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Mathematical Models of Interactions between Species: Peaceful Co-existence of Vampires and Humans Based on the Models Derived from Fiction Literature and Films

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Abstract

Our paper presents a new approach to mathematical modeling of intertemporal interactions between species based on differential equations. It employs the example of interactions between vampires and humans using several types of vampire behavior described in popular fiction literature, comic books, films and TV series. Although mathematical modeling enables us to reject most of the popular scenarios embedded in popular literature and films, it appears that several popular culture sources outline the models describing plausible and peaceful vampire and human co-existence.

Keywords: intertemporal interactions, predator-prey models, differential equations, vampires, co-existence of species

1. Introduction: Dynamics of growth in human population

Assume that the world's population is to follow the exponential growth rate *x(t)*, and by the end of 2011 (x_1) reaches 7 billion people (t_1). This dynamics (which

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became a reality at the end of October 2011, when humanity reached the 7th billion) can be expressed by: $dx/tx=kx$ (1)

where k represents the coefficient of the population growth.

Using the method of division of variables we would arrive to the following solution: $x(t) = x_0 e^{k(t-to)}$ (2)

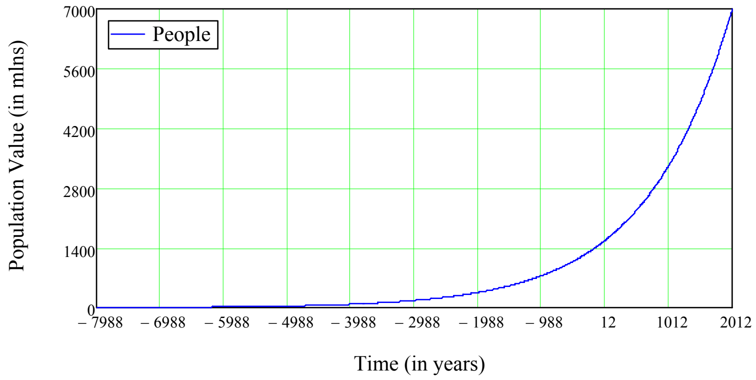
where x_0 is the total volume of population at the initial time period t_0.

It seems logical to assume that the exponential phase in the growth of our planet's population started at the moment the first civilizations formed themselves (i.e. the level of socialization allowed for the reproduction of the human species regardless the caprices of nature). It was scientifically proven that the first civilizations on Earth were those dating back to around 8000 B.C. (e.g. Egyptian, Sumerian, Assyrian, Babylonian, Helenian, Minoan, Indian and Chinese civilizations) (Edwards et al., 1971).

According to Maddison (2006), 10 thousand years ago the population of Earth was about four million people. The initial conditions are formalized as follows: $t_0 = -8000$, $x_0 = 4$ million people. The coefficient of the world's population growth is: $k = ln\ (x_1/x_0)/\ t_1 - t_0 = ln\ (1 + T_{ry}) = 7.46 \cdot 10^{-4}$ (3)

where T_{ry} is the annual growth rate of population (Figure 1).

Figure 1: Exponential growth model of world's population from the 8000 B.C.



A simple calculation of the annual growth rate of Earth's population in accordance with this dynamics yields the number 0.075%. This is 15 times less than the average population growth in 2010 (Kapitsa, 2010).

Figure 2: Logarithmic scale of Earth's population growth

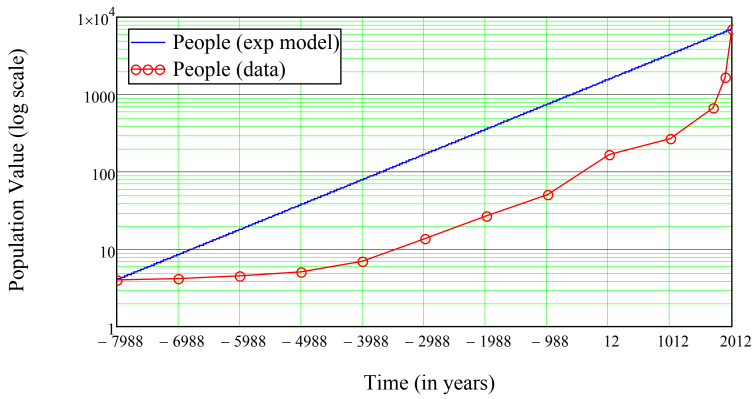


Figure 2 above depicts the logarithmic scale of the exponential dynamics and the actual dynamics built using the values of population growth starting from 8000 B.C (McEvedy and Jones, 1978). It is obvious that there is some hidden factor preventing human population from the explosive growth.

2. Vampires in the model of human population growth: predator-prey model

Consider introducing vampires into the model of population growth presented in (1). Vampires, the man's natural predators, are often described in legends and folklore. The vampirical theme has found its way into research literature becoming the key topic for popular science articles and even several academic papers (see e.g. Hartl and Mehlmann, 1982; Hartl and Mehlmann, 1983, Hartl et al., 1992; or Efthimiou and Gandhi, 2007).

The word "vampire" is considered to come from the Hungarian language where it is spelled "vampir". In Slavic languages, the word "vampire" exists in a quite similar form in Russian, Polish, Czech, Serbian and Bulgarian languages and is thought to come from the old Greek root "pi" (which means "to drink"). The first myths and legends about vampires have probably existed since the dawn of human history. In the 19th century, ancient Mesopotamian texts dating back to 4000 B.C. were translated into English revealing some mentioning of "seven spirits" that are very much like the description of vampires as we think of them today (Campbell Thompson, 1904).

