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A New Method for Solving Two Stage Supply Chain Fuzzy Inventory Problem

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Abstract

In this paper, we develop fuzzy integrated production-inventory-marketing model to determine the relevant profit-maximizing decision variable values. The model proposed is based on the joint total profit of both the vendor and the buyer, and it finds out the optimal ordering, pricing and shipment policies. Shortages are not allowed. Production rate, ordering and holding cost of buyer and vendor are taken as triangular fuzzy numbers. Graded mean integration representation method is used for defuzzification. The numerical example suggests that it is more beneficial for the buyer and the vendor to cooperate with each other when the demand is more prices sensitive.

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1 Introduction

The goal of many research efforts related to the supply chain management is to present models to reduce operational costs. The supply chain management has enabled numerous firms to enjoy great advantages by integrating all activities associated with the flow of material, information and capital between suppliers of raw materials and the ultimate customers. The benefits of a properly managed supply include reduced costs, faster product delivery, greater efficiency and lower costs for both the business and its customers. In the increasingly fierce competitive environment in today's global markets, the supply chain coordination is becoming a key component. If no coordination exists, the supply chain members act independently to maximize their own profits. In traditional inventory management, the inventory and shipment polices for the vendor and the buyer in a two-echelon supply chain are managed independently. The optimal lot size for the buyer may not result in an optimal policy for the vendor, and vice versa. To overcome this difficulty, the integrated vendor-buyer model has been developed, where the joint total relevant cost for both the buyer and the vendor is minimized. Determining the ordering and shipment policies results in a reduction of the total inventory cost of the system if the determination is based on the integrated total cost function rather than the buyer's or vendor's individual cost function.

Goyal [7] first introduced the idea of a joint total cost for a single-vendor and a single-buyer scenario assuming an infinite production rate for the vendor and lot-for-lot policy for the shipments from the vendor to the buyer. Then Goyal [8] introduced to the efforts of generalizing the problem by relaxing the assumption of lot-for-lot. He assumed that the production lot is shipped in a number of equal-size shipment. Goyal [9] developed a model where the shipment size increases by a factor equal to the ratio of the production rate to the demand rate. He formulated the problem and developed an optimal expression for the first shipment size as a function of the number of shipments. Hill [10] generalized Goyal's model [9] by taking the geometric growth factor as a decision variable. The basic Joint Economic Lot Sizing (JELS) models have been extended in many different directions.

Lau and Lau [11] investigated a joint pricing-inventory model (not including setup costs). Viswanathan and Wang [18] later developed a model of single-vendor, single-retailer distribution channel, where the retailers face a price-sensitive deterministic demand. Ray et al.[17] who introduced an integrated marketing-inventory model for two pricing policies, price as a decision variable and mark-up pricing. Bakal et al.[2] presented two inventory models with a price-sensitive demand. Two different pricing strategies were also investigated, where (i) the firm chooses to offer a single price in all markets selected and (ii) a different price is set for each market.

This paper is based on Mohsen S. Sajdieh, Mohammad R. Akbari Jokar [14] in which the Production rate, ordering and holding cost of buyer and vendor are taken as triangular fuzzy numbers. Here we consider a supply chain for a product which consists of a single vendor and single buyer in fuzzy Environment. The final demand for this product is assumed to be deterministic but price sensitive. The lots delivered from the vendor to the buyer are equal-sized batches. As soon as the on-hand inventory at the buyer drops to the reorder point, an order of size $\tilde{Q}$ is released by the buyer. The vendor manufactures the product at the production rate $\tilde{P}$ and in lot sizes which are a multiple n of $\tilde{Q}$. The objective is to determine the number of shipments$\tilde{n}$, the selling price $\tilde{\delta}$ as well as the order size$\tilde{Q}$, so that the total profits of the vendor and the buyer are maximized.

2. Methodology

2.1. Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions is a generalized fuzzy number $\tilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0,1],

(ii) $\mu_A = 0, -\infty < x \leq a_1$,

(iii) $\mu_A = L(x)$ is strictly increasing on $[a_1,a_2]$

(iv) $\mu_A = w_A, a_2 \leq x \leq a_3$

(v) $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$

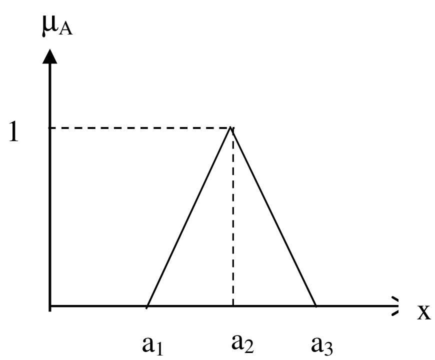
(vi) $\mu_A = 0, a_4 \leq x < \infty$

where $0 < w_A \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$; When $w_A=1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

2.2. Triangular Fuzzy Number

The fuzzy set $\breve{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2}, a_2 \leq x \leq a_3 \\ 0, \text{otherwise} \end{cases}$$



2.3. The Function Principle

The function principle was introduced by Chen [6] to treat fuzzy arithmetical operations. This principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3,)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} + \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \text{ x } \tilde{B} = (c_1, c_2, c_3)$
where $T = \{ a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$ $c_1 = \min T$, $c_2 = a_2b_2$, $c_3 = \text{Max } T$
If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then
$\tilde{A} \text{ x } \tilde{B} = (a_1b_1, a_2b_2, a_3b_3)$

(iii) $-\tilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} - \tilde{B} = (a_1-b_3, a_1-b_2, a_3-b_1)$
where $a_1,a_2,a_3, b_1, b_2, b_3$ are any real numbers

(iv) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = (1/b_3, 1/b_2, 1/b_1)$ where b_1, b_2, b_3 are all non zero positive real number, then the division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$

