Applied Mathematical Sciences, Vol. 7, 2013, no. 132, 6593 - 6600
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ams.2013.310603

Laplace Adomian Decomposition Method for Solving Newell-Whitehead-Segel Equation

P. Pue-on

Department of Mathematics, Faculty of Science
Mahasarakham University, Mahasarakham, 44150, Thailand
prapart.p@msu.ac.th

Abstract

In this manuscript, the Laplace Adomian decomposition method (LADM) is presented to solve Newell-Whitehead-Segel equation. The method can be applied to linear and nonlinear problems. Some examples have been carried out in order to illustrate the efficiency and reliability of the method.

Mathematics Subject Classification: 35G25

Keywords: Newell-Whitehead-Segel equation, Laplace Adomian decomposition method

1 Introduction

In natural phenomena, nonequilibrium systems are usually shown in many extended states: uniform, oscillatory, chaotic, and pattern states. Many stripe patterns, e.g., ripples in sand, stripes of seashells, occure in a variety of spatially extended systems which can be described by a set of equation called amplitude equations. One of the most important of amplitude equations is the Newell-Whitehead-Segel equation which describes the appearance of the stripe pattern in two dimensional systems. Moreover, this equation was applied to a number of problem in a variety systems, e.g.,Rayleigh-Benard convection, Faraday instability, nonlinear optics, chemical reactions and biological systems. In recent years various methods and techniques are developed to solve this nonlinear, parabolic partial differential equation. The approximate solutions of the Newell-Whitehead-Segel equation were presented by Adomian decomposition [10], differential transformation method [1], $(\frac{G'}{G})$ expansion method [2], reduce differential transformation [3], lattice Bolzmann scheme [11], finite difference scheme [6] and so on

Laplace Adomian decomposition method(LADM) is semi-analytic method which is first introduced by Suheil A. Khuri [9]. This method has been successfully used to solve differential equations. There are many literatures improved concerning Laplace Adomian decomposition method [7], [8] and the related modification to investigate various scientific model [8] [12]. The major advantage of this method is its capability of combining the two powerful method to obtain exact solution for nonlinear equation

In this paper a reliable Laplace Adomian decomposition method is applied for solving Newell-Whitehead-Segel equation. The method can be employed to linear and nonlinear problems. Moreover, some examples are illustrative for demonstrating the advantage of the method.

2 Laplace Adomian Decomposition Method

Let us consider the initial value problem in Newell-Whitehead-Segel equation in the form

$$\begin{cases} u_t(x,t) = ku_{xx}(x,t) + au(x,t) - bu^m(x,t), \\ u(x,0) = f(x), \end{cases} \tag{1}$$

where a and b are real numbers and p and m are positive integers.
By applying the Laplace transform on both sides of the equation and using the linearity of the Laplace transform gives:

$$\mathcal{L}[u_t(x,t)] = k\mathcal{L}[u_{xx}] + a\mathcal{L}[u(x,t)] - b\mathcal{L}[u^m] \tag{2}$$

Because of the differentiation property of Laplace transform, Eq. (2) can be written as

$$\begin{aligned} s\mathcal{L}[u(x,t)] - u(x,0) &= k\mathcal{L}[u_{xx}] + a\mathcal{L}[u(x,t)] - b\mathcal{L}[u^m] \\ s\mathcal{L}[u(x,t)] - f(x) &= k\mathcal{L}[u_{xx}] + a\mathcal{L}[u(x,t)] - b\mathcal{L}[u^m] \\ \mathcal{L}[u(x,t)] &= \frac{f(x)}{s-a} + \frac{k}{s-a}\mathcal{L}[u_{xx}] - \frac{b}{s-a}\mathcal{L}[u^m] \end{aligned} \tag{3}$$

The Laplace Adomian decomposition method represents solution as an infinite series of components given below

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{4}$$

and the nonlinear term $F(u) = u^m$, $m > 1$ can be presented by an infinite series

$$F(u) = \sum_{n=0}^{\infty} A_n \tag{5}$$

where the components A_n are Adomain polynomials [5] of $u_0, u_1, \ldots, u_n$ which can be calculated by formula

$$A_n = \frac{1}{n!}\Big[\frac{d^n}{d\lambda^n} F\Big(\sum_{i=0}^{\infty} \lambda^i u_i\Big)\Big]_{\lambda=0} \tag{6}$$