The fact that vampires constituted a threat to humans throughout the history of mankind (whether this threat was real or imaginary one) can be illustrated by the examples of recent archaeological findings at ancient burial sites where some human remains showed signs of being staked, strapped or gagged with a stone, a typical way to slay the vampire, as the legends have it (New Scientist, 2009).

Suppose the vampire population is denoted by the function $y(t)$, $y_0=1$. Vampires act as natural predators for humans. The human population dynamics can therefore be presented as the following function: $dx/dt = kx - v(x)y$ (4)
where the equation $v(x)$ is the rate at which humans are killed by vampires.
Assume that the number of any vampire's victims is growing proportionally. Thence, the function $v(x)$ can be presented as the following: $v(x) = a \cdot x$ (5)
where $a>0$ is the coefficient of the human's lethal interaction with a vampire (a human is either killed by a vampire or is turned into a vampire).
As a result, the differential equation describing the growth rate of human population can be formulated as the following: $dx/dt = x(k-ay)$ (6)
Assume the dynamics of vampire's population change to be $y(t)$. The growth of vampire population will be determined by the quality and quantity of interactions with humans. After selecting its victim, any vampire can kill it by simply draining its blood, turning it into a new vampire or feeding on it but leaving it to live. Let us also introduce vampire slayers into the model. The slayers regulate the population of vampires by periodically killing vampires. The equation will then be modified to look like as the following: $dy/dt = baxy-cy$ (7)
where $0<b\leq 1$ is the coefficient reflecting the rate with which humans are turned into vampires, $c\geq 0$ is the coefficient of lethal outcome of the interaction between a vampire and vampire slayer.
Consider a Lotka-Volterra system (8). This system is classified as a "predator-prey" type model (Volterra, 1931):

$$\begin{cases} \dfrac{dx}{dt} = x(k - ay) \\ \dfrac{dy}{dt} = y(bax - c) \end{cases} \tag{8}$$

The system allows for the stationary solution, meaning that there is a pair of solutions for the system that creates a state when human and vampire populations can co-exist in time without any change in numbers. In order to find the solutions for these two populations, x_s and y_s, we have to solve the following system putting it equal to zero:

$$\begin{cases} x(k - ay_s) = 0 \\ y(bax_s - c) = 0 \end{cases} \tag{9}$$

As a result, in the stationary case the initial system breaks down into two independent equations yielding the following parameters:

$$(x_s, y_s) = \left(\frac{c}{ba} \quad \frac{k}{a} \right) \tag{10}$$

It is obvious from a stationary case that the size of human population is determined by the effectiveness of slaying vampires by vampire hunters c and the number of cases when the humans are turned into vampires ba. The size of vampire population depends on the growth rate of human population k and vampires' thirst for human blood a. The stationary solution shows that when vampires are capable of restraining their blood thirst, the size of both populations can be rather high in mutual co-existence. The system is held in balance by the existence of vampire slayers.

The model described in (8) represents a system of ordinary differential equations which can be solved by using iterative numerical methods. The most widely-used ones are a family of the Range-Kutta methods (Butcher, 2008) that represent the modified and corrected Euler's method with a higher degree of precision. The time-step algorithm includes the integration of differential equation from the initial to the final condition and computing the value of equation at the next step through the previous one.

$$y_{i+1} = y_i + \int_{t_i}^{t_{i+1}} g(t, y)dt \tag{11}$$

The basic idea of the Runge-Kutta algorithms lies in substituting the function *g(t,y)* that depends on the unknown function *y(t)* by certain approximation. The more precise the approximation of the integral, the more accurately one can determine y_{i+1}. Integrals can be approximated using either the rectangle method (2nd degree of precision) or Simpson's rule of numerical approximation of definite integrals (3rd degree of precision). The price one has to pay for the higher degree of precision would be the necessity to get the approximation of the integral in three points.

Running numerous experimental calculations it was established that the best ratio of precision and the volume of calculations is yielded by the fourth-order Runge-Kutta method. The formulae of calculations using the fourth-order Runge-Kutta method are presented below:

$$x_{i+1} = x_i + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right] \text{ (12)}$$

$$y_{i+1} = y_i + \frac{1}{6}\left[m_1 + 2m_2 + 2m_3 + m_4\right]$$

$$k_1 = f(t_i, x_i, y_i)\Delta t$$

$$m_1 = g(t_i, x_i, y_i)\Delta t$$

$$k_2 = f(t_i + \frac{\Delta t}{2}, x_i + \frac{k_1}{2}, y_i + \frac{m_1}{2})\Delta t$$

$$m_2 = g(t_i + \frac{\Delta t}{2}, x_i + \frac{k_1}{2}, y_i + \frac{m_1}{2})\Delta t$$

$$k_3 = f(t_i + \frac{\Delta t}{2}, x_i + \frac{k_2}{2}, y_i + \frac{m_2}{2})\Delta t$$

$$m_3 = g(t_i + \frac{\Delta t}{2}, x_i + \frac{k_2}{2}, y_i + \frac{m_2}{2})\Delta t$$

$$k_4 = f(t_i + \Delta t, x_i + k_3, y_i + m_3)\Delta t$$

$$m_4 = g(t_i + \Delta t, x_i + k_3, y_i + m_3)\Delta t$$

$$f(t_i, x_i, y_i) = x_i(k - ay_i)$$

$$g(t_i, x_i, y_i) = y_i(bax_i - c)$$

We employ the fourth-order Runge-Kutta method using the function *rkfixed (V,a, b,n D)* in order to solve the system described above [18]. The function has five arguments: *V*- the vector of initial values of the functions (border conditions), *[a,b]* - coordinates of the beginning and the end of the computation interval, *n* - the number of the network segments, *D* - the vector of the first derivatives of the system. The function *rkfixed()* yields a matrix consisting of *(n+1)* rows and three columns.