(v) For any real number K, $K\tilde{A} = (Ka_1, Ka_2, Ka_3)$ if $K > 0$

$K\tilde{A} = (Ka_3, Ka_2, Ka_1)$ if $K < 0$

2.4. Graded Mean Integration Representation Method

If $\tilde{A} = (a_1, a_2, a_3, a_4, w_A)_{LR}$ is a generalized fuzzy number then the defuzzified Value $P(\tilde{A})$ by graded mean integration representation method is given by

$$p(\tilde{A}) = \int_0^{w_a} h\left[\frac{L^{-1}(h) + R^{-1}(h)}{2}\right]dh \Big/ \int_0^{w_{A1}} hdh, \text{ with } 0 < h \le w_A \text{ and } 0 < w_A \le 1.$$

If $\tilde{A} = (a_1, a_2, a_3)$ is a triangular number then the graded mean integration representation of $\tilde{A}$ by above formula is

$$p(A) = 1/2\frac{\int_0^1 h[a_1 + h(a_2 - a_1) + a_3 - h(a_3 - a_2)]dh}{\int_0^1 hdh} = \frac{a_1 + 4a_2 + a_3}{6}$$

2.5. Notations

P	- Production rate of the vendor
Q	- Order quantity of the Buyer
A_v	- Setup cost of the vendor
A_b	- Ordering Cost of the Buyer
c	- the buyer's unit purchasing price
δ	- Unit selling price of the buyer
D	- Demand rate as a function of unit selling price
h_v	- inventory holding cost for the vendor per year
h_b	- Inventory holding cost for the buyer per year
n	– Number of shipments
$\tilde{P} = (P_1,P_2,P_3)$	- Fuzzy Production rate of the vendor
$\tilde{Q} = (Q_1,Q_2,Q_3)$	- Fuzzy order quantity of the Buyer
$\widetilde{A_v} = (A_{v_1}, A_{v_2}, A_{v_3})$	- Fuzzy setup cost of the vendor
$\widetilde{A_b} = (A_{b_1}, A_{b_2}, A_{b_3})$	- Fuzzy ordering Cost of the Buyer
$\tilde{\delta} = (\delta_1,\delta_2,\delta_3)$	- Fuzzy unit selling price of the buyer
$\tilde{D} = (D_1,D_2,D_3)$	- Fuzzy demand rate as a function of unit selling price
$\widetilde{h_v} = (h_{v_1}, h_{v_2}, h_{v_3})$	- Fuzzy inventory holding cost for the vendor per year
$\widetilde{h_b} = (h_{b_1}, h_{b_2}, h_{b_3})$	- Fuzzy inventory holding cost for the buyer per year
$\tilde{n} = (n_1,n_2, n_3)$	- number of shipments

2.6. Assumptions

i. The model deals with a single vendor and a single buyer for a single product.

ii. The buyer faces a linear Demand $\tilde{D}(\tilde{\delta})$ as a function of selling price $\tilde{\delta}$.

iii. A finite production rate for the vendor is considered which is greater than the demand rate.
iv. The inventory is continuously reviewed. The buyer orders a lot of size $\tilde{Q}$, when the on-hand inventory reaches the reorder point.
v. The vendor manufactures a production batch n$\tilde{Q}$ at one setup. However, the size of shipment delivered to the buyer is $\tilde{Q}$.
vi. The inventory holding cost at the buyer is higher than that at the vendor. i.e., $\widetilde{h_b} > \widetilde{h_v}$.
vii. Shortage is not allowed.
viii. The time horizon is infinite.

3. Fuzzy Mathematical Model

The optimal policy of the integrated system is derived. However, for comparative purposes, we first obtain the buyer and the vendor policies, if each party optimizes its profit independently. The policies and profits are then compared to the case of integrated system when they co-operate, particularly in information sharing.

We assume that the buyer faces a linear demand $\widetilde{D}\,(\tilde{\delta}) = a - b\tilde{\delta}$ (a > b > 0) as a function of its unit selling price. As $\widetilde{D}\,(\tilde{\delta}) > 0$, the maximum selling price is a/b, i.e., $\tilde{\delta} < a/b$. The buyer's yearly profit is equal to the gross revenue minus the sum of purchasing, order processing, and inventory holding costs. The buyer wishes to maximize its yearly profit function, $\widetilde{TP_B}$, through the optimal choice of selling price and order quantity, i.e.,

$\max_{\delta,Q} \widetilde{TP_B}(\check{\delta}\,,\check{Q}) = (a - b\check{\delta})(\check{\delta} - c) - \frac{(a-b\check{\delta})\widetilde{A_b}}{\check{Q}} - \frac{\widetilde{h_b}\,\check{Q}}{2}$ such that., $\tilde{\delta} < a/b$ and Q>0 ....(3.1)

The buyer's selling price $\tilde{\delta}$ determines the annual demand $\widetilde{D}\,(\tilde{\delta}) = a - b\tilde{\delta}$. The optimal order size is then the economic order quantity,

$$\widetilde{Q^*} = \sqrt{2\,(a - b\tilde{\delta})\,\widetilde{A_b}/\,\widetilde{h_b}}$$

Substituting the optimal order size into Eq.(3.1)

$$\max_{\delta,Q} \widetilde{TP_B}(\check{\delta}\,,\check{Q}) = (a - b\check{\delta})(\check{\delta} - c) - \frac{(a - b\check{\delta})\widetilde{A_b}}{\check{Q}} - \frac{\widetilde{h_b}\,\check{Q}}{2}$$

$$= (a - b\check{\delta})(\check{\delta} - c) - \frac{(a - b\check{\delta})\widetilde{A_b}}{\sqrt{2\,(a - b\tilde{\delta})\,\widetilde{A_b}/\,\widetilde{h_b}}} - \frac{\widetilde{h_b}\sqrt{2\,(a - b\tilde{\delta})\,\widetilde{A_b}/\,\widetilde{h_b}}}{2}$$

