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On Common Fixed Point for Converse Commuting Maps in IFMS

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Abstract

In this paper, we introduce the notion of converse commuting maps in IFMS. Also, we obtain common fixed point theorems for converse commuting maps in IFMS.

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1. Introduction

Several authors([1],[2],[3],[4]) studied and developed the concept in different direction and proved some fixed point in fuzzy metric space. Also, Lii[5] proved common fixed point for converse commuting on metric spaces, and Vijayaraju et.al.[8] obtained some common fixed point theorems in fuzzy metric space. Recently, Park et.al.[7] introduced the IFMS, and proved common fixed point theorem in IFMS. Also, Popa[6] proved some properties and a general fixed point theorem for converse commuting multivalued maps in symmetric spaces.

In this paper, we introduce the notion of converse commuting maps in IFMS. Also, we obtain common fixed point theorems for converse commuting maps in IFMS.

2. Preliminaries

In this part, we recall some definitions, properties and known results in the IFMS as following :

Let us recall(see [9]) that a continuous $t-$norm is a operation $* : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions: (a)$*$ is commutative and associative, (b)$*$ is continuous, (c)$a * 1 = a$ for all $a \in [0,1]$, (d)$a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a,b,c,d \in [0,1])$. Also, a continuous $t-$conorm is a operation $\diamond : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions: (a)$\diamond$ is commutative and associative, (b)$\diamond$ is continuous, (c)$a \diamond 0 = a$ for all $a \in [0,1]$, (d)$a \diamond b \geq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a,b,c,d \in [0,1])$.

Definition 2.1. ([7])The $5-$tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space(Shortly, IFMS) if X is an arbitrary set, $*$ is a continuous $t-$norm, $\diamond$ is a continuous $t-$conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that

(a)$M(x, y, t) > 0$,
(b)$M(x, y, t) = 1$ if and only if $x = y$,
(c)$M(x, y, t) = M(y, x, t)$,
(d)$M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(e)$M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous,
(f)$N(x, y, t) > 0$,
(g)$N(x, y, t) = 0$ if and only if $x = y$,
(h)$N(x, y, t) = N(y, x, t)$,
(i)$N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
(j)$N(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Note that (M, N) is called an IFM on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Definition 2.2. Let A, B be self maps on IFMS X.

(a)The pair (A, B) is called a commuting if $ABx = BAx$ for $x \in X$.
(b)The pair (A, B) is said to be converse commuting if $ABx = BAx$ implies $Ax = Bx$.

In this part, $C(A, B)$ denotes the set of converse commuting points of A and B.

3. Main results

Theorem 3.1. *Let A, B, S, T be self maps on IFMS X such that the pairs (A, S) and (B, T) are converse commuting maps satisfying*

$$\begin{aligned}
&\phi\{M(Sx, Ty, t), M(Ax, By, t), M(Ax, Sx, t), M(By, Ty, t),\\
&\qquad M(Ax, Ty, t), M(By, Sx, t)\} \geq 0 \qquad (3.1)\\
&\psi\{N(Sx, Ty, t), N(Ax, By, t), N(Ax, Sx, t), N(By, Ty, t),\\
&\qquad N(Ax, Ty, t), N(By, Sx, t)\} \leq 1
\end{aligned}$$

for every $x, y \in X$, $t > 0$, $\phi : [0,1]^6 \to [0,1]$ and $\psi : [0,1]^6 \to [0,1]$ are continuous functions satisfying

(a)$\phi(t, t, 1, 1, t, t) \geq 0$ or $\phi(t, t, t, 1, t, t) \geq 0$ implies that $t = 1$, and $\psi(t, t, 0, 0, t, t) \leq 1$ or $\psi(t, t, t, 0, t, t) \leq 1$implies that $t = 0$ for all $t > 0$.

If A and S, B and T have a commuting, then A, B, S and T have a unique common fixed point in IFMS X.

Proof. Let $u \in C(A,S)$ and $v \in C(B,T)$, then if $ASu = SAu$, then $Au = Su$. Hence $M(Au, Su, t) = 1$, $N(Au, Su, t) = 0$ and $AAu = ASu = SAu$. Also, $M(AAu, SAu, t) = 1$ and $N(AAu, SAu, t) = 0$. Similarly, if $BTv = TBv$, then $Bv = Tv$. Hence $M(Bv, Tv, t) = 1$, $N(Bv, Tv, t) = 0$ and $BBv = BTv = TBv$. Also, $M(BBv, TBv, t) = 1$ and $N(BBv, TBv, t) = 0$.