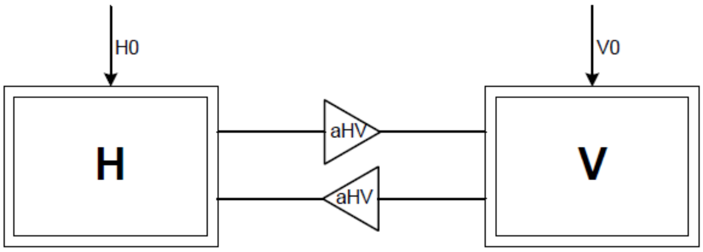
3. Evaluating the possibility of vampires' and humans' co-existence in the real world based on popular literature, comic books and films

Starting from Bram Stoker's "Dracula", the theme of vampirism has been widely exploited by many authors: Anne Rice, Stephen King, Stephenie Meyer, Elizabeth Kostova or Charlaine Harris, just to name a few. In addition, vampires often appear in comic books and films and TV series based on these books (e.g. "Blade" or "Buffy the Vampire Slayer"). We reviewed popular literature, comic books, and films on vampires and identified five types of scenarios describing vampires and humans interactions. These scenarios were used to draw models of vampire-human confrontation using the predator-prey model described and defined above.

3.1. Scenario 1: The Stoker-King model

Bram Stoker's "Dracula" and Stephen King's "'Salem's Lot" describe interactions between vampires and humans in the following way: a vampire selects a human victim and gets into its proximity (it typically happens after dark and the vampire needs the victim to invite her/him in). Often the vampire does not require permission to enter the victim's premises and attacks the sleeping victim (Stoker, 1897; McNally and Florescu, 1994). The vampire bites the victim and drinks the victim's blood, then returns to feed for 4-5 consecutive days, whereupon the victim dies, is buried and rises to become another vampire (unless a wooden stake is put through its heart). Vampires usually need to feed every day, so more and more human beings are constantly turned into vampires (Stoker, 1897; King, 1975).
Assume the events described in "Dracula" were real. How would things evolve given the Stoker-King model dynamics described in both sources? Let us take 1897 as the starting point (i.e. the year Stoker's novel was first published). In 1897, the world population was about 1 650 million people (UN, 1999).
The initial conditions of the Stoker-King model are the following: 1 vampire, 1 650 million people, there are no organized groups of vampire slayers. The date we choose for the first vampire has little bearing on our argument and therefore can be set arbitrarily. The model can be presented in a form of a diagram (Diagram 1).

Diagram 1: The Stoker-King model



where H0 denotes humans and V denotes vampires, H0 is the initial state of human population, v0 is the initial state of vampire population and the aHV describes an interaction between a human and a vampire (with a as the coefficient of a lethal outcome for vampire-human interaction for humans).

Let us calibrate the parameters of this specific (vampiric) case of predator-prey model. The calculation period is set at 1 year with a step of 5 days ($t = 0 \ldots 73$). The coefficient of human population growth k for the given period is very small and can be neglected, therefore $k=0$. The coefficient of lethal outcome for humans interacting with vampires can be calculated according to the scenario presented in the Stoker-King model $y_0(t) = y_0 q^t$, where $y_0=1$, $q=2$. The probability of a human being turned into a vampire is very high, thence $b=1$. Jonathan Harker and Abraham van Helsing could not be, by all means, considered very efficient vampire slayers, therefore we can put $c=0$.

The resulting model is presented in a form of the following Cauchy problem:

$$\begin{cases} \dfrac{dx}{dt} = -axy \\ \dfrac{dy}{dt} = axy \\ x(0) = 1.65 \cdot 10^9 \\ y(0) = 1 \end{cases} \tag{13}$$

Due to the fact that the total sum of humans and vampires does not change in time (human population does not grow and humans gradually become vampires), the predator-prey model is diminished to a simple problem of an epidemic outbreak (Munz et al., 2009).

It can be assumed that for any moment t there holds an equality $x(t)+y(t)=x_0+y_0$, where $x_0=1.65 \cdot 10^9$. The system of differential equations can be presented in a form of a single differential equation: $dy/dt = ax(t)y(t) = ay(t)[y_0+x_0-y(t)]$ (14) with the initial condition $y(0)= y_0=1$.

This differential equation belongs to the class of logistic equations (e.g. the Verhulst equation that describes the growth of population). Let us solve the Cauchy problem for this equation:

$$\frac{dy}{y[x_0 + y_0 - y]} = adt \Rightarrow \frac{dy}{y} + \frac{dy}{y_0 + x_0 - y} = (y_0 + x_0)adt \Rightarrow \tag{15}$$

$$\ln(y) - \ln(y_0 + x_0 - y) = \ln y_0 - \ln(y_0 + x_0 - y_0) + a(y_0 + x_0)(t - t_0) \Rightarrow$$

$$\ln\left[\frac{y}{y_0+x_0-y}\right]=\ln\left[\frac{y_0}{x_0}\right]+a(y_0+x_0)(t-t_0)\Rightarrow\frac{y}{y_0+x_0-y}=\frac{y_0}{x_0}e^{a(y_0+x_0)(t-t_0)}$$