$$= (a - b\check{\delta})(\check{\delta} - c) - \sqrt{\frac{(a - b\check{\delta})^2\,\widetilde{A_b}^2\,\widetilde{h_b}}{2\,(a - b\tilde{\delta})\,\widetilde{A_b}}} - \sqrt{\frac{2\,(a - b\tilde{\delta})\,\widetilde{A_b}\,\widetilde{h_b}^{\,2}}{4\,\widetilde{h_b}}}$$

$$= \left(a - b\check{\delta}\right)\left(\check{\delta} - c\right) - \sqrt{\frac{\left(a - b\tilde{\delta}\right)\widetilde{A_b}\widetilde{h_b}}{2}} - \sqrt{\frac{\left(a - b\tilde{\delta}\right)\widetilde{A_b}\widetilde{h_b}}{2}}$$

$$= \left(a - b\check{\delta}\right)\left(\check{\delta} - c\right) - \sqrt{2\left(a - b\tilde{\delta}\right)\widetilde{A_b}\widetilde{h_b}}$$

Using the approximation developed by Qin et al.,[16] the expression above can be rewritten as,

$$\widetilde{TP_B}\left(\check{\delta}\right) = \left(a - b\check{\delta}\right)\left(\check{\delta} - c\right) - \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,\left(d_0\,\check{\delta}^2 + d_1\,\check{\delta} + d_2\right)$$

Where $d_0 = (-8 + 4\sqrt{2})(b/a)^2$ $d_1 = (12 - 7\sqrt{2})(b/a)$ and $d_2 = 3\sqrt{2} - 4$

Taking the second derivative of TP_B with respect to $\check{\delta}$, we have,

$$\frac{\partial \widetilde{TP_B}\left(\check{\delta}\right)}{\partial \check{\delta}} = a - 2b\check{\delta} + bc - \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\;\left(2d_0\,\check{\delta} + d_1\right)$$

$$\frac{\partial^2 \widetilde{TP_B}^{\,2}\left(\check{\delta}\right)}{\partial \check{\delta}^{\,2}} = -2b - 2\sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\;\left(d_0\right)$$

The expression above is negative if $a^3 > 11\,\widetilde{A_b}\widetilde{h_b}b^2$. In practice, a is usually very large (see Qin et al., [16]) and therefore the buyer's profit function is concave in $\check{\delta}$. The optimal selling price is then determined by equating the first derivative of $\widetilde{TP_B}$ to zero.

$$\frac{\partial \widetilde{TP_B}\left(\check{\delta}\right)}{\partial \check{\delta}} = a - 2b\check{\delta} + bc - \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\;\left(2d_0\,\check{\delta} + d_1\right)$$

$$0 = a - 2b\check{\delta} + bc - \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\;\left(2d_0\,\check{\delta} + d_1\right)$$

$$2b\check{\delta} + 2\sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,\left(d_0\,\check{\delta}\right) = a + bc - \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\;\left(d_1\right)$$

$$\check{\delta} = \frac{a + bc - \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\;\left(d_1\right)}{2b + 2\sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,\left(d_0\right)} \qquad \ldots\ldots(3.2)$$

The optimal order quantity can be obtained as,

$$\tilde{Q}^* = \sqrt{2\left(a - b\tilde{\delta}\right)\widetilde{A_b}/\,\widetilde{h_b}}$$

$$= \sqrt{\frac{2\left\{a - b\left(\dfrac{a + bc - \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\;\left(d_1\right)}{2b + 2\sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,\left(d_0\right)}\right)\right\}\widetilde{A_b}}{\widetilde{h_b}}}$$

$$= \sqrt{\frac{2\,\widetilde{A_b}\left\{a\left(2b + 2\sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,\left(d_0\right)\right) - b\left(a + bc - \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\;\left(d_1\right)\right)\right\}}{\widetilde{h_b}\,2\left(b + \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,\left(d_0\right)\right)}}$$

$$= \sqrt{\frac{\widetilde{A_b}\left(2ab + 2ad_0\sqrt{2\,\widetilde{A_b}\widetilde{h_b}a} - ab - b^2c + bd_1\sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\right)}{\widetilde{h_b}\left(b + \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,(d_0)\right)}}$$

$$= \sqrt{\frac{\widetilde{A_b}\left(ab - b^2c + \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,(2ad_0 + bd_1)\right)}{\widetilde{h_b}\left(b + \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,(d_0)\right)}}$$

$$\tilde{Q}^* = \sqrt{\frac{b(a - bc)\,\widetilde{A_b} + \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\;\widetilde{A_b}(2ad_0 + bd_1)}{\widetilde{h_b}\left(b + \sqrt{2\,\widetilde{A_b}\widetilde{h_b}a}\,(d_0)\right)}} \qquad \ldots\ldots\ldots(3.3)$$

When the buyer's order quantity and the selling price are adopted, the orders are received by the vendor at a known interval $\tilde{Q}^*/\widetilde{D}\,(\tilde{\delta})$,
The vendor's average inventory can then be obtained as follows:

$$\widetilde{AI_V} = \frac{\widetilde{D}\,(\tilde{\delta})}{\tilde{n}\tilde{Q}^*}\left\{\left[\tilde{n}\tilde{Q}^*\left(\frac{\tilde{Q}^*}{\tilde{P}} + (\tilde{n}-1)\frac{\tilde{Q}^*}{\widetilde{D}\,(\tilde{\delta})}\right) - \frac{\tilde{n}^2\tilde{Q}^{*2}}{2\,\tilde{P}}\right] - \left[\frac{\tilde{Q}^{*2}}{\widetilde{D}\,(\tilde{\delta})}(1 + 2 + \cdots + (\tilde{n}-1))\right]\right\}$$

$$\widetilde{AI_V} = \frac{\tilde{Q}^*}{2}\left[(\tilde{n}-1)\left(1 - \frac{\widetilde{D}\,(\tilde{\delta})}{\tilde{P}}\right) + \frac{\widetilde{D}\,(\tilde{\delta})}{\tilde{P}}\right] \qquad \ldots\ldots\ldots.(3.4)$$