First, we prove that $Au = Bv$. If $Au \neq Bv$, then $M(Au, Bv, t) < 1$, $N(Au, Bv, t) > 0$. Using (3.1) with $x = u, y = v$, we get

$$\phi\{M(Su,Tv,t), M(Su,Tv,t), 1, 1, M(Su,Tv,t), M(Tv,Su,t)\} \geq 0$$
$$\psi\{N(Su,Tv,t), N(Su,Tv,t), 0, 0, N(Su,Tv,t), N(Tv,Su,t)\} \leq 1.$$

Also,ϕ, ψ satisfies (a), so $M(Su, Tv, t) = 1$ and $N(Su, Tv, t) = 0$. This is a contradiction. Hence $Su = Tv$. Therefore $Au = Bv$.

Now, we prove that $A^2u = Au$. Suppose that $A^2u \neq Au$, then $M(AAu, Au, t) < 1$ and $N(AAu, Au, t) > 0$. Using (3.1) with $x = Au, y = v$, we have

$$\begin{aligned}\phi\{&M(SAu,Tv,t), M(AAu,Bv,t), M(AAu,Su,t), 1,\\ &M(AAu,Tv,t), M(Bv,SAu,t)\} \geq 0\\ \psi\{&N(SAu,Tv,t), N(AAu,Bv,t), N(AAu,Su,t), 0,\\ &N(AAu,Tv,t), N(Bv,SAu,t)\} \leq 1.\end{aligned}$$

As $Au = Su$ and $AAu = ASu = SAu$ and ϕ, ψ satisfies (a),

$$\begin{aligned}\phi\{&M(AAu,Au,t), M(AAu,Au,t), M(AAu,Au,t), 1,\\ &M(AAu,Au,t), M(AAu,Au,t)\} \geq 0\\ \psi\{&N(AAu,Au,t), N(AAu,Au,t), N(AAu,Au,t), 0,\\ &N(AAu,Au,t), N(AAu,Au,t)\} \leq 1.\end{aligned}$$

But ϕ, ψ satisfies (a), hence $M(AAu, Au, t) = 1$ and $N(AAu, Au, t) = 0$. Therefore $AAu = Au$. This is a contradiction. and hence $A^2u = Au = Bv$. Similarly, we can show that $B^2v = Bv$.

On the other hand, since $AAu = ASu = SAu$ and $Bv = BBv = BTv = TBv$, putting $Au = z = Bv$, then $AAu = Az = z$, $BBv = Bz = z$, $TBv = Tz = z$ and $SAu = Sz = z$. Hence z is a common fixed point of A, B, S and T and uniqueness of fixed point follows easily from condition (3.1) of Theorem 3.1. □

Example 3.2. Let X be an IFMS in which $X = R^+$, $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ for all $a, b \in [0, 1]$ such that $M(x, y, t) = \frac{t}{t+d(x,y)}$, $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ for all $t > 0$. Define the maps A, B, S and T on X by

$$Ax = \begin{cases} 2x-1 & \text{if } x \leq 2, \\ 2x & \text{if } x > 2, \end{cases} \qquad Bx = \begin{cases} 3x-2 & \text{if } x \leq 2, \\ x+1 & \text{if } x > 2, \end{cases}$$
$$Sx = \begin{cases} x^2 & \text{if } x < 2, \\ 2x & \text{if } x \geq 2, \end{cases} \qquad Tx = \begin{cases} 2x^2-1 & \text{if } x \leq 2, \\ x & \text{if otherwise.} \end{cases}$$

Define $\phi : [0,1]^6 \to [0,1]$ and $\psi : [0,1]^6 \to [0,1]$ by

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6) = \min\{t_1, t_2, t_3, t_4, t_5, t_6\}$$

and

$$\psi(t_1, t_2, t_3, t_4, t_5, t_6) = \max\{t_1, t_2, t_3, t_4, t_5, t_6\}.$$

Here the pairs (A, S) and (B, T) are converse commuting and the contractive condition is satisfied. Hence 1 is a unique common fixed point of A, B, S and T.

Corollary 3.3. *Let X be an IFMS and let $A, S : X \to X$ be self mappings such that the pair (A, S) is converse commuting satisfying*

$$\begin{aligned} &\phi\{M(Sx, Sy, t), M(Ax, Ay, t), M(Ax, Sx, t), \\ &\qquad\qquad M(Ay, Sy, t), M(Ax, Sy, t), M(Ax, Sy, t)\} \geq 0 \\ &\psi\{N(Sx, Sy, t), N(Ax, Ay, t), N(Ax, Sx, t), \\ &\qquad\qquad N(Ay, Sy, t), N(Ax, Sy, t), N(Ax, Sy, t)\} \leq 1 \end{aligned}$$

for every $x, y \in X$, $t > 0$ and ϕ, ψ are satisfies (a). Then A and S have a unique common fixed point in X.

Proof. Suppose that $A = B$ and $S = T$ in the equation (3.1) of Theorem 3.1 in IFMS, then we get this corollary. □

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