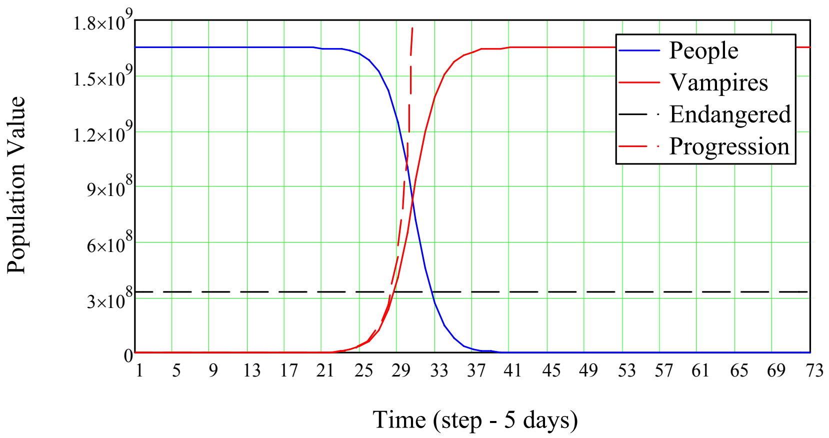
By solving the problem above we get the following equation:

$$y(t)=\frac{(x_0+y_0)y_0}{y_0+x_0e^{-\alpha(y_0+x_0)(t-t_0)}},\ t\geq 0 \quad (16)$$

The equation clearly shows that with passing time the number of vampires grows and very soon there are no humans left: $\lim_{t\to\infty} y(t)=x_0+y_0$

The solution to this problem is presented below (Figure 3). It is clearly visible that the human population is drastically reduced by 80% by the 165th day from the moment when the first vampire arrives. This means that the human population reaches its critical value and practically becomes extinct (following the definitions of "Critically Endangered species" by the International Union for Conservation of Nature (see IUCN, 2012)). At that precise moment, the world will be inhabited by 1 384 million vampires and 266 million people.

Figure 3: The change in the numbers of humans and vampires in time (1 step = 5 days) in the Stoker-King model



Let us observe the speed with which vampire population grows. In order to do that, an analysis of the following magnitude should be carried out: *dy/dt*.

$$\frac{dy}{dt}=\frac{a(y_0+x_0)^2y_0x_0e^{-a(y_0+x_0)(t-t_0)}}{(y_0+x_0e^{-a(y_0+x_0)(t-t_0)})^2} \quad (17)$$

The results are shown on Figure 4 that follows.

Figure 4: The change in vampires' growth dynamics (1 step = 5 days) in Stoker-King model

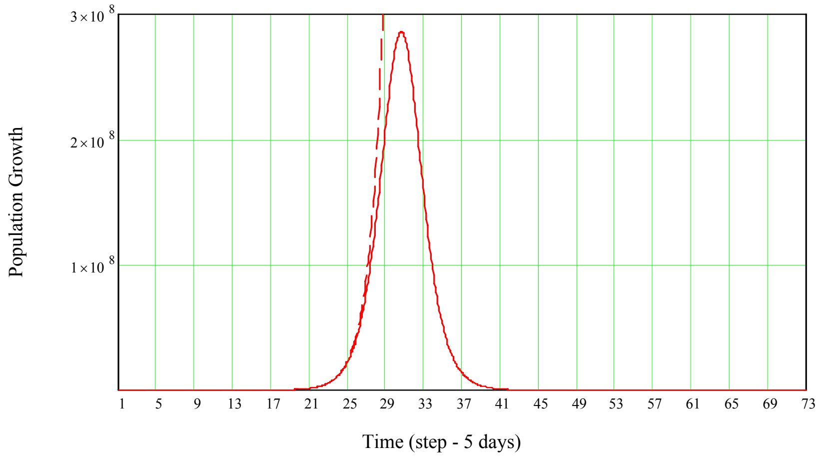
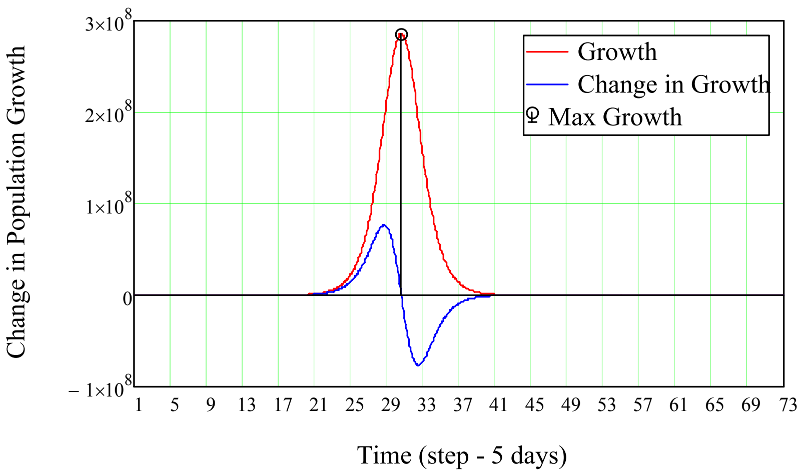


Figure 4 clearly shows that the growth of vampire population is extreme: at first, the number of vampires jumps up abruptly, but then slows down and declines.
In order to determine the moment of time when the speed of vampire population's growth reaches its maximal values, we need to take a look at the following magnitude: d^2y/dt^2

$$\frac{d^2y}{dt^2} = \frac{-a^2(y_0+x_0)^3\, y_0 x_0 e^{-a(y_0+x_0)(t-t_0)}\left[y_0 - x_0 e^{-a(y_0+x_0)(t-t_0)}\right]}{\left[y_0 + x_0 e^{-a(y_0+x_0)(t-t_0)}\right]^3} \tag{18}$$

Figure 5 illustrates how that the speed of vampire population's growth accelerates until the point denoted by t_{max} and then slows down.