Hence, the vendor's yearly profit function is,

$$\max_n \widetilde{TP_v}(\tilde{n}) = ac - bc\check{\delta} - \frac{(a - b\check{\delta})\widetilde{A_v}}{\tilde{n}\tilde{Q}} - \frac{\widetilde{h_v}\,\tilde{Q}}{2}\left[\tilde{n}\left(1 - \frac{a - b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a - b\check{\delta})}{\tilde{P}}\right] \ldots(3.5)$$

such that n is integer.
It can easily be shown that $\widetilde{TP_v}(\tilde{n})$ is concave in n.
Optimality conditions for n^*.

$$\tilde{n}^*\,(\tilde{n}^* - 1) \le \frac{2(a - b\check{\delta})\tilde{P}\widetilde{A_v}}{\widetilde{h_v}\,\tilde{Q}^2\,(\tilde{P} - a - b\check{\delta})} \le \tilde{n}^*\,(\tilde{n}^* - 1) \quad \ldots\ldots.(3.6)$$

If the buyer is free to choose its own marketing and ordering policies $(\check{\delta}, \tilde{Q})$, and the vendor is free to choose its number of shipment n, then it is straight forward that the total system profit under individual optimization, $\widetilde{TP_I}(\check{\delta}, \tilde{Q}, \tilde{n})$ is equal to the sum of buyer's and the vendor's profits.

$$\widetilde{TP_I}(\check{\delta}, \tilde{Q}, \tilde{n}) = \widetilde{TP_B}(\check{\delta}) + \widetilde{TP_v}(\tilde{n})$$

Suppose that both parties decide to cooperate and agree to follow the jointly optimal integrated policy. The cost stemming from the purchasing price is an internal transfer of money from one supply chain member (the vendor) to another supply chain member (the buyer).

Therefore, it is not a cost of the whole supply chain. The total system profit under joint optimization, $\widetilde{TP}_J(\check{\delta},\tilde{Q},\tilde{n})$ is going to be maximized,

$$\max_{\check{\delta},\tilde{Q},n} \widetilde{TP}_J(\check{\delta},\tilde{Q},\tilde{n}) = a\check{\delta} - b\check{\delta}^2 - \frac{(a-b\check{\delta})(\widetilde{A_v} + \tilde{n}\,\widetilde{A_b})}{\tilde{n}\tilde{Q}} - \frac{\widetilde{h_b}\tilde{Q}}{2} - \frac{\widetilde{h_v}\,\tilde{Q}}{2}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right] \dots\dots (3.7)$$

3.1 Solution Procedure for joint model

The total system profit is concave in $\tilde{Q}$ for given values of the buyer's selling price $\check{\delta}$ and the number of shipments $\tilde{n}$. The optimal order quantity can then be obtained as,

$$\frac{\partial \widetilde{TP}_J}{\partial \tilde{Q}} = 0$$

$$\frac{(a-b\check{\delta})(\widetilde{A_v} + \text{n}\,\widetilde{A_b})}{\tilde{n}\tilde{Q}^2} - \frac{\widetilde{h_b}}{2} - \frac{\widetilde{h_v}}{2}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right] = 0$$

$$\frac{(a-b\check{\delta})(\widetilde{A_v} + \widetilde{A_b}/\tilde{n})}{\tilde{Q}^2} = \frac{\widetilde{h_b}}{2} + \frac{\widetilde{h_v}}{2}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]$$

$$\frac{2\,(a-b\check{\delta})(\widetilde{A_v} + \widetilde{A_b}/\tilde{n})}{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]} = \tilde{Q}^2$$

$$\widetilde{Q^*} = \sqrt{\frac{2\,(a-b\check{\delta})(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})}{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]}} \quad \dots\dots\dots (3.8)$$

Substituting expression (3.8) into the cost function (3.7), we obtain

$$\widetilde{TP}_J(\check{\delta},\tilde{n}) = a\check{\delta} - b\check{\delta}^2 - \frac{(a-b\check{\delta})(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})}{\sqrt{\frac{2\,(a-b\check{\delta})(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})}{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]}}} - \frac{1}{2}\sqrt{\frac{2\,(a-b\check{\delta})(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})}{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]}}\left\{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]\right\}$$

$$\widetilde{TP}_J(\check{\delta},\tilde{n}) = a\check{\delta} - b\check{\delta}^2 - \sqrt{\frac{(a-b\check{\delta})^2(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})^2\left\{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]\right\}}{2(a-b\check{\delta})(\widetilde{A_v} + \widetilde{A_b}/\tilde{n})}} - \sqrt{\frac{(a-b\check{\delta})(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})\left\{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]\right\}}{2}}$$

$$= a\check{\delta} - b\check{\delta}^2 - \sqrt{\frac{(a-b\check{\delta})(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})\left\{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]\right\}}{2}} - \sqrt{\frac{(a-b\check{\delta})(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})\left\{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]\right\}}{2}}$$

$$\widetilde{TP}_J(\check{\delta},\tilde{n}) = a\check{\delta} - b\check{\delta}^2 - \sqrt{2(a-b\check{\delta})(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})\left\{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]\right\}} \quad \text{........(3.9)}$$

For a given value of $\check{\delta}$, maximizing $\widetilde{TP}_J$ is equivalent to minimizing the following expression:

$$\widetilde{TP}_J^{\,I} = 2(a-b\check{\delta})(\widetilde{A_b} + \widetilde{A_v}/\tilde{n})\left\{\widetilde{h_b} + \widetilde{h_v}\left[\tilde{n}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - 1 + \frac{2(a-b\check{\delta})}{\tilde{P}}\right]\right\} \quad \text{... (3.10)}$$