Specific algorithms were seen in [4, 5] to formulate Adomian polynomials. The following algorithm:

$$\begin{cases} A_0 = F(u_0), \\ A_1 = F'(u_0)u_1, \\ A_2 = F'(u_0)u_2 + \frac{1}{2}F''(u_0)u_1^2, \\ A_3 = F'(u_0)u_3 + F''(u_0)u_1u_2 + \frac{1}{3!}F'''(u_0)u_1^3, \\ \quad \vdots \end{cases} \tag{7}$$

can be used to construct Adomian polynomial. By substituting (4) and (5) into (3), one get

$$\mathcal{L}\Big[\sum_{n=0}^{\infty} u_n(x,t)\Big] = \frac{f(x)}{s-a} + \frac{k}{s-a}\mathcal{L}\Big[\Big(\sum_{n=0}^{\infty} u_n(x,t)\Big)_{xx}\Big] - \frac{b}{s-a}\mathcal{L}\Big[\sum_{n=0}^{\infty} A_n\Big]$$

By comparing the both sides of the Eq. (8) we have

$$\begin{aligned} \mathcal{L}\Big[u_0(x,t)\Big] &= \frac{f(x)}{s-a} \\ \mathcal{L}\Big[u_1(x,t)\Big] &= \frac{k}{s-a}\mathcal{L}\Big[\Big(u_0(x,t)\Big)_{xx}\Big] - \frac{b}{s-a}\mathcal{L}\Big[A_0\Big] \\ \mathcal{L}\Big[u_2(x,t)\Big] &= \frac{k}{s-a}\mathcal{L}\Big[\Big(u_1(x,t)\Big)_{xx}\Big] - \frac{b}{s-a}\mathcal{L}\Big[A_1\Big] \end{aligned}$$

In general, the recursive relation is given by

$$\mathcal{L}\Big[u_{n+1}(x,t)\Big] = \frac{k}{s-a}\mathcal{L}\Big[\Big(u_n(x,t)\Big)_{xx}\Big] - \frac{b}{s-a}\mathcal{L}\Big[A_n\Big], \quad n \geq 0$$

Taking inverse Laplace transform, hence one get recusive relation

$$u_0 = \mathcal{L}^{-1}\Big[\frac{f(x)}{s-a}\Big] \tag{8}$$

$$u_{n+1}(x,t) = \mathcal{L}^{-1}\Big[\frac{k}{s-a}\mathcal{L}\Big[\Big(u_n(x,t)\Big)_{xx}\Big] - \frac{b}{s-a}\mathcal{L}\Big[A_n\Big]\Big], \quad n \geq 0 \tag{9}$$

From (7), (8) and (9), we can determine the components $u_n(x,t)$, and hence the series solution of $u(x,t)$ in (4) can be immediately obtained. For numerical purposes, the $n-$term approximant

$$\Psi_n = \sum_{k=0}^{n} u_n(x,t), \tag{10}$$

can be used to approximate the exact solution.

3 Illustrative Examples

In this section, some initial value problems are presented to show the advantages of the proposed method which can be applied to linear and nonlinear problem.

Example 1. Consider linear Newell-Whitehead-Segel equation

$$u_t = u_{xx} - 3u \tag{11}$$

subject to initial condition

$$u(x,0) = e^{2x} \tag{12}$$

By taking the Lapalce transform on both sides of (11), then using the differentiation property of Laplace transform one obtains

$$s\mathcal{L}[u] - u(x,0) = \mathcal{L}[u_{xx}] - 3\mathcal{L}[u].$$

Applying initial condition (12) becomes

$$\mathcal{L}[u] = \frac{e^{2x}}{s+3} + \frac{1}{s+3}\mathcal{L}[u_{xx}] \tag{13}$$

The inverse Laplace transformation is applied to Eq.(13) we get

$$\begin{aligned} u(x,t) &= \mathcal{L}^{-1}\Big[\frac{e^{2x}}{s+3}\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s+3}\mathcal{L}[u_{xx}]\Big] \\ &= e^{2x}e^{-3t} + \mathcal{L}^{-1}\Big[\frac{1}{s+3}\mathcal{L}[u_{xx}]\Big] \end{aligned} \tag{14}$$

The Laplace Adomian decomposition defines the solution $u(x,t)$ by the series

$$u(x,t) = \sum_{n=0}^{\infty} u_n \tag{15}$$

So, the term u_{xx} can be defined by an infinite series

$$u_{xx}(x,t) = \sum_{n=0}^{\infty} (u_n)_{xx} \tag{16}$$

Substituting (15) and (16) into both sides of Eq.(14) gives

$$\sum_{n=0}^{\infty} u_n = e^{2x}e^{-3t} + \mathcal{L}^{-1}\Big[\frac{1}{s+3}\mathcal{L}\Big[\sum_{n=0}^{\infty}(u_n)_{xx}\Big]\Big]$$

The recursive relation is defined by

$$u_0 = e^{2x}e^{-3t} \tag{17}$$

$$u_{n+1} = \mathcal{L}^{-1}\Big[\frac{1}{s+3}\mathcal{L}\Big[\sum_{n=0}^{\infty}(u_n)_{xx}\Big]\Big],\ n \geq 0 \tag{18}$$