Figure 5: The change of speed of growth for vampire's population (1 step = 5 days) in the Stoker-King model



The maximal growth of the number of vampires (infected humans) will be observed in a moment of time t_{max}:

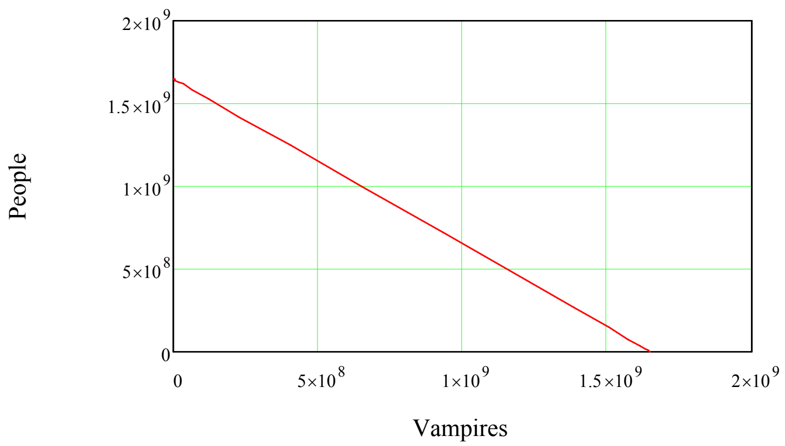
$$t_{\max} = \frac{\ln(x_0 / y_0)}{a(y_0 + x_0)} + t_0 \tag{19}$$

where t_{max}= 153 is the day (153[rd] day) when the number of vampires is the highest, $x(t_{max})=825$ million is the number of vampires in a moment of time t_{max}, $x'(t_{max})=286$ million is the number of newly turned vampires in day t_{max}.

Figure 6 shows the phase diagram of both populations. It is apparent that the increase in one population (vampires) inevitably leads to the decrease in another (humans). When the number of vampires reaches the number of human population, the humans disappear altogether. The presence of vampires in the Stoker–King model brings the mankind to the brink of extinction.

The Stoker-King model describes the "explosive" growth of vampire population. Within the two months of Dracula's arrival to England (or Kurt Barlow's arrival to New England), there would have been 4 thousand vampires in operation. The model analyzed in this scenario is very similar to an epidemic outbreak caused by a deadly virus (e.g. Ebola or SARS). According to the Stoker-King model, vampires need just half a year to take up man's place in nature. Therefore, the co-existence of humans and vampires seems highly unrealistic.

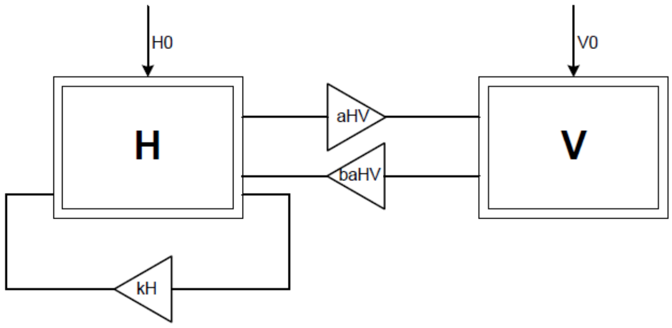
Figure 6: Phase diagram of vampire (z^2) and human (z^1) populations in the Stoker-King model.



3.2. Scenario 2: The Rice model

Anne Rice "Vampire Chronicles" describes the world with vampires, where vampires still need to feed on human beings (like in the Stoker-King model) but do so discretely (Rice, 1997). The vampire can attack a human being, feed on it and leave it to live. In some cases (if they are too hungry), vampires kill their victims by draining their blood. The vampire cannot easily turn the human into another vampire (in order to do so, the victim's permission needs to be gained, it needs to drink some of vampire's blood and the whole process is painful for both of them and takes several days, so it happens very rarely). Vampires do not need to feed every day: some blood once a week or so is enough to survive. The initial conditions of the Rice model are the following: 2 vampires, 982 million people, there are no organized groups of vampire slayers (Diagram 2).

Diagram 2: The Rice model



where H denotes humans and V denotes vampires, H0 is the initial state of human population, kH denotes the exponential growth of human population, v0 is the initial state of vampire population, and aHV and baHV both describe interactions between a human and a vampire (with a as the coefficient of a lethal outcome for vampire-human interaction for humans and b as the coefficient describing the rate with which humans are turned into vampires).

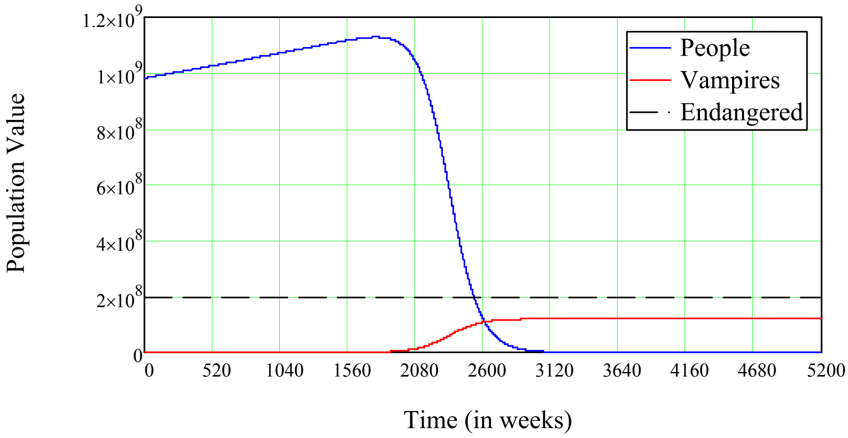
Assume the events described in “Vampire Chronicles” were real. How would things evolve given the Rice model dynamics described in her literary works? Let us take 1791 as a starting point (a year Lestat made Lui a vampire). In 1791, the world population was about 982 million people (UN, 1999).