Taking the first and second partial derivatives of $\widetilde{TP}_J^{\,I}$ with respect to $\tilde{n}$, we obtain

$$\frac{\partial \widetilde{TP}_J^{\,I}}{\partial \tilde{n}} = 2(a-b\check{\delta})\left[\widetilde{A_b}\,\widetilde{h_v}\left(1-\frac{a-b\check{\delta}}{\tilde{P}}\right) - \frac{\widetilde{A_v}\,\widetilde{h_b}}{\tilde{n}^2} + \frac{\widetilde{A_v}\,\widetilde{h_v}}{\tilde{n}^2} - \frac{2\widetilde{A_v}\,\widetilde{h_v}}{\tilde{n}^2\tilde{P}}(a-b\check{\delta})\right]$$

$$\frac{\partial \widetilde{TP}_J^{\,I}}{\partial \tilde{n}} = \frac{2(a-b\check{\delta})\left[\widetilde{A_b}\,\widetilde{h_v}(\tilde{P}-a-b\check{\delta})n^2 - \widetilde{A_v}\,\widetilde{h_b}\tilde{P} + \widetilde{A_v}\,\widetilde{h_v}\tilde{P} - 2\widetilde{A_v}\,\widetilde{h_v}(a-b\check{\delta})\right]}{\tilde{n}^2\tilde{P}}$$

$$\frac{\partial \widetilde{TP}_J{}^I}{\partial \tilde{n}} = \frac{2\left(a - b\check{\delta}\right)\left[\widetilde{A_b}\,\widetilde{h_v}\left(\tilde{P} - a - b\check{\delta}\right)n^2 - 2\widetilde{A_v}\,\widetilde{h_v}\left(a - b\check{\delta}\right) - \widetilde{A_v}\left(\widetilde{h_b} - \widetilde{h_v}\right)\tilde{P}\right]}{\tilde{n}^2\tilde{P}}$$

$$\frac{\partial^2 \widetilde{TP}_J{}^I}{\partial \tilde{n}^2} = 2\left(a - b\check{\delta}\right)\left[\frac{2\widetilde{A_v}\,\widetilde{h_b}}{\tilde{n}^3} - \frac{2\widetilde{A_v}\,\widetilde{h_v}}{\tilde{n}^3} + \frac{4\widetilde{A_v}\,\widetilde{h_v}\left(a - b\check{\delta}\right)}{\tilde{n}^3\tilde{P}}\right]$$

$$\frac{\partial^2 \widetilde{TP}_J{}^I}{\partial \tilde{n}^2} = \frac{4\left(a - b\check{\delta}\right)\left[\widetilde{A_v}\,\widetilde{h_b}\tilde{P} - \widetilde{A_v}\,\widetilde{h_v}\tilde{P} + 4\widetilde{A_v}\,\widetilde{h_v}\left(a - b\check{\delta}\right)\right]}{\tilde{n}^3\tilde{P}}$$

$$\frac{\partial^2 \widetilde{TP}_J{}^I}{\partial \tilde{n}^2} = \frac{4\left(a - b\check{\delta}\right)\widetilde{A_v}\left[2\,\widetilde{h_v}\left(a - b\check{\delta}\right) + \tilde{P}\left(\widetilde{h_b} - \widetilde{h_v}\right)\right]}{\tilde{n}^3\tilde{P}}$$

As $\widetilde{h_b} > \widetilde{h_v}$ *and* $\check{\delta} < a/b$, the second derivative is positive. Consequently, $\widetilde{TP}_J{}^I$ is strictly convex $\tilde{n}$. solving $\partial \widetilde{TP}_J{}^I / \partial \tilde{n} = 0$.

$$\frac{2\left(a - b\check{\delta}\right)\left[\widetilde{A_b}\,\widetilde{h_v}\left(\tilde{P} - a - b\check{\delta}\right)\tilde{n}^2 - 2\widetilde{A_v}\,\widetilde{h_v}\left(a - b\check{\delta}\right) - \widetilde{A_v}\left(\widetilde{h_b} - \widetilde{h_v}\right)\tilde{P}\right]}{\tilde{n}^2\tilde{P}} = 0$$

$$\widetilde{A_b}\,\widetilde{h_v}\left(\tilde{P} - a - b\check{\delta}\right)\tilde{n}^2 - 2\widetilde{A_v}\,\widetilde{h_v}\left(a - b\check{\delta}\right) - \widetilde{A_v}\left(\widetilde{h_b} - \widetilde{h_v}\right)\tilde{P} = 0$$

$$\tilde{n}^2 = \frac{\widetilde{A_v}\left[\tilde{P}\left(\widetilde{h_b} - \widetilde{h_v}\right) + 2\,\widetilde{h_v}\left(a - b\check{\delta}\right)\right]}{\widetilde{A_b}\,\widetilde{h_v}\left(\tilde{P} - a - b\check{\delta}\right)}$$

$$\tilde{n} = \sqrt{\frac{\widetilde{A_v}\left[\tilde{P}\left(\widetilde{h_b} - \widetilde{h_v}\right) + 2\,\widetilde{h_v}\left(a - b\check{\delta}\right)\right]}{\widetilde{A_b}\,\widetilde{h_v}\left(\tilde{P} - a - b\check{\delta}\right)}}$$

As can be seen, the number of shipments is a non-increasing function of $\check{\delta}$. The minimum number of shipments can then be established when $\check{\delta} < a/b$,

$$\tilde{n}_{min} = \max\left\{\left\lfloor\sqrt{\frac{\widetilde{A_v}\left(\widetilde{h_b} - \widetilde{h_v}\right)}{\widetilde{A_b}\,\widetilde{h_v}}}\right\rfloor,\ 1\right\} \quad \ldots.(3.11)$$

The maximum number of shipments can also be obtained when $\check{\delta} = 0$.

$$\tilde{n}_{max} = \left\lceil\sqrt{\frac{\widetilde{A_v}\tilde{P}\left(\widetilde{h_b} - \widetilde{h_v}\right) + 2\widetilde{A_v}\,\widetilde{h_v}a}{\widetilde{A_b}\,\widetilde{h_v}\left(\tilde{P} - a\right)}}\right\rceil \quad \ldots.(3.12)$$

These two values are used in the solution algorithm as the upper and the lower bounds of the number of shipments.

