2.append(d2[i])

stroffspring1 = ""
stroffspring2 = ""
strflip1 = ""
strflip2 = ""
flipoffspring1 =[]
flipoffspring2 = []

for x in offspring1:
stroffspring1 = stroffspring1 + x

for x in offspring2:
stroffspring2 = stroffspring2 + x


for i in range(len(offspring1)):
if offspring1[i] =="0":
flipoffspring1.append("1")
elif offspring1[i] =="1":
flipoffspring1.append("0")

for i in range(len(offspring2)):
if offspring2[i] =="0":
flipoffspring2.append("1")
elif offspring2[i] =="1":
flipoffspring2.append("0")

for x in flipoffspring1:
strflip1 = strflip1 + x


for x in flipoffspring2:
strflip2 = strflip2 + x
```

```
intoffspring1 = int(stroffspring1, base =2)
intoffspring2 = int(stroffspring2, base =2)

intflip1 = int(strflip1, base =2)
intflip2 = int(strflip2, base =2)

Pts= max (intflip1,intflip2)
lt.append (Pts)
li=[]
uti=[]
ali=[]
#stopping time
tau=[T for i in range(n1+1)]
for j in reversed(range(m)):
E=sum(lt)/(len(lt))
for i in range (n1):
tj=j*dt
W = np.random.standard_normal(size = n1)
W = np.cumsum(W)*np.sqrt(dt)
X = (p-0.5*sigma**2)*tj + sigma*W
Stj1=S0*np.exp(X)
li.append(Stj1)
Pt= np.max (K-Stj1, 0)
uti.append(Pt)
ints1 = [int(float(num)) for num in uti]
ali.append(ints1)
for aa in ali:


for bb in al:
aa!=bb
for hh in aa:
x11 = bin(hh)
for gg in bb:
y11 = bin(gg)
d11 = list(x11)
d22 = list(y11)


del d11[0:2]
del d22[0:2]
if len(d11)%2 != 0:
```

```
d11.insert(0,'0')
if len(d22)%2 != 0:
d22.insert(0,'0')

offspring11 = []
offspring22 = []
m11 = len(d11)
m111 = m11/2
m1111 = int(m111)
m22 = len(d11)
n111 = len(d22)/2
n1111 = int(n111)


for i in range(n1111):
offspring11.append(d22[i])

for i in range(m1111,m22):
offspring11.append(d11[i])


for i in range(m1111):
offspring22.append(d11[i])

for i in range(n1111,len(d22)):
offspring22.append(d22[i])

stroffspring11 = ""
stroffspring22 = ""
strflip11 = ""
strflip22 = ""
flipoffspring11 =[]
flipoffspring22 = []

for xi in offspring11:
stroffspring11 = stroffspring11 + xi

for xi in offspring22:
stroffspring22 = stroffspring22 + xi


for i in range(len(offspring11)):
```

```
if offspring11[i] =="0":
flipoffspring11.append("1")
elif offspring11[i] =="1":
flipoffspring11.append("0")

for i in range(len(offspring22)):
if offspring22[i] =="0":
flipoffspring22.append("1")
elif offspring22[i] =="1":
flipoffspring22.append("0")

for xi in flipoffspring11:
strflip11 = strflip11 + xi


for xi in flipoffspring22:
strflip22 = strflip22 + xi


intoffspring11 = int(stroffspring11, base =2)
intoffspring22 = int(stroffspring22, base =2)

intflip11 = int(strflip11, base =2)
intflip22 = int(strflip22, base =2)

Pts1= max (intflip11,intflip22)

lt[i]=max(Pts1,math.exp (-p*dt)*E)
if lt[i]==Pts1:
tau[i]=tj

print  'the optimal stopping time is', sum(tau)/n1
print 'the option price is',sum(lt)/(len(lt))
```

4 Results and Discussion

4.1 Discussion

From the code above, the output gives the option price as \$ 4.12 and the optimal stopping time as 3.6 years. When equation (2) is used to calculate the underlying asset price at time $\tau = 3.6$, we obtain the the price of the underlying asset as \$ 116.23. This means at $\tau = 3.6$years, the option seller will make a profit of \$ 3.77 since the price at that time will rise to \$ 116.23.

We use the same numerical values to find the option price using the Black Scholes model. We note S_0= \$ 100, K = \$ 120, $\sigma = 0.35$, $\rho = 10\%$ and using $\tau = 3.6$years which was the optimal stopping time obtained in our GA, we now use the equation,

$$\text{Put} = N(-d_2)Ke^{-\rho(\tau)} - N(-d_1)S$$

to price the American put option. $N(\cdot)$ is the standard normal cumulative distribution function,

$$d_1 = \frac{1}{\sigma\sqrt{\tau}}\left[\ln\left(\frac{S}{K}\right) + \left(\rho + \frac{\sigma^2}{2}\right)(\tau)\right]$$

and

$$d_2 = \frac{1}{\sigma\sqrt{\tau}}\left[\ln\left(\frac{S}{K}\right) + \left(\rho - \frac{\sigma^2}{2}\right)(\tau)\right].$$

We programmed this formula in python 2.7.3 software using equation (2) and the formula for the put option to obtain \$ 11.91 at $\tau = 3.6$ years.

5 Results

We found the option price of the Black Scholes Model to be \$ 11.91 in the 3.6th year whiles the GA gives an option price to be \$ 4.12 in the 3.6th year without change in conditions.

This means that it is better to price an American put option using GA than using Black Scholes model. From the results obtained, early exercise is the best for an American put option when the underlying asset price is Geometric Brownian Motion. Also any investor will prefer a minimum option price under the same conditions than a higher price in option. Option investors should practise the use of GA to obtain a perfect minimum price in option which will give them the opportunity to gain more and make option pricing more interesting.

From pricing of option using GA, if the holder of the option pays an option price of \$ 4.12 at $t = 0$ and exercises at time $\tau = 3.6$, then the option holder gets intrinsic value of \$ 3.77. The seller of the option buys α assets and β bonds at time $t = 0$ with the \$ 4.12 received to obtain the same payoff as the buyer at the exercise time.

6 Conclusion

We used Geometric Brownian motion to simulate the underlying asset. GA and Black Scholes model were used to calculate the option price and the optimal stopping time of a fixed term American put option with the help of Python 2.7.3 software for the programming. We analysed the performance of Black Scholes model and the performance of GA to determine the one that gives the minimum price of a fixed term American put option under the same conditions. For an American option, using our GA, we assigned values giving it a starting time and a stopping time. If it is possible to exercise early then we assign a stopping time in which the option could be exercised.

We have proposed a GA to price options and find the optimal stopping time. A perfect price of an American put option was obtained using GA which was lower than the American put option using the Black Scholes model. For American put option exercising early is the best way. The results are encouraging and that our GA approach performed better than the Black Scholes model.

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