Let us calibrate the parameters of this specific predator-prey model. The calculation period is set to 100 years with a step of 7 days *(t=0…5200)*. The coefficient of human population growth is calculated as $k = \dfrac{\ln(x_1 / x_0)}{t_1 - t_0}$ where где $x_1=1520$ million people at a moment of time $t_1=1891$, $x_0=982$ million people at $t_0=1791$. Humans do not necessarily die or become vampires after their encounter with vampires, thence the coefficient of lethal outcome a will be considerably lower than in the Stoker-King model and is therefore denoted as $0.1 \cdot a$. The probability of a human turned into a vampire is quite low and can be denoted as $b=0.1$. There are no efficient groups of vampire slayers, therefore we can put $c=0$. The resulting simplified model is presented in a form of the following Cauchy problem:

$$\begin{cases} \dfrac{dx}{dt} = x(k - ay) \\ \dfrac{dy}{dt} = baxy \\ x(0) = 982 \cdot 10^6 \\ y(0) = 2 \end{cases} \tag{20}$$

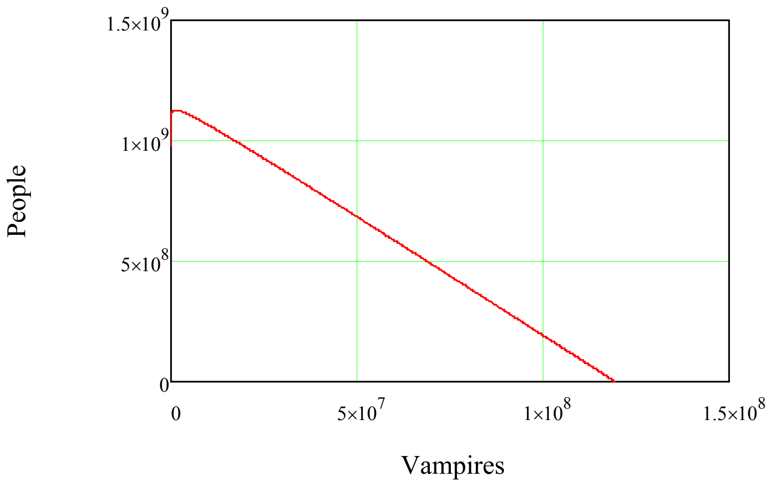
The following system is solved using the Range-Kutta method. The results are presented in a graphical form on Figure 7 that follows.

Figure 7: The change in the numbers of humans and vampires in time (1 step = 7 days) in the Rice model.



It is apparent that in spite of the presence of vampires, the human population in the Rice model grows in the beginning. However, when the number of vampires reaches its critical mass, the human population starts to shrink and after 48.7 years is almost extinct. The number of vampires at this moment is equal to 100 million. Figure 8 depicts the phase diagram of the system. It shows a clear pattern: when the vampire population is small, the human population is growing at its natural rate of reproduction. However, when the number of vampires starts to rise, the human population is diminishing proportionally to the increase in vampire population.

Figure 8: Phase diagram of vampire (z^2) and human (z^1) populations in the Rice model.



When compared to the Stoker-King model, the Rice model merely delays the total extinction of mankind. It is not possible to find equilibrium or stationary solution for this system. According to the Rice model, the co-existence of humans and vampires is possible for a short period of time. However, as time passes, all humans will be extinct or turned into vampires. Therefore, the co-existence of humans and vampires described by Anne Rice also seems unrealistic.

3.3. Scenario 3: The Harris-Meyer-Kostova model

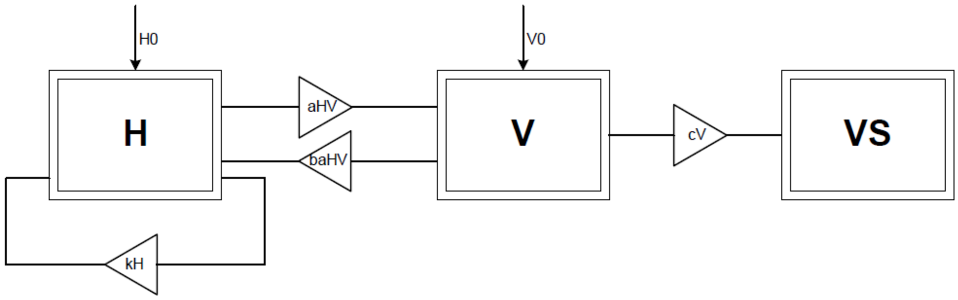
In the books of Stephenie Meyer's "Twilight series", Charlaine Harris's "Sookie Stockhouse (Southern Vampire) series", "True Blood" (TV series) and Elizabeth Kostova's "The Historian" there is a world drawn where vampires peacefully co-exist with humans.

For instance, in Stephenie Meyer's "Twilight series" vampires can tolerate the sunlight, interact with humans (even fall in love with them) and drink animals' blood to survive (Meyer, 2005). Of course, they have to live in secrecy and pretend to be human beings. In "True Blood" TV series, however, a world is shown where vampires and humans live side-by-side and are aware of each other. Vampires can buy synthetic blood of different blood types that is sold in bottles and can be bought in every grocery store, bar or gas station (Harris, 2001). They cannot walk during daytime, so they usually come out at night. Humans also find use of vampires' essence – vampires' blood (called "V") is a powerful hallucinogenic drug that is sought by humans and traded on the black market (sometimes humans capture vampires with the help of silver chains or harnesses and then kill them by draining their blood). Some humans seek sex with vampires (vampires are stronger and faster than humans and can provide superb erotic experience). There is a possibility to turn a human being into a vampire, but it takes time and effort.