The other components of the solution can easily calculated by using the above recursive relation

$$\begin{aligned}
u_1 &= \mathcal{L}^{-1}\Big[\frac{1}{s+3}\mathcal{L}\Big[(u_0)_{xx}\Big]\Big] \\
&= 4e^{2x}\mathcal{L}^{-1}\Big[\frac{1}{(s+3)^2}\Big] \\
&= 4te^{2x}e^{-3t} \\
u_2 &= \mathcal{L}^{-1}\Big[\frac{1}{s+3}\mathcal{L}\Big[(u_1)_{xx}\Big]\Big] \\
&= 16e^{2x}\mathcal{L}^{-1}\Big[\frac{1}{(s+3)^3}\Big] \\
&= 8t^2e^{2x}e^{-3t} \\
u_3 &= \mathcal{L}^{-1}\Big[\frac{1}{s+3}\mathcal{L}\Big[(u_2)_{xx}\Big]\Big] \\
&= 64e^{2x}\mathcal{L}^{-1}\Big[\frac{1}{(s+3)^4}\Big] \\
&= \frac{32}{3}t^3e^{2x}e^{-3t} \\
&\vdots
\end{aligned}$$

Using (15), hence the series solution is expressed by

$$\begin{aligned}
u(x,t) &= u_0 + u_1 + u_2 + u_3 + \ldots \\
&= e^{2x}e^{-3t}(1 + 4t + 8t^2 + \frac{32}{3}t^3 + \ldots) \\
&= e^{2x}e^{-3t}e^{4t} \\
&= e^{2x+t}
\end{aligned}$$

Example 2. Consider nonlinear Newell-Whitehead-Segel equation

$$u_t = 5u_{xx} + 2u + u^2 \tag{19}$$

subject to initial condition

$$u(x,0) = \alpha \tag{20}$$

where α is arbitrary constant. By taking the Lapalce transform on both sides of (19), then using the differentiation property of Laplace transform one obtains

$$s\mathcal{L}[u] - u(x,0) = 5\mathcal{L}[u_{xx}] + 2\mathcal{L}[u] + \mathcal{L}[u^2].$$

Applying initial condition (20) becomes

$$\mathcal{L}[u] = \frac{\alpha}{s-2} + \frac{5}{s-2}\mathcal{L}[u_{xx}] + \frac{1}{s-2}\mathcal{L}[u^2] \tag{21}$$

The inverse Laplace transformation is applied to Eq.(21) we get

$$\begin{aligned} u(x,t) &= \mathcal{L}^{-1}\Big[\frac{\alpha}{s-2}\Big] + \mathcal{L}^{-1}\Big[\frac{5}{s-2}\mathcal{L}[u_{xx}]\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s-2}\mathcal{L}[u^2]\Big] \\ &= \alpha e^{2t} + \mathcal{L}^{-1}\Big[\frac{5}{s-2}\mathcal{L}[u_{xx}]\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s-2}\mathcal{L}[u^2]\Big] \end{aligned} \tag{22}$$

As before we defines the solution $u(x,t)$ by the series

$$u(x,t) = \sum_{n=0}^{\infty} u_n \tag{23}$$

and u_{xx} can be defined by an infinite series

$$u_{xx}(x,t) = \sum_{n=0}^{\infty} (u_n)_{xx} \tag{24}$$

The nonlinear term $F(u) = u^2$ is decomposed in term of Adomian polynomials

$$u^2 = \sum_{n=0}^{\infty} A_n \tag{25}$$

Substituting (23), (24) and (25) into both sides of Eq.(22) gives

$$\sum_{n=0}^{\infty} u_n = \alpha e^{2t} + \mathcal{L}^{-1}\Big[\frac{5}{s-2}\mathcal{L}\Big[\sum_{n=0}^{\infty}(u_n)_{xx}\Big]\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s-2}\mathcal{L}\Big[\sum_{n=0}^{\infty} A_n\Big]\Big]$$

The recursive relation is defined by

$$u_0 = \alpha e^{2t} \tag{26}$$

$$u_{n+1} = \mathcal{L}^{-1}\Big[\frac{5}{s-2}\mathcal{L}\Big[(u_n)_{xx}\Big]\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s-2}\mathcal{L}\Big[A_n\Big]\Big], \; n \geq 0 \tag{27}$$