3.2 Defuzzification of this model

Using Graded mean integration representation method, We will get the crisp value of Selling price (δ), Order size (Q), number of shipments (n) for both individual and Joint model, Total profit function for vendor (TP_v), Total profit function for buyer (TP_B), Total system profit under individual optimization (TP_I), the joint total profit of vendor (TP_{VJ}), the joint total profit of buyer (TP_{BJ}), Total system profit under joint optimization (TP_J).

For a given value of n, TP_J can be rewritten as

$$TP_J(D) = m_1D + m_2D^2 - \sqrt{m_3D + m_4D^2}$$

Where, $m_1 = a/b \qquad m_2 = -1/b,$

$m_3 = 2\,(A_b + A_v/n)\{h_b + (n-1)h_v\}$
$m_4 = 2\,h_v(A_b + A_v/n)(2-\mathrm{n})h_v/P$ and $D(\delta) = a - b\delta$
There is a one-to-one relationship between price and demand. Therefore, we base our analysis on the identification of the optimal value of demand, rather than the optimal value of price. The first and second Partial derivative of $TP_J(D)$, with respect to D are as follows.

$$\frac{\partial TP_J(D)}{\partial D} = m_1 + 2\,m_2 D - \frac{m_3 + 2m_4 D}{2\sqrt{m_3 D + m_4 D^2}}$$

$$\frac{\partial^2 TP_J(D)}{\partial D^2} = 2\,m_2 + \frac{{m_3}^2}{4}(m_3 D + m_4 D^2)^{-3/2}$$

Three cases can occur for $TP_J(D)$, depending on the number of shipments.

Case 1: n=1

Hence $m_4 > 0$, and therefore there are two saddle points, SP_1 and SP_2. The total profit function is convex when $SP_1 < D < SP_2$, and is concave when $D \leq SP_1$ or $D \geq SP_2$. The optimal value of the demand is then
$D^* = LO_1$ if $LO_1 < a$, and It is $D^* = a$ if $LO_1 \geq a$.

Case 2: n=2

Hence, $m_4=0$, and therefore there is a saddle point, $SP = b^{2/3} m_3^{1/3}/4$. The total profit function is convex when $D < SP$, and is concave when $D \geq SP$. Because the total profit function is zero at D = 0, there is no more than one local optimal amount for the demand. The optimal value of the demand is then
$D^* = LO_2$ if $LO_2 < a$, and It is $D^* = a$ if $LO_2 \geq a$.

Case 3: $\boldsymbol{n} \geq 3$

Hence, $m_4 < 0$, and therefore there are two saddle points, SP_1 and SP_2. The total profit function is concave when $SP_2 < D < SP_1$ and it is convex when $D \leq SP_2$ or $D \geq SP_1$. Moreover, m_3,t > 0 and thus $SP_1 > 0$ and $SP_2 > 0$. The optimal value of the demand is then, $D^* = LO_3$ if $LO_3 < a$, and It is $D^* = a$ if $LO_3 \geq a$.
As no closed form solution exists for the local-optimal values of the demand, we use a numerical method to find LO_i, i=1,2,3.

4 Solution Algorithms

We establish the following algorithm to find the optimal values of the three decision variables for the integrated model.

1. Initialize by setting $TP_J(D)^{opt} = 0$.
   Calculate n_{min} & n_{max} equ(11) and equ(12) respectively. Set $n = n_{min}$.
2. If n =1 , then determine LO_1. If $LO_1 < a$, then set $D = LO_1$. Otherwise, D=a.
3. If n =2 , then determine LO_2. If $LO_2 < a$, then set $D = LO_2$. Otherwise, D=a.
4. If $n \geq 3$, then determine LO_3. If $LO_3 < a$, then set $D = LO_3$. Otherwise, D=a.
5. Set $\delta = (a - D)/b$
6. Compute the value of order quantity using equation (8).
7. Calculate TP_J using equation (7).
8. If $TP_J > TP_J^{opt}$, *then set* $TP_J^{opt} = TP_J(\delta, Q, n)$, $n_{opt} = n$, $\delta_{opt} = \delta$, $Q_{opt} = Q$

9. Increment n by 1. If $n \leq n_{max}$ .then goto step 3.
10. The current solution is globally optimal.

5. Numerical Example

We consider an example with the following data:

$\tilde{P}$ = (3100 , 3200, 3300) / year $\widetilde{A_v}$ = (Rs.300, Rs.400, Rs.500) / setup

$\widetilde{A_b}$ = (Rs.20, Rs.25, Rs.30) / order $\widetilde{h_v}$ = (Rs.3, Rs.4, Rs.5) / unit / year

$\widetilde{h_b}$ = (Rs.4, Rs.5, Rs.6) / unit / year a=1500,b={10,20,30,….,100},c=Rs.5/unit.

We analyze the effect of demand's price-sensitivity. The effect is evaluated by the impact on the benefits of vendor-buyer coordination, as well as the impact on the decision variables. In order to gain insight into this effect, different levels of b have been considered, i.e., b={10,20,30,….,100}. The percentage improvement obtained by joint optimization compared to individual optimization is defined in order to clarify the benefits of coordination.

i.e., $PI = \frac{(TP_J - TP_I)}{TP_I} X\ 100$

TP_J and TP_I represent the total system profit under joint and individual optimization, respectively. The net benefit under joint optimization should be shared by both parties in some equitable fashion. In order to encourage the buyer to cooperate with the vendor, a judicious method is essential for allocating profit. A proposed way is that the joint total profit/cost be allocated to the buyer and the vendor as follows (see Ouyang et al., [15]; Wu and Ouyang, [18]; Goyal, [7])

$TP_{VJ} = \frac{TP_V(n)}{TP_I(\delta,Q,n)} TP_J(\delta, Q, n)$ & $TP_{BJ} = \frac{TP_B(n)}{TP_I(\delta,Q,n)} TP_J(\delta, Q, n)$

Where $TP_{VJ}\ and\ TP_{BJ}$ are the profit of the vendor and the buyer under a coordinated supply chain, respectively.