In Elizabeth Kostova's novel "The Historian", vampires are rare although real and do not reveal themselves to humans too often. Their food ratios are limited and they spend lots of time brooding in their well-hidden tombs (Kostova, 2005).

"Sookie Stackhouse (Southern Vampire) Series" by Charlaine Harris comes with an interesting concept of vampires "coming out" in the 2000s: vampires have ultimately decided to reveal themselves to humans (a concept totally unacceptable in the works of Stephanie Meyer) and co-exist with them peacefully exerting their citizens' rights (see e.g. Harris, 2001). Assume that at the time of the events described in the first book of the series, "Dead Until Dark" (2001), the world's vampire hypothetical population was around five million (the population of the state of Louisiana in 2001 we arbitrarily use in our model). The initial conditions of the Harris-Meyer-Kostova model are the following: five million vampires, 6 159 million people, there are organized groups of vampire "drainers". The model can be presented in a form of a diagram (Diagram 3).

Diagram 3: The Harris-Meyer-Kostova model



where H denotes humans, V denotes vampires and VS denotes vampire slayers. H0 is the initial state of human population, kH denotes the exponential growth of human population, v0 is the initial state of vampire population, aHV and baHV both describe interactions between a human and a vampire (with a as the coefficient of a lethal outcome for vampire-human interaction for humans and b as the coefficient describing the rate with which humans are turned into vampires) and cV denotes the death rate for vampires.

Let us calibrate the parameters of this specific case of predator-prey model. The calculation period is set at 100 years with a step of 1 year *(t=2001…2101)*. The coefficient of human population growth is calculated as $k = \frac{\ln(x_1 / x_0)}{t_1 - t_0}$ where $x_1 = 7000$ million people at a moment of time $t_1=2012$, $x_0=6150$ million of people at time $t_0=2001$. Humans almost always come out alive from their encounters with vampires, hence the coefficient of lethal outcome *a* will be low and is denoted by $0.01 \cdot a$. The probability of a human being turned into a vampire is similar to the on in the Rice model and equals to $b=0.1$. There are numerous groups of vampire "drainers" (although the number of drained vampires is relatively low and would not lead to their total extinction), therefore we can put $c>0$ (*c* is calculated similarly to the coefficient *k*). The resulting model is presented in the initial set-up of predator-prey framework:

$$\begin{cases} \frac{dx}{dt} = x(k - ay) \\ \frac{dy}{dt} = y(bax - c) \\ x(0) = 6150 \cdot 10^6 \\ y(0) = 5 \cdot 10^6 \end{cases} \tag{21}$$

The model allows for a stationary solution: there are system parameters (x_s, y_s) that would stabilize the populations of humans and vampires in time. In order to find the stabilized populations of both spices, x_s and y_s, an equality described in (10) might be employed: $(x_s, y_s) = (7704\ 8)$ million individuals. Figure 9 that follows shows the stationary solution presented on a logarithmic scale.

Figure 9: Figure with stationary solution presented on a logarithmic scale for the vampire (y_s) and human (x_s) populations in the Harris-Meyer-Kostova model.

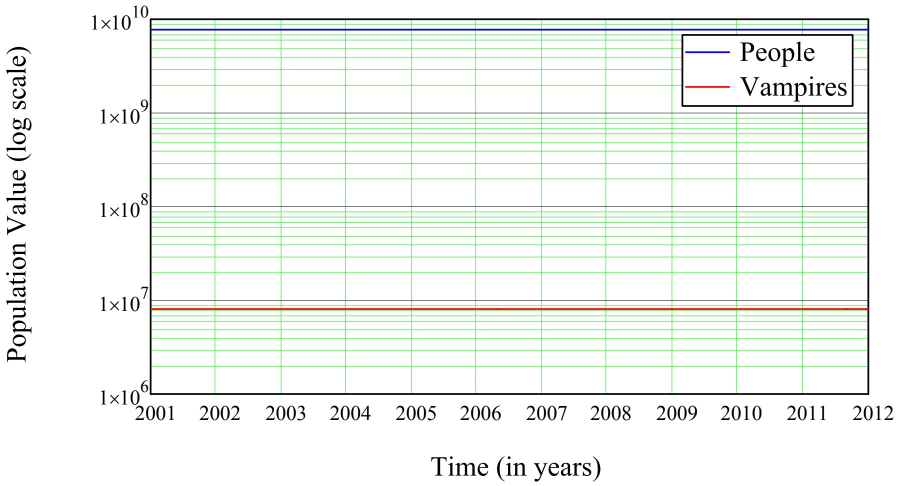
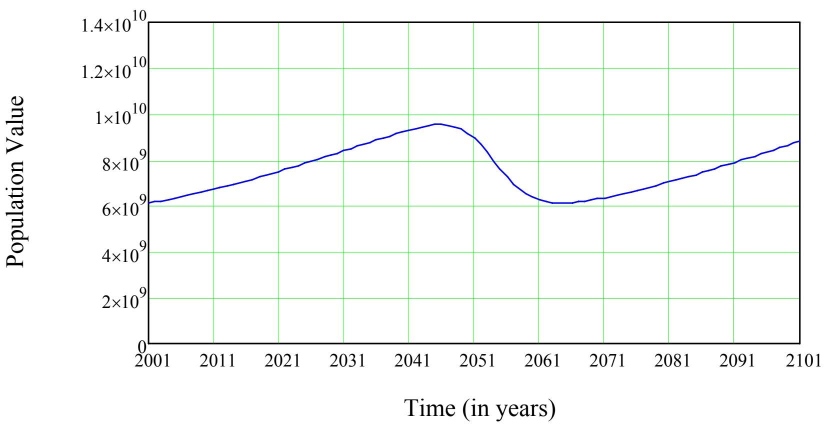
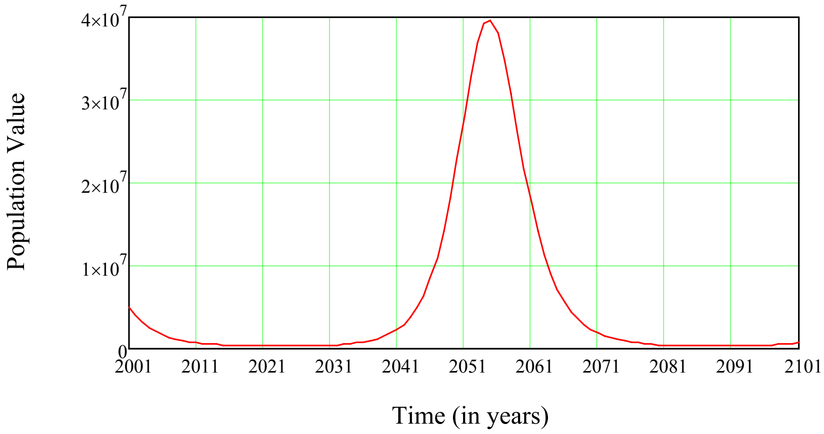