The other components of the solution can easily calculated by using the above recursive relation

$$
\begin{aligned}
u_1 &= \mathcal{L}^{-1}\Big[\frac{5}{s-2}\mathcal{L}\Big[(u_0)_{xx}\Big]\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s-2}\mathcal{L}\Big[A_0\Big]\Big] \\
&= \alpha^2\mathcal{L}^{-1}\Big[\frac{1}{(s-2)(s-4)}\Big] \\
&= \frac{\alpha^2}{2}e^{2t}(e^{2t}-1) \\
u_2 &= \mathcal{L}^{-1}\Big[\frac{5}{s-2}\mathcal{L}\Big[(u_1)_{xx}\Big]\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s-2}\mathcal{L}\Big[A_1\Big]\Big] \\
&= \alpha^3\mathcal{L}^{-1}\Big[\frac{1}{(s-2)(s-6)} - \frac{1}{(s-2)(s-4)}\Big] \\
&= \frac{\alpha^3}{4}e^{2t}(e^{2t}-1)^2 \\
u_3 &= \mathcal{L}^{-1}\Big[\frac{5}{s-2}\mathcal{L}\Big[(u_2)_{xx}\Big]\Big] + \mathcal{L}^{-1}\Big[\frac{1}{s-2}\mathcal{L}\Big[A_2\Big]\Big] \\
&= \frac{\alpha^4}{4}\mathcal{L}^{-1}\Big[\frac{3}{(s-2)(s-8)} - \frac{6}{(s-2)(s-6)} + \frac{3}{(s-2)(s-4)}\Big] \\
&= \frac{\alpha^4}{8}e^{2t}(e^{2t}-1)^3 \\
&\vdots
\end{aligned}
$$

Using (23), hence the series solution is expressed by

$$
\begin{aligned}
u(x,t) &= u_0 + u_1 + u_2 + \ldots \\
&= e^{2t}\Big[\alpha + \frac{\alpha^2}{2}(e^{2t}-1) + \frac{\alpha^3}{4}(e^{2t}-1)^2 + \frac{\alpha^4}{8}(e^{2t}-1)^3 + \ldots\Big] \\
&= e^{2t}\Big[\frac{\alpha}{1-\frac{\alpha(e^{2t}-1)}{2}}\Big] \\
&= \frac{2\alpha e^{2t}}{2+\alpha(1-e^{2t})}
\end{aligned}
$$

4 Conclusion

Laplace Adomian decomposition method is a powerful device to solve many functional equations. Here we have successfully used the method for solving Newell-Whitehead-Segel equation. In above examples, it is demonstrated that the method has the ability of applying to linear and nonlinear problem .

ACKNOWLEDGEMENTS. This work has been financially supported by Faculty of Science, Mahasarakham University.

References

[1] A. Aasaraai, Analytic solution for Newell-Whitehead-Segel equation by differential transform method, *Middle-East J. Sci. Res.* **10**(2), (2011), 270-273.

[2] A. Malik, F. Chand, H.Kumar, S.C. Mishra, Exact solutions of some physical models using the $(\frac{G'}{G})$ expansion method, *Pramana J. Phys.***78**(4), (2012), 513-529.

[3] A. Saravanan, N. Magesh, A comparison between the reduced differential transform method and the Adomian decomposition method for the Newell-Whitehead-Segel equation,*J. Egyptian Math. Soc,* **21**(2013), 259-265.

[4] A.M. Wazwaz, *A First Course in Integral Equations,* World Scientific, Singapore, 1997.

[5] A.M. Wazwaz, A new algorithm for calculating Adomian polynomials for nonlinear operators, *Appl Math Comput,* **111**(1) (2000), 53-69.

[6] J.E. Macías-Díaz, J. Ruiz-Ramírez, A non-standard symmetry-preserving method to compute bounded solutions of a generalized Newell-Whitehead-Segel equation, *Appl Numer. Math,* **61**, (2011), 630-640.

[7] J. Fadaei, Application of Laplace-Adomian decomposition method on linear and nonlinear system of PDEs, *Appl Math Sci,* **5**(27), (2011), 1307-1315.

[8] M. Hussain and M. Khan, Modified Laplace decomposition method, *Appl Math Sci,* **4**(36), (2010), 1769-1783.

[9] S.A. Khuri, A Laplace decomposition algorithm applied to a class of nonlinear differential equations, *J. Appl. Math,***1**(4), (2001), 141-155.

[10] S.A. Manaa, An Approximate solution to the Newell-Whitehead-Segel equation by the Adomian decomposition method, *Raf. J. Comp. Math.* **8**(1), (2011), 171-180.

[11] Q. Li, Z. Zheng, S. Wang, J. Liu, Lattice Bolzmann model for nonlinear heat equations, *Adv. Neural Netw.*(2012), 140-148.

[12] Y. Fu-Kang, H. Wang-Yi and S. Jun-Qiang, Modified Laplaced decomposition method for Lane-Emden type differential equations, *Int. J. Appl Phys Math,* **3**(2)(2013),98-102.

Received: October 27, 2013