Table 1 Decision variables under individual optimization (Fuzzy environment)

b	δ	Q	n	TP_V	TP_B	TP_I
10	(77.3647, 77.5463, 77.7284)	(69.3534, 85.1196, 104.3512)	(5,5,5)	(1686.1606, 2376.3957, 2899.1850)	(51787.7684, 52136.8799, 52486.4407)	(53473.929, 54513.2756, 55385.6257)
20	(39.8763, 40.0620, 40.2486)	(67.7799, 83.5918, 102.8448)	(5,5,5)	(1564.5032, 2265.8689, 2772.5776)	(23737.1227, 24081.9638, 24427.7517)	(25301.6259, 26347.8327, 27200.3293)
30	(27.3881, 27.5780, 27.7692)	(66.2429, 82.0158, 101.1584)	(5,5,5)	(1443.8753, 2154.4877, 2667.297)	(14436.9866, 14777.2382, 15118.9846)	(15880.8619, 16931.7259, 17780.2816)
40	(21.1765, 21.3777, 21.5811)	(64.4762, 80.3051, 99.4159)	(5,5,5)	(1311.969, 2036.6662, 2558.0574.)	(9815.6552, 10160.3219, 10507.2754)	(11127.6242, 12196.9881, 13065.3328)

50	(17.4296, 17.6328, 17.8387)	(62.8597, 78.6358, 97.6137)	(5,5,5)	(1190.2566, 1924.7807, 2452.4911)	(7082.3292, 7418.4389, 7757.4656)	(8272.5858, 9343.2196, 10209.9567)
60	(14.9368, 15.1432, 15.3528)	(61.1714, 76.9031, 95.7499)	(5,5,5)	(1066.2828, 1811.8613, 2346.3997)	(5285.5135, 5614.2543, 5946.6138)	(6351.7963, 7426.1156, 8293.0135)
70	(13.1699, 13.3827, 13.5994)	(59.2943, 75.0474, 93.8128)	(5,5,5)	(934.404, 1694.546, 2184.5234)	(4019.0081, 4345.9919, 4674.7692)	(4954.0081, 6040.5379, 6859.2931)
80	(11.8467, 12.0651, 12.2883)	(57.3586, 73.1294, 91.8018)	(5,5,5)	(801.8107, 1577.2079, 2128.9781)	(3093.4189, 3412.7115, 3737.4727)	(3895.2296, 4989.9194, 5866.4508)
90	(10.8134, 11.0354, 11.2629)	(55.4505, 71.1909, 89.7211)	(5,5,5)	(674.5607, 1462.6759, 2020.0148)	(2391.7997, 2702.8709, 3020.3061)	(3066.3604, 4165.5468, 5040.3209)
100	(9.9948, 10.2224, 10.4562)	(53.3515, 69.1204, 87.5467)	(5,5,5)	(538.3163, 1344.7951, 1913.6252)	(1845.0526, 2149.4529, 2461.3064)	(2383.3689, 3494.248, 4374.9316)

Table 2. Decision variables under JOINT optimization (Fuzzy environment)

b	δ	Q	n	TP_{VJ}	TP_{BJ}	TP_J
10	(75.3986, 75.4894, 75.6997)	(86.4734, 110.9304, 142.0357)	(4,4,4)	(1623.9883, 2378.7981, 3007.9717)	(49878.244, 52189.587, 54455.8999)	(53343.4407, 54568.3851, 55480.4419)
20	(37.0456, 37.9840, 38.8456)	(84.8850, 110.5327, 143.8255)	(4,4,4)	(1382.1513, 2266.5584, 3125.9944)	(20970.4239, 24171.5772, 27541.5294)	(24029.9738, 26445.8778, 28526.7954)
30	(25.0042, 25.4899, 26.0104)	(84.7074, 110.1110, 142.7618)	(4,4,4)	(1239.4374, 2172.5076, 3114.5141)	(12392.8575, 14900.8335, 17653.9361)	(15262.7762, 17073.3411, 18542.5328)
40	(18.9878, 19.3545, 19.7699)	(84.0133, 109.3122, 141.7612)	(4,4,4)	(1079.1573, 2068.5469, 3145.7908)	(8073.8615, 10319.3654, 12921.4031)	(10746.8819, 12387.9124, 13684.2818)
50	(14.9095, 15.5359, 16.1628)	(82.3714, 109.0921, 143.6390)	(4,4,4)	(874.8073, 1973.2120, 3341.3241)	(5205.326, 7605.1017, 10568.9299)	(7504.0501, 9578.3138, 11270.7402)
60	(12.8654, 13.0943, 13.3349)	(83.4258, 108.3417, 140.3765)	(4,4,4)	(811.4129, 1880.0158, 3235.0424)	(4022.1356, 5828.4386, 8198.7513)	(6310.7633, 7705.4544, 8757.387)
70	(11.0040, 11.2249, 11.4572)	(83.2748, 108.3552, 140.5938)	(4,4,4)	(678.1471, 1772.0791, 3268.602)	(2917.2423, 4544.8405, 6994.6408)	(4978.1569, 6369.2073, 7412.453)