Figure 10: The change in the number of humans in the Harris-Meyer-Kostova model (cyclical nature)



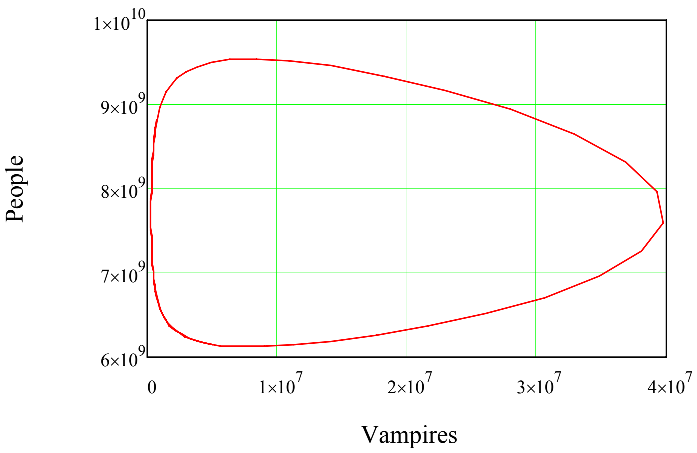
This stationary solution for 2001 cannot be found with the chosen population growth coefficient k and can be reached applying some conditions only after 2012. The deviations in the number of people and vampires from the stationary state at the initial period of time are quite small which points at the fact that the system might be stable and auto-cyclical. This is proved by the further calculations (Figures 10-11).

Figure 11: The change in the number of vampires in the Harris-Meyer-Kostova model (cyclical nature)



It is apparent from Figures 10 and 11 that the human population will be growing until 2046 when it reaches its peak of 9.6 billion people, whereupon it will be declining until 2065 until it reaches its bottom at 6.12 billion people. This process will repeat itself continuously. The vampire population will be declining until 2023 when it reaches its minimum of 289 thousand vampires, whereupon it will be growing until 2055 until it reaches its peak at 397 million vampires. This process will repeat itself continuously. Figure 12 shows the phase diagram of the cyclical system of human-vampire co-existence.

Figure 12: Phase diagram of vampire (z^2) and human (z^1) populations in the Harris-Meyer-Kostova model



Under certain conditions, the Harris-Meyer-Kostova model seems plausible and allows for the existence of vampires in our world. Peaceful co-existence of two spices is a reality. However, this symbiosis is very fragile and whenever the growth rate of human population slows down, the blood thirst of vampires accelerates, or vampire drainers become too greedy, the whole system lies in ruins with just one population remaining.

4. Conclusions

It appears that although vampire-human interactions would in most cases lead to great imbalances in the ecosystems, there are several cases that might actually convey plausible models of co-existence between humans and vampires.
In total, three different models were defined, calibrated and analyzed. The Stoker-King model (based on Bram Stoker's "Dracula" and Stephen King's "'Salem's Lot") described the "explosive" rate of growth in vampire population that would lead to exterminating 80% of the human population on the 165th day of the first vampire's arrival. The scenario is similar to severe epidemic outbreaks and would lead first to the complete extinction of humans and then to the death of all vampires. The Rice model (based on Anne Rice's "Vampire Chronicles") would merely delay the total extinction of mankind by vampires by 48 years with respect to the first model and therefore cannot be considered as realistic.
Unlike the previous two, the Harris-Meyer-Kostova model (based on Charlaine Harris's "Southern Vampire Series", Stephenie Meyer's "Twilight saga" and Elizabeth Kostova's "The Historian") allows for the peaceful (and totally unnoticeable) existence of vampires in our world. However, the system is very fragile and some coordination is required to keep things in balance.
Overall, although mathematical principles enabled us to doubt the realism of many human and vampire encounters described in the literature, several sources provide what might be an acceptable description of the situation in which vampires and humans co-exist in a world that is very similar to the one we live in.

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