80	(9.7897, 9.9256, 10.0749)	(83.0757, 107.63, 139.0837)	(4,4,4)	(564.2088, 1696.4684, 3427.3333)	(2176.7411, 3670.7636, 6016.7668)	(4128.0361, 5367.232, 6270.7316)
90	(8.7089, 8.8243, 8.9405)	(83.1781, 107.6194, 138.994)	(4,4,4)	(454.235, 1611.2611, 3590.2527)	(1610.5877, 2977.4408, 5368.1102)	(3394.0462, 4588.7019, 5449.9644)
100	(7.8405, 8.0596, 8.2707)	(81.5381, 106.6146, 139.2099)	(4,4,4)	(317.0331, 1526.4435, 4010.7434)	(1086.6153, 2439.7906, 5158.6216)	(2576.5485, 3966.2342, 4995.2734)

Table 3 Decision variables under individual optimization (after defuzzification)

B	δ	Q	n	TP_V	TP_B	TP_I
10	77.5464	85.6972	5	2348.4881	52136.9548	54485.4428
20	40.0621	84.1653	5	2233.4261	24082.1216	26315.5477
30	27.5782	82.5774	5	2121.5205	14777.4873	16898.0078
40	21.3781	80.8521	5	2002.7819	10160.7030	12163.4849
50	17.6332	79.1694	5	1890.3117	7418.9251	9309.2368
60	15.1937	77.4223	5	1776.6879	5614.8574	7391.5454
70	13.3833	75.5494	5	1649.5186	4346.3901	6029.2421
80	12.0659	73.6130	5	1539.9367	3413.6229	4953.5597
90	11.0363	71.6559	5	1424.2132	2703.9316	4128.1447
100	10.2234	69.5633	5	1305.1870	2150.6951	3455.8821

Table 4 Decision variables under JOINT optimization (after defuzzification)

b	δ	Q	n	TP_{VJ}	TP_{BJ}	TP_J
10	75.5093	112.0384	4	2357.8587	52182.0820	54516.2372
20	37.9712	111.8069	4	2262.3966	24199.7103	26390.0467
30	25.4957	111.3189	4	2173.9970	14941.6879	17016.4456
40	19.3626	110.5039	4	2083.1893	10378.7877	12330.4689
50	15.5360	110.3965	4	2018.1632	7699.1104	9514.6742
60	13.0962	109.5282	4	1927.7531	5920.4402	7648.3280
70	11.2268	109.5482	4	1839.1776	4681.8742	6311.2398
80	9.9278	108.7799	4	1796.2359	3812.7604	5311.2826
90	8.8244	108.7749	4	1748.2553	3148.0768	4533.1364
100	8.0583	107.8677	4	1738.9251	2667.3999	3906.1264

6. Conclusion

In this paper, we developed a fuzzy integrated production-inventory-marketing model for two-stage supply chains. Here we find out the optimal ordering, pricing and shipment policies. We found the crisp value of number of shipments n, the selling price δ and order size Q by using graded mean integration method.

References

[1] Abad, P.L., 1996. Optimal pricing and lot-sizing under conditions of perishability and partial backordering. Management Science 42 (8), 1093-1104.
[2] Bakal, I.S., Geunes, J., Romeijn, H.E., 2008. Market selection decisions for inventory models with price-sensitive demand. Journal of Global Optimization 41, 633-657.
[3] Ben Daya, M., Darwish, M., Ernogral, K., 2008.The joint economic lot sizing problem review and extensions. European Journal of Operational Research185, 726-742.
[4] Chan, C.K., Kingsman, B.G., 2007. Coordination in a single-vendor mulit-buyer supply chain by synchronizing delivery and production cycles. Transportation Research Part E 43,90-111.
[5] Chang, H.C., Ouyang, L.Y., Wu, K.S., Ho. C.H., 2006. Integrated vendor-buyer cooperative inventory models with controllable lead time and ordering cost reduction. European Journal of Operational Research 170,481-495.
[6] Chen, S.H., Operations on fuzzy numbers with function principle, Tamkang Journal of Management Sciences, 6(1), (1985), pp 13 – 26.
[7] Goyal, S.K., 1976. An integrated inventory model for a single supplier-single customer problem. International journal of Production Research 15(1), 107-111.
[8] Goyal, S.K., 1988. A joint economic-lot-size model for purchaser and vendor; a comment. Decision science 19,236-241.
[9] Goyal, S.K.,1995. A one-vendor multi-buyer integrated inventory model: a comment. European Journal of Operational Research 82, 209-210.
[10] Hill, R.M., 1997. The single-vendor single-buyer integrated production-inventory model with a generalized policy. European Journal of Operational Research 97, 493-499.
[11] Lau, A.H.L., Lau, H.S., 2003. Effects of a demand-curve's shape on the optimal solutions of a multi-echelon inventory/pricing model. European journal of Operational Research 147. 530-548.
[12] Liu, L., Parlar, M., Zhu, S.X., 2007. Pricing and lead time decisions in decentralized supply chains. Management Science 53 (5), 713-725.
[13] Majumder, P., Srinivasan, A., 2006. Leader location, cooperation, and coordination in serial supply chains. Production and Operations Management 15(1), 22-39.
[14] Mohsen S. Sajadieh., Mohammad R. Akbari Jokar., 2009. Optimizing shipment, ordering and pricing policies in a two-stage supply chain with price-sensitive demand. Transportation Research Part E 45, 564-571.

[15] Ouyang, L., Wu, K., Ho, C., 2004. Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time. International Journal of Production Economics 92, 255-266.
[16] Qin, Y., Tang, H., Chonghui, G., 2007. Channel coordination and volume discounts with price-sensitive demand. International Journal of Production Economics 105, 43-53.
[17] Ray, S., Gerchak, Y., Jewkers,, E.M., 2005. Joint pricing and inventory policies for make-to-stock products with deterministic price-sensitive demand. International Journal of Production Economics 97, 143-158.
[18] Viswanathan, S., Wang, Q., 2003. Discount pricing decisions in distribution channels with price-sensitive demand. European journal of operational Research 149, 571-587.
[19] Wu, k., Ouyang, L., 2003. An integrated single-vendor single buyer inventory system with shortage derived algebraically. Production planning and Control 14(6). 555-